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Coexistence Phenomena of Chaotic and Non-Chaotic Oscillations in a Three-Dimensional Chaotic Circuit

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Abstract— In this study, chaotic behavior on a simple chaotic circuit with multi-driving points is investigated. Modified chaotic circuit which have diodes with both polarities and a piecewise-linear resistor with several segments are shown. A design scheme of a piecewise-linear element constructed by using electrical elements is proposed and shown. Both chaotic and non-chaotic oscillations in the same parameters can be confirmed simultaneously in the proposed chaotic circuit. Further some experimental results and piecewise-linear characteristic of the nonlinear element by PSPICE simulation are also shown.

1. Introduction

Many types of chaotic systems and circuits have already been proposed and investigated. They have interesting features, if the negative resistor as an active element are assumed to be constructed in parallel or series connection with nonlinear functions. Complex and strange nonlinear structures yield a wide variety of chaotic phenomena. It is an important idea that how does nonlinearity lead to various kinds of behaviors. There are famous chaotic attractors such as double-scroll[1], n–double scroll and scroll grid attractors[2]. They are also constructed by making several equilibrium points. A mechanism and search of chaotic regions in piecewise-linear systems were investigated[3]. The purpose of our study is to clarify coexistent both phenomena of chaotic and non-chaotic in chaotic systems with several equilibrium points.

In this study, we adopt a three-dimensional autonomous chaotic circuit, and chaotic behavior on a simple chaotic circuit with multi-driving points is investigated. This circuit consists of three memory elements, some diodes and designed negative resistors. It is well known that the chaotic circuit proposed by Inaba *et al.*[4] is asymmetric with respect to the origin and it behaves chaotic. In general, a linear region of the negative resistor realized by an OP amp on a circuit experiment is only used. We modified this chaotic circuit to include diodes with both polarities and with some minor changes. We change a characteristic of the negative resistor to a continuous five-segment piecewise-linear resistor. Our proposed system is symmetric with respect to the origin. It is similar to a three-dimensional chaotic system[5] at a glance, however it is different from stability at the origin, i.e., it is oscillatory at the origin because of negative resistance. Generally speaking, it can be constructed a concept for complicated structure of chaotic attractors to make many equilibrium points. Piecewise segment design scheme constructed by electrical elements is proposed. Then, several computer simulation results in the proposed circuit are shown. We can confirm simultaneously both chaotic and non-chaotic oscillations in the same parameters. Further some experimental results and piecewise-linear characteristic of the nonlinear element by PSPICE simulation are also shown. For the future research, we make a proposal idea to realize chaotic complex motions in coupled systems and to apply several chaotic applications.

2. Model Description

There are several chaotic circuits constructed by three-dimensional autonomous systems. In this study we adopted a modified three-dimensional chaotic system. One of the well-known chaotic circuit proposed by Inaba and Saito[4] is shown in Fig. 1. It is also well known that it can be behave chaotic, and it shapes Rössler type chaotic attractors. The circuit equations are described by

$$\begin{cases} L_1 \dfrac{di_{L1}}{dt} &= v \\ L_2 \dfrac{di_{L2}}{dt} &= v - v_d(i_{L2}) \\ C\dfrac{dv}{dt} &= -(i_{L1} + i_{L2}) - gv \end{cases} \qquad (1)$$

where g is a linear negative conductance value of N_R if we consider the negative resistor as an ideal. The variable $v_d(i_{L2})$ is a function depending on the current through their diodes D in Fig. 1(a), which determines their chaotic dynamics. The $i - v$ characteristic of one diode is approximated by two-segment piecewise-linear functions as shown in Fig. 2(a). The part of diodes with both polarities is constructed by some diodes, and their threshold voltages can be set as $+V_d$ and $-V_d$ respectively. Its characteristic is shown in Fig. 2(b) which is described as three-segment piecewise-linear functions by

