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Phase-Inversion-Waves on Coupled Oscillators by Inductors as a Cross

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1. Introduction

Many kinds of digital circuits of for various applications have been designed and developed. These circuits use signals generated by oscillators. Therefore, oscillators always keep an important position, and various oscillators still continued to be designed and put to practical use. Various researches about systems of coupled oscillators have been carried out up to now. And, a lot of interesting phenomena have been reported [1]-[2]. But, discoveries of many new phenomena still continue. Therefore, coupled oscillators catch interest of researchers.

Recently, we discovered continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in coupled van der Pol oscillators by inductors as a ladder [3]-[5]. This phenomenon is observed in steady state. We call this phenomenon as "phase-inversion-wave". And, the mechanisms of "propagation," "disappearance," "reflection in the middle of the array" and "reflection at an edge of the array," which were the basic characters of the phase-inversion-waves were clarified [3].

In this study, four ladders of van der Pol oscillators are coupled by an oscillator and four inductors as a cross. Each ladder is composed by van der Pol oscillators which are coupled by inductors. The phase-inversion-waves are observed in this system. We investigate various phenomena of the phase-inversion-waves such as "reflection," "disappearance" and "penetration" at a coupling point of four ladders, by changing initial values and parameters. The observed phenomena are classified.

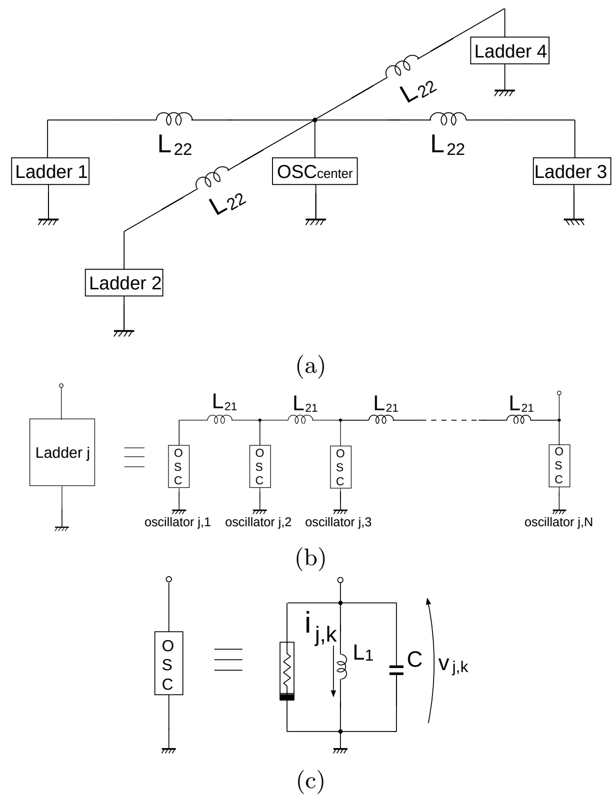


Figure 1: Circuit Model. (a) Coupled ladders system. (b) Coupled oscillators as a ladder. (c) Van der Pol oscillator.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1. N van der Pol oscillators are coupled by coupling inductors L_{21} and L_{22}. We carried out computer calculations for the cases of $N = 8$. In the computer calculations, we assume the $v - i$ characteristics of the nonlinear negative resistors in each circuit as the following function.

$$i_r(v_{j,k}) = -g_1 v_{j,k} + g_3 v_{j,k}^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are written as:

[Center Oscillator]

$$\dot{x}_c = y_c \tag{2}$$

$$\dot{y}_c = -x_c + \alpha_2 (\sum_{i=1}^{M} x_{i,N} - M x_c) + \varepsilon (y_c - \frac{1}{3} y_c^3)$$

