

## Information Transmission Using Signal Processing by Coupled Chaotic Circuits

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### Abstract

In this study, information transmission with coupled chaotic circuits are considered. The coupled chaotic circuits are used as a receiver, and digital signals are transmitted by amplitude shift keying. The result of this study suggests that it is possible to transmit information by the coupled chaotic circuits.

### 1. Introduction

Studies on engineering applications of chaos, such as chaos communication systems and chaos cryptosystems, attract many researchers' attentions. Because chaotic signals possess infinite information, it could be possible to create novel engineering systems with great advantages. However, at this moment, there are only a few systems with some kinds of advantages over conventional engineering systems. Hence, basic researches on chaotic phenomena with future engineering applications in mind are still quite important.

On the other hand, the authors have researched about many oscillatory circuits connected by a resistor [1]-[3]. Using this connection of circuits, chaotic signals generated from chaotic circuits change the state of synchronization to minimize the total of currents flowing in the coupled resistor. Moreover, we investigated phenomena of the coupled chaotic circuits when a sinusoidal input signal is added [4]. The coupled system changes the state of synchronization against the input signal with very limited range of angular frequency. Therefore, this coupled system is useful for signal processing of input signals.

In this study, information transmission with coupled chaotic circuits can be considered. Chaotic signals in the coupled system synchronize depending on the amplitude of input signals. We can transmit digital signals by amplitude shift keying. Computer simulations of digital signal transmission are carried out. The result of this study suggests that it is possible to transmit information by the system with this coupled chaotic circuits.

### 2. Coupled chaotic circuits

At first, the coupled system with chaotic circuits investigated in the previous research is explained [4]. Figure 1 shows the circuit model. The circuit in Fig. 1(a) is a three-dimensional autonomous circuit generating chaotic signal and was proposed by Inaba and Mori [5]. In the circuit in Fig. 1(b), two Inaba's circuits are coupled by one coupling resistor  $R$  and an input signal is added to the coupling resistor as a current  $I_3$ .

We assume the  $i-v$  characteristics of the diodes in the circuit by the two-segment piecewise linear function as Eq. (1).

$$v_d(i_k) = 0.5(r_d i_k + E - |r_d i_k - E|). \quad (1)$$

By using the variables and the parameters in Eq. (2), the circuit equations are normalized as Eq. (3).

$$\begin{aligned} I_k &= \sqrt{\frac{C}{L_1}} E x_k, \quad i_k = \sqrt{\frac{C}{L_1}} E y_k, \\ v_k &= E z_k, \quad t = \sqrt{L_1 C} \tau, \quad \alpha = \frac{L_1}{L_2}, \end{aligned} \quad (2)$$

$$\beta = r \sqrt{\frac{C}{L_1}}, \quad \gamma = R \sqrt{\frac{C}{L_1}}, \quad \delta = r_d \sqrt{\frac{C}{L_1}}.$$

$$\begin{aligned} \frac{dx_k}{d\tau} &= \beta(x_k + y_k) - z_k - \gamma \sum_{j=1}^3 x_j \\ \frac{dy_k}{d\tau} &= \alpha \{ \beta(x_k + y_k) - z_k - f_d(y_k) \} \end{aligned} \quad (3)$$

$$\frac{dz_k}{d\tau} = x_k + y_k$$

$$(k = 1, 2).$$

In this study, we consider the case that a simple sinusoidal signal is input. Namely, we use the following sinusoidal function with the normalized amplitude  $A_m$  and the normalized angular frequency  $\omega$  as the input  $x_3$ .

$$x_3(\tau) = A_m \sin \omega \tau. \quad (4)$$

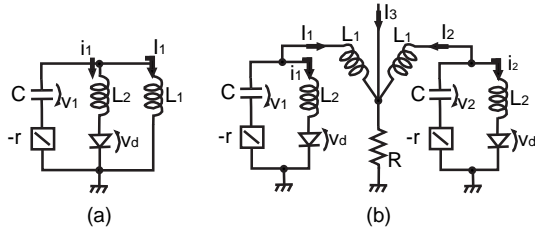


Figure 1: Chaotic circuit and the coupled chaotic circuits.  $\alpha = 7.0$ ,  $\beta = 0.14$  and  $\delta = 100$ .

## 2.1. Basic response to sinusoidal input signal

Figure 2 shows the computer simulated results of Eq. (3) using the fourth-order Runge-Kutta method. At first, in order to understand the typical response of the coupled system, we fix the angular frequency of the input signal as  $\omega = 1.037$  which is close to the value of the averaged angular frequency of the chaotic signals obtained from the chaotic circuits. Further, the amplitude of the input signal  $A_m$  is chosen using the averaged amplitude of the chaotic signals as a standard value.