$$v_d(i_{L2}) = \frac{1}{2}\{|r_s i_{L2} + V_d| - |r_s i_{L2} - V_d|\} \tag{2}$$

Furthermore the circuit equation (1) can be rewritten by changing the following variables and parameters as follows:

$$\begin{aligned} & i_{L1} = \sqrt{\frac{C}{L_1}} V_d\, x\ ,\quad i_{L2} = \sqrt{\frac{C}{L_1}} V_d\, y\ , \\ & v = V_d\, z\ ,\quad t = \sqrt{L_1 C}\ d\tau\ ,\quad \text{“}\,.\,\text{”} = \frac{d}{d\tau}\ , \\ & \beta = \frac{L_1}{L_2}\ ,\quad \gamma = g\sqrt{\frac{L_1}{C}}\ ,\quad \delta = r_s\sqrt{\frac{C}{L_1}} \end{aligned} \tag{3}$$

The circuit equation can be normalized if we chose V_d for a threshold voltage value of the diodes. Then the circuit equations can be rewritten by

$$\left\{ \begin{aligned} \dot{x} &= z \\ \dot{y} &= \beta\,(z - f(y)) \\ \dot{z} &= -(x + y) - \gamma z \end{aligned} \right. \tag{4}$$

where

$$f(y) = \frac{1}{2}\{|\delta y + 1| - |\delta y - 1|\} \tag{5}$$

In this study, we consider the negative resistor N_R included in the circuit as a piecewise-linear characteristic shown in Fig. 3. The piecewise-linear resistor can be easily constructed by a connection of some components in parallel as shown in Fig. 4(a)–(c), and its compound characteristic of N_R can be illustrated as shown in (d). Let us consider that the part of negative resistance in Eq.(4) replaces to $h(z)$, then it was rewritten by

$$\left\{ \begin{aligned} \dot{x} &= z \\ \dot{y} &= \beta(z - f(y)) \\ \dot{z} &= -(x + y) - h(z) \end{aligned} \right. \tag{6}$$

where $h(z)$ is a function of z, which is designed by five-segment piecewise-linear as shown in Fig. 5. This characteristic is described with a canonical form as follows:

$$\begin{aligned} h(z) \ &\equiv m_0\gamma^* z + \frac{\gamma^*}{2} \\ &\{(m_0 - m_1)(|z - Bp_2| - |z + Bp_2|) \\ &+(m_1 - m_2)(|z - Bp_1| - |z + Bp_1|)\} \end{aligned} \tag{7}$$

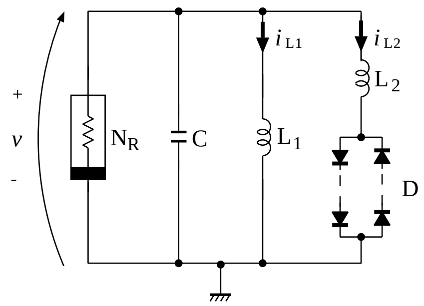

Figure 1: Schematic figure of a chaotic circuit. A basic model was proposed by Inaba *et al.*

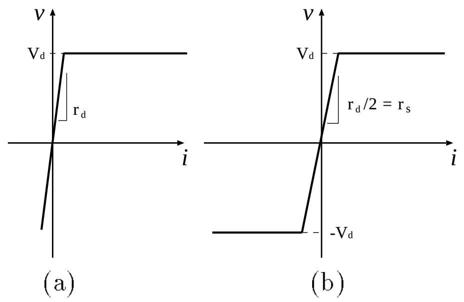

Figure 2: Examples of characteristic of a typical diode in (a) and a diode model with both polarities in (b).