[Edge Oscillators]

$$\dot{x}_{j,1} = y_{j,1} \tag{3}$$

$$\dot{y}_{j,1} = -x_{j,1} + \alpha_1 (x_{j,2} - x_{j,1}) + \varepsilon (y_{j,1} - \frac{1}{3} y_{j,1}^3)$$

$$(j = 1 \sim 4)$$

[Middle Oscillators]

$$\dot{x}_{j,k} = y_{j,k} \tag{4}$$

$$\dot{y}_{j,k} = -x_{j,k} + \alpha_1(x_{j,k+1} - 2x_{j,k} + x_{j,k-1}) + \varepsilon(y_{j,k} - \frac{1}{3}y_{j,k}^3)$$

$$(j = 1 \sim 4, \quad k = 2 \sim N-1)$$

[Adjacent Oscillators of Center Oscillator]

$$\dot{x}_{j,N} = y_{j,N} \tag{5}$$

$$\dot{y}_{j,N} = -x_{j,N} + \alpha_1(x_{j,N-1} - x_{j,N}) + \alpha_2(x_c - x_{j,N}) + \varepsilon(y_{j,N} - \frac{1}{3}y_{j,N}^3)$$

$$(j = 1 \sim 4)$$

where

$$t = \sqrt{L_1 C}\tau,\ i_{j,k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_{j,k},\ v_{j,k} = \sqrt{\frac{g_1}{3g_3}}y_{j,k},$$

$$\alpha_1 = \frac{L_1}{L_{21}},\ \alpha_2 = \frac{L_1}{L_{22}},\ \varepsilon = g_1\sqrt{\frac{L_1}{C}},\ \frac{d}{d\tau} = \text{“}\cdot\text{”}.$$

It should be noted that α corresponds to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $N = 8$, $\alpha_1 = 0.050$, $\varepsilon = 0.250$ and $\Delta\tau = 0.01$ and calculate (3)-(2) by using the fourth-order Runge-Kutta method.

3. Phase-Inversion-Waves

Value of α_2 and oscillator which are generated phase-inversion-waves are changed. And we observe phase-inversion-waves in this circuit. Further, we fix that a pair of phase-inversion-waves is generated at edge oscillators.

Complex phenomena are observed in domain of α_2 which is not written in tables. These phenomena can not been classified.

3.1. When phase-inversion-waves are generated at $OSC_{1,1}$

Observed phenomena are classified into 3 patterns by changing α_2(see Table 1).

1−(a) The phase-inversion-waves propagate to the oscillator of center and reflect. The phase-inversion-waves do not penetrate to other ladders and continuously exist.

1−(b) The phase-inversion-waves propagate to the oscillator of center. And the phase-inversion-waves reflect at the center and penetrate to other three ladders. The phase-inversion-waves are continuously existing on all ladders.

1−(c) The phase-inversion-waves propagate to the oscillator of center and disappear.

Table 1: Phase-inversion-waves generated at $OSC_{1,1}$

domain number	α_2	sample figure
1−(a)	∼0.008	2(a)
1−(b)	0.084∼0.104 0.111∼0.136	2(b)
1−(c)	0.168∼	2(c)