Figure 2(a) shows the result for the case of  $A_m = 0$ , namely no input signal is added. The two chaotic signals almost synchronize at anti-phase. Figure 2(b) shows the result for the case of  $A_m = 1.17$ , namely the sinusoidal signal with the averaged amplitude and the averaged angular frequency of the chaotic signal is input. In this case, the two chaotic signal  $x_1$  and  $x_2$  and the input signal  $x_3$  almost synchronize at three-phase. Figure 2(c) shows the result for the case of  $A_m = 2.34$ , namely the sinusoidal signal with about doubled amplitude of the chaotic signal is input. Even in this case, the shapes of the chaotic attractors observed in the two chaotic circuits are not influenced by the input signal. And the two chaotic signals almost synchronize at in-phase and they almost synchronize to the input signal at anti-phase.

We show the circuit experimental results in Fig. 3. We can observe similar results to the computer simulated results.

## 2.2. Response to sinusoidal input signal

In order to investigate the synchronization in detail, we define the phase difference of the two chaotic signals. By using the time when the chaotic signals take extrema as shown in Fig. 4, the phase difference  $\theta$ [deg] is defined as follows.

$$\theta = \frac{\tau_{20} - \tau_{10}}{\tau_{11} - \tau_{10}} \times 360. \quad (5)$$

Figure 5(a) shows the results for the case that the angular frequency of the input is fixed as  $\omega = 1.037$  and the amplitude  $A_m$  is varied continuously. In the figure, 1500 data of  $\theta$  are plotted after the system settles to the steady state for each  $A_m$ . Figure 5(b) shows the

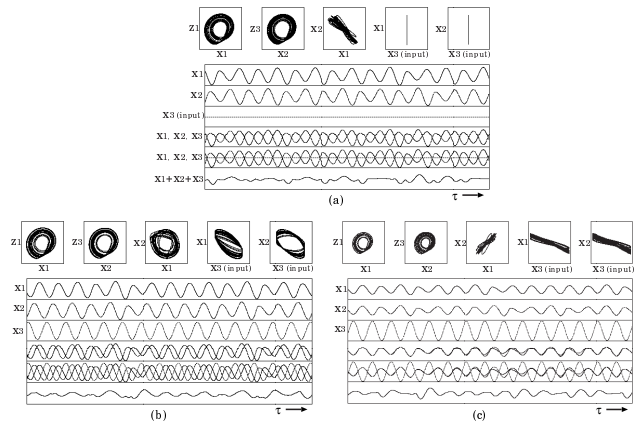


Figure 2: Computer simulated results.  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$  and  $\omega = 1.037$ . (a)  $A_m = 0$ . (b)  $A_m = 1.17$ . (c)  $A_m = 2.34$ .

average of  $\theta$  in Fig. 5(a). Because the fluctuation caused by chaotic feature is averaged, we can see the variation of  $\theta$  more clearly. The results in Fig. 5 suggest that it is possible to distinguish the amplitude of the input sinusoidal signal by the phase difference between the two chaotic signals.

Next, we fix the amplitude of the input signal as  $A_m = 1.17$  and vary the angular frequency  $\omega$ . Figure 6(a) shows 1500 data of  $\theta$  and Fig. 6(b) shows their average. In this case the coupled system responds to the input clearly only for very limited values of angular frequency. Namely, as we can see from Fig. 6(b), the two chaotic signals almost synchronize at anti-phase for wide parameter region and they synchronize to the input signal at three-phase only around  $\omega = 1.037$ . The results in Fig. 6 suggest that the coupled system could be useful as a filter to distinguish the frequency of the input sinusoidal signal.

## 3. Signal detecting by coupled chaotic circuits

As the results shown previously, the coupled chaotic circuits respond to only a certain limited signal. Figure 7 shows how the coupled system can detect a sinusoidal signal from the multiplexed signal consisting of two sinusoidal signals with different angular frequencies. In this simulations, two sinusoidal signals are added as Eq. (6), and the coupled chaotic circuits detect the first sinusoidal signal.

$$x_3(\tau) = A_m \sin \omega \tau + A_m \sin(\omega + \Delta\omega)\tau. \quad (6)$$

If the phase difference  $\theta$  of the two chaotic signals is around 120 degree, it means that the coupled system can detect the first sinusoidal signal from the multiplexed sig-