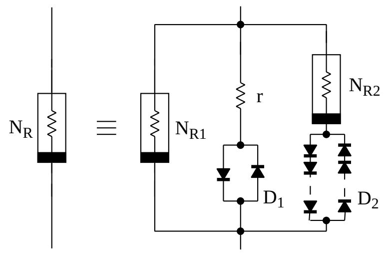

Figure 3: Nonlinear resistor model N_R.

where γ^* is a basic parameter, hence values $m_k(k = 0,1,2)$ are the ratio to the value γ^*. This circuit model is symmetric with respect to the origin and it has five equilibrium points which three are unstable and others are stable. A feature of this model is to possess two nonlinear components, such as diodes with switching role and piecewise-linear negative resistors with active elements for their oscillations.

3. Simulation Results

Some simulation results are shown for the parameters $\gamma^* = 0.20$ and $\gamma^* = 0.50$ respectively. Other parameters are $\beta = 10.0$, $\delta = 100$, and construction of the piecewise-linear characteristics form are realized by $Bp_1 = 0.1$, $Bp_2 = 0.3$, $m_0 = -1.0$, $m_1 = 1.0$ and $m_2 = -1.0$. It is noticed that threshold voltage of diodes with both

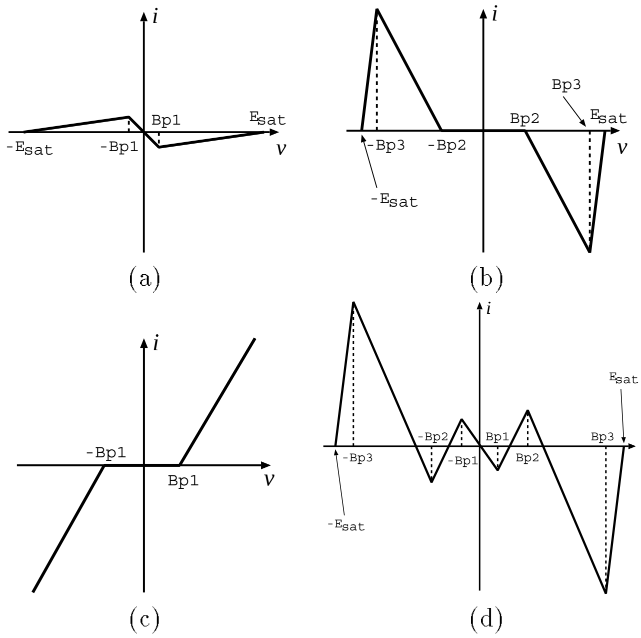

Figure 4: Design method for v–i characteristic of a piecewise-linear resistor in the chaotic circuit. (a) NR_1, (b) NR_2 and ideal diodes with both polarities, (c) ideal diodes and one resistor, (d) piecewise-linear characteristic of NR: compound characteristic of (a) to (c) connected in parallel.

polarities including in the chaotic circuit is normalized as $V_d = 1.0$. Hence we chose V_d between breakpoints Bp_2 and Bp_3. Because the value of breakpoint Bp_3 is larger than enough V_d, it can be ignored in computer calculation. Therefore five-segment region is only drawn in Fig. 5 and this canonical form of five-segment piecewise-linear function was also only described in (7). We can change values γ^* and m_k as control parameters. In order to be accuracy calculation, the boundary of switching of the diodes is calculated by using bisection method. Figure 6 shows some attractors when the initial conditions are changed. Both figures (a-1) and (a-2) does not behave chaotic for the parameter $\gamma^* = 0.20$. We can only observe one periodic oscillations in this parameter. On the other hand, we can confirm that chaotic and non-chaotic oscillations are coexisting for the parameter $\gamma^* = 0.50$. Chaotic attractors were confirmed on such parameters. Simultaneously we can also observe the non-chaotic oscillation in the same parameters. This reason is that solution of oscillation behaves just like a van der Pol oscillator because oscillation acts on a region around the area $|z| \leq 0.2$ Two oscillatory phenomena coexist in this proposed circuit, i.e., a chaotic oscillation and a non-chaotic oscillation. This feature is very interesting because variety of chaotic phenomena was emerged if we

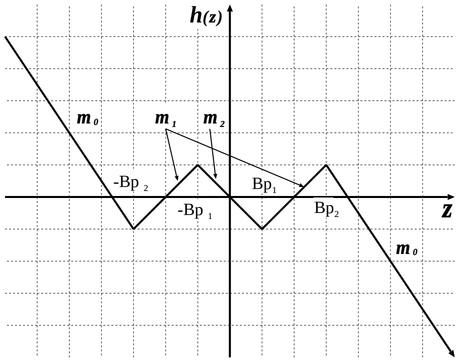

Figure 5: Piecewise-linear characteristics of the nonlinear resistor.

couple these systems as like as a network, a ring and so on. It is expected to be utilized for several applications by these coupled systems.

The circuit experiment results by using PSPICE simulator are also shown. In order to realize the non-linear characteristic of N_R, we designed a piecewise-linear resistor constructed by using some OP amps(TL082CP) and resistors. The details technique is explained in [6]. Figure 7 shows a schematic design of N_R. The nonlinear resistor is realized by two OP amps, some diodes and DC voltage which is used for setting threshold voltage strictly, and resistors. Figure 8 shows piecewise-linear characteristic realized on the PSPICE simulation. Normally, the voltage of capacitor C oscillates in the area of the threshold voltage between around $\pm\ V_d$ ($V_d = 5$[V]). A schematic design of a whole circuit on the SPICE simulation is shown in Fig. 9. Circuit parameters are all illustrated on a schematic figure. Chaotic and non-chaotic attractors corresponding to the figure 6 are shown in Fig. 10. We can also confirm that both chaotic and non-chaotic attractors are simultaneously observed in the same circuit parameters.

4. Conclusions

In this study, we have investigated an chaotic behavior and coexisting phenomena of both chaotic and non-chaotic oscillations in the same parameters on a chaotic circuit whose resistor has a piecewise linear characteristic. Namely, we could observe multi-driving of chaotic and non-chaotic oscillations. Design of a new chaotic system has been proposed and some simulation results have been shown. These chaotic behavior observed in this study are expected to yield new chaotic phenomena in several coupled chaotic systems. We have proposed a chaotic system with nonlinear resistors connected in parallel, however we expect to confirm very strange and complex behavior on coupled such nonlinear elements in

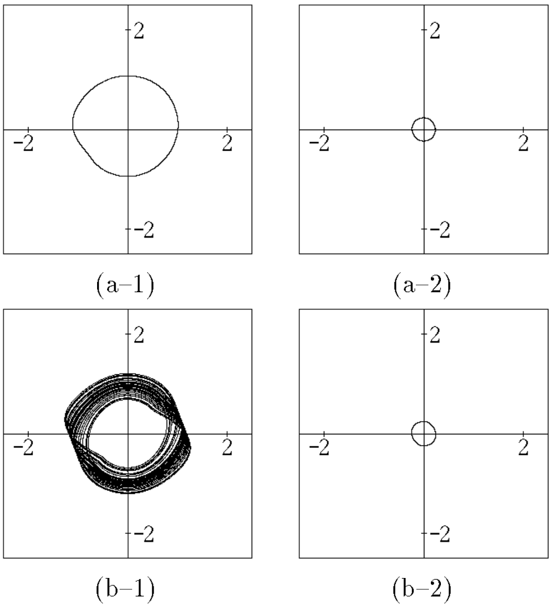

Figure 6: The $z-x$ plane projection of attractors on the chaotic circuit for the parameter γ^* is 0.20 (a–1, a–2) and 0.50 (b–1, b–2) respectively. The other parameters are $\beta = 10.0$, $\delta = 100$, $Bp_1 = 0.1$, $Bp_2 = 0.3$, $m_0 = -1.0$, $m_1 = 1.0$ and $m_2 = -1.0$. Both chaotic and non-chaotic attractors have all the same parameters, however the initial conditions are just only different.

series because it has been considered that coupled continuous nonlinear elements yield a variety of complex characteristics.