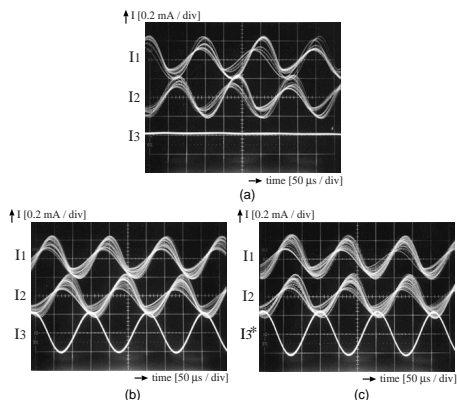


Figure 3: Circuit experimental results.  $L_1=100\text{mH}$ ,  $L_2=10\text{mH}$ ,  $C=33\text{nF}$  and  $R=320\Omega$ . (a)  $A_m = 0$  (no input signal). (b)  $A_m = 1.17$  (averaged amplitude of the chaotic signals). (c)  $A_m = 2.34$  (doubled amplitude of the chaotic signals). \*In (c) vertical scale of  $I_3$  is doubled.

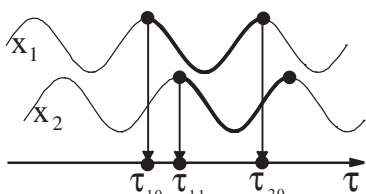


Figure 4: Definition of the phase difference  $\theta$ .

nal. This result suggests that more than 10% difference of angular frequency of input signals is needed.

Figure 8 shows how the coupled system can detect a sinusoidal signal when  $N$  sinusoidal signals are multiplexed as an input signal;

$$x_3(\tau) = \sum_{i=1}^N A_m \sin\{\omega + (i-1)\Delta\omega\}\tau. \quad (7)$$

The coupled chaotic circuits detect the first sinusoidal signal from the multiplexed signal for  $N \leq 10$ .

#### 4. Information transmission

In this section, an example of information transmission using the signal processing by the coupled chaotic circuits is shown.

In this simulations, 4 ASK signals are multiplexed and input to the coupled chaotic circuits. At first, four pairs of coupled chaotic circuits are connected as shown in Fig. 9. These coupled chaotic circuits are tuned to oscillate with the averaged angular frequencies  $\omega + (i-1)\Delta\omega$ . The simulated results are shown in Fig. 10(a), We can see that each pair of coupled chaotic circuits can decodes the corresponding ASK signal. Next, we consider

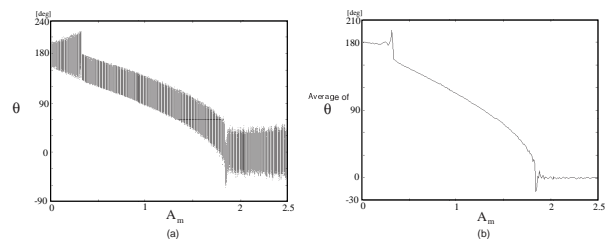


Figure 5: Phase difference  $\theta$  as varying  $A_m$ .  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$  and  $\omega = 1.037$ . (a)  $\theta$ . (b) Average of  $\theta$ .

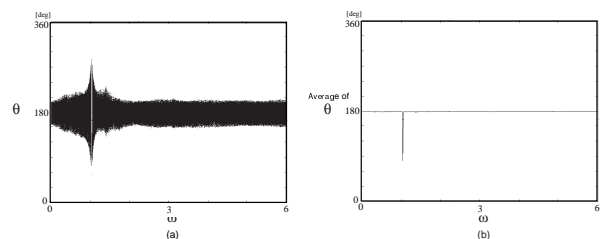


Figure 6: Phase difference  $\theta$  as varying  $\omega$ .  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$  and  $A_m = 1.17$ . (a)  $\theta$ . (b) Average of  $\theta$ .

the case that four pairs of the coupled chaotic circuits are not connected, namely each pair works as an detector dependently. The results are shown in Fig. 10(b). In this case, the coupled chaotic circuits cannot decode any information from the multiplexed signal.

#### 5. Conclusions

In this research, we have investigated that the coupled chaotic circuits could detect sinusoidal signals. Furthermore, we showed the example of information transmission using signal processing by the coupled chaotic circuits. It suggests that multiplexed information can be transmitted by using this coupled system.

It is necessary to consider the tolerance over the noise of a system in detail. Moreover, the response speed of this system is slow and is not practical. It is necessary to consider how to improve.