References

[1] L.O.Chua, *et al.*, "The Double Scroll Family," IEEE Trans. Circuits Syst. I, vol. 33, no. 11, pp. 1072–1118, 1986.

[2] M.E.Yalçin, J.A.Suykens, *et al.*, "Families of Scroll Grid Attractors," Int. J. of Bif. and Chaos, vol. 12, no. 1, pp. 23–41, 2002.

[3] T.Yang, L.O.Chua, "Piecewise–Linear Chaotic Systems with a Single Equilibrium Point," Int. J. of Bif. and Chaos, vol. 10, no. 9, pp. 2015–2060, 2000.

[4] N.Inaba, T.Saito, *et al.*, "Chaotic Phenomena in a Circuit with a Linear Negative Resistance and an Ideal Switch of Diode," IEICE Trans. Circ. and Syst., vol. E70, no. 8, pp. 744–754, 1987.

[5] M.Kojima, Y.Nishio, *et al.*, "Simultaneous Asynchronous Oscillation of Chaos and Torus in Coupled Chaotic Circuits," Proc. of NOLTA2000, vol. 2, pp. 567–570, 2000.

[6] M.P.Kennedy, "Robust OP Amp Realization of Chua's Circuit," Frequenz, vol. 46, pp. 66–80, 1992.

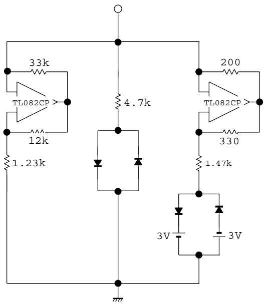

Figure 7: Schematic design for non-linear resistor N_R in the proposed chaotic circuit.

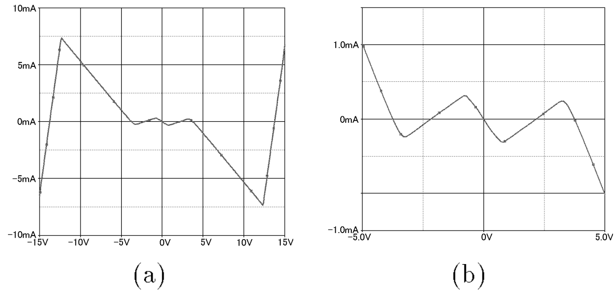

Figure 8: Piecewise-linear characteristic of the non-linear resistor N_R realized by several circuit elements on the PSPICE simulator. The model type of an OP amp used in simulations is TL082CP. (a) the whole characteristic, (b) enlargement area between −5 and +5[V].

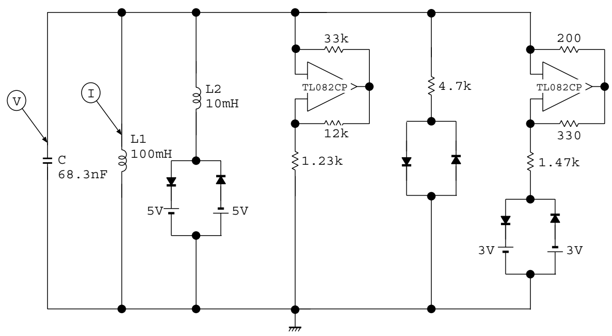

Figure 9: Schematic design of a whole chaotic circuit on the PSPICE simulation.

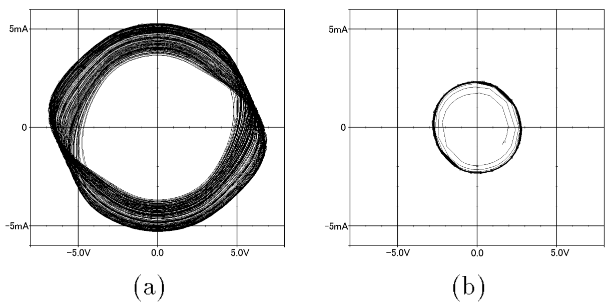

Figure 10: PSPICE simulation results.