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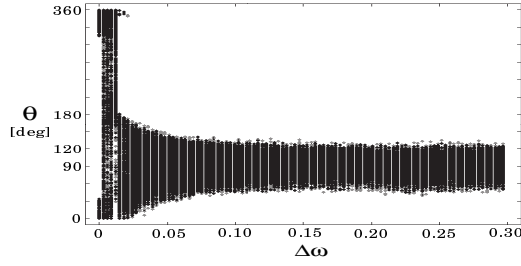


Figure 7: Phase difference  $\theta$  as varying  $\Delta\omega$ .  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$ ,  $A_m = 1.17$ ,  $\omega_1 = 1.035$  and  $\omega_2 = 1.035 + \Delta\omega$ .

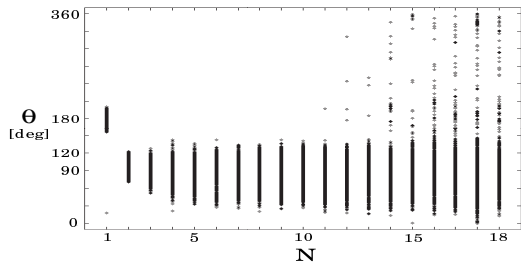


Figure 8: Phase difference  $\theta$  when  $N$  sinusoidal signals are multiplexed as an input signal.  $\Delta\omega = 0.2$ ,  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$  and  $A_m = 1.17$ .

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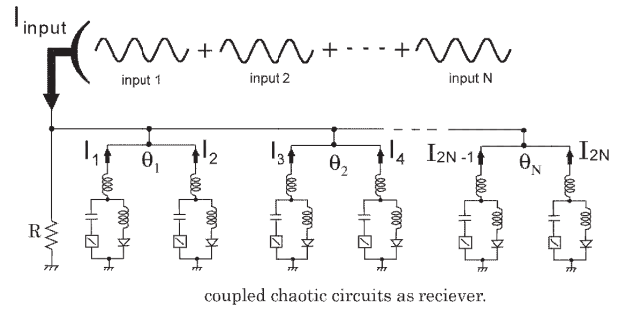


Figure 9: Circuits for information transmission.

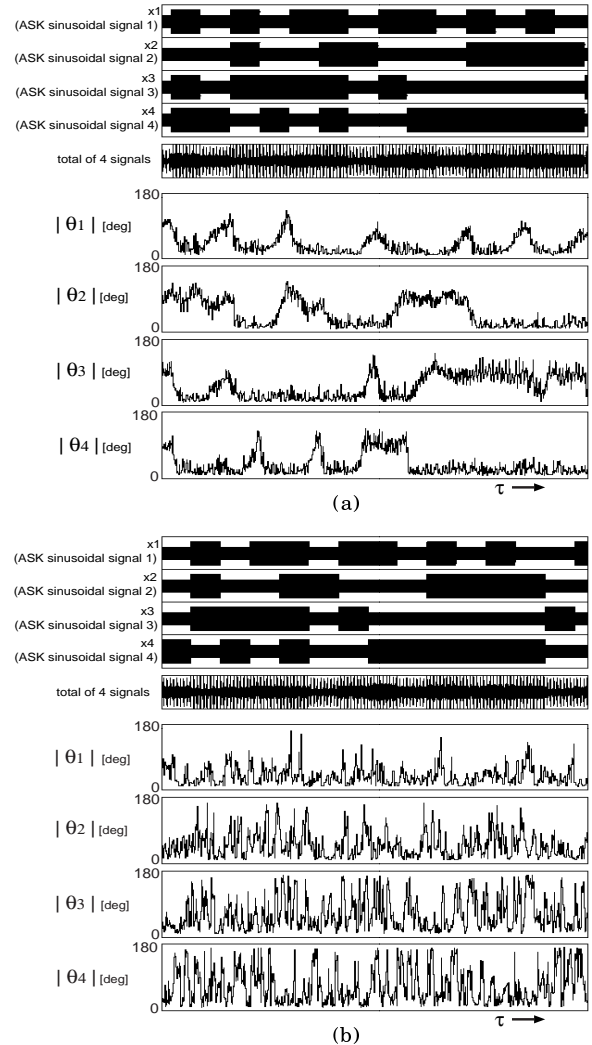


Figure 10: Information Transmission by multiplexing 4 ASK signals. (a) Coupled chaotic circuits are connected, (b) not connected.  $\alpha = 7.0$ ,  $\beta = 0.14$ ,  $\gamma = 0.03$ ,  $\delta = 100$ ,  $A_m = 1.17$  or  $2.34$  and  $\Delta\omega = 0.05$ .