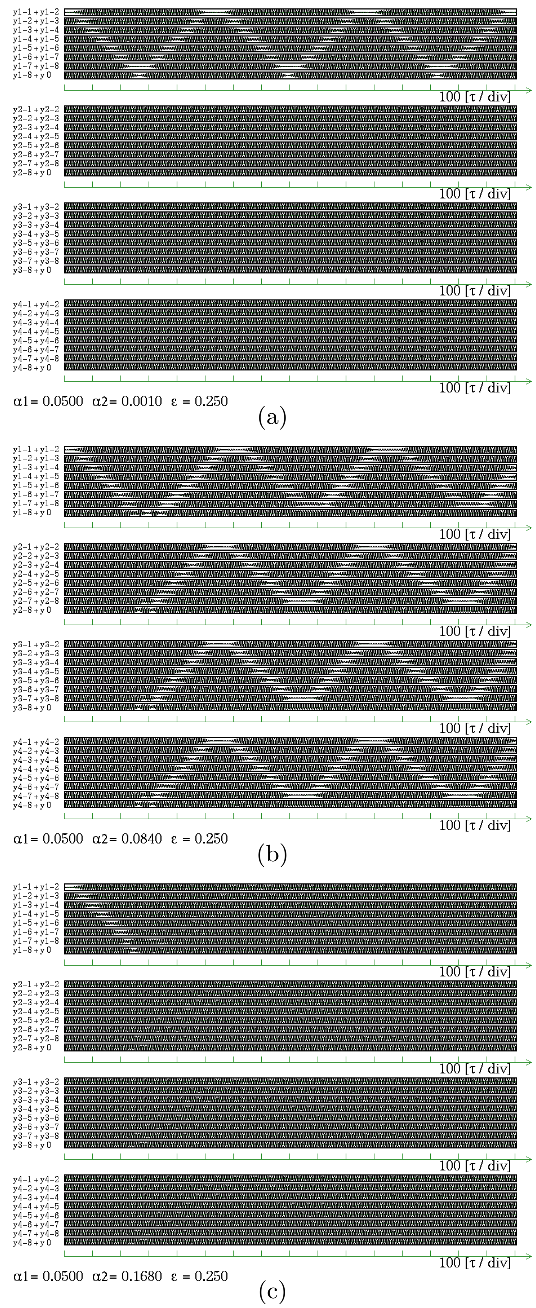


Figure 2: Phase-inversion-waves generated at $OSC_{1,1}$. (a) $\alpha_2 = 0.001$. (b) $\alpha_2 = 0.084$. (c) $\alpha_2 = 0.168$.

3.2. When phase-inversion-waves are generated at $\mathbf{OSC}_{1,1}$ and $\mathbf{OSC}_{2,1}$

Observed phenomena are classified into 4 patterns by changing α_2(see Table 2).

2−(a) The phase-inversion-waves propagate to the oscillator of center and reflect. The phase-inversion-waves do not penetrate to other ladders and continuously exist.

2−(b) The phase-inversion-waves propagate to the oscillator of center. And the phase-inversion-waves do not reflect and penetrate to other two ladders. The phase-inversion-waves continuously exist on two ladders.

2−(c) The phase-inversion-waves propagate to the oscillator of center. And the phase-inversion-waves reflect at the oscillator of center and penetrate to other two ladders.

2−(d) The phase-inversion-waves become extinction while the phase-inversion-waves are propagating.

Table 2: Phase-inversion-waves generated at $OSC_{1,1}$ and $OSC_{2,1}$

domain number	α_2	sample figure
2−(a)	∼0.008	3(a)
2−(b)	0.071∼0.216	3(b)
2−(c)	0.237∼0.354	3(c)
2−(d)	0.381∼0.426 0.531∼	3(d)

3.3. When phase-inversion-waves are generated at $\mathbf{OSC}_{1,1}$, $\mathbf{OSC}_{2,1}$ and $\mathbf{OSC}_{3,1}$

Observed phenomena are classified into 2 patterns by changing α_2(see Table 3).

3−(a) The phase-inversion-waves propagate to the oscillator of center and reflect. And the phase-inversion-waves continuously exist.

3−(b) The phase-inversion-waves propagate to the oscillator of center. And the phase-inversion-waves reflect and penetrate to other one ladder.

Table 3: Phase-inversion-waves generated at $OSC_{1,1}$, $OSC_{2,1}$ and $OSC_{3,1}$

domain number	α_2	sample figure
3−(a)	∼0.008	4(a)
3−(b)	0.059∼	4(b)

3.4. When phase-inversion-waves are generated at all oscillators of edges

Observed phenomena are classified into 2 patterns by changing α_2(see Table 4).

4−(a) The phase-inversion-waves propagate to the oscillator of center and reflect. And the phase-inversion-waves continuously exist.

4−(b) The phase-inversion-waves become extinction while the phase-inversion-waves are propagating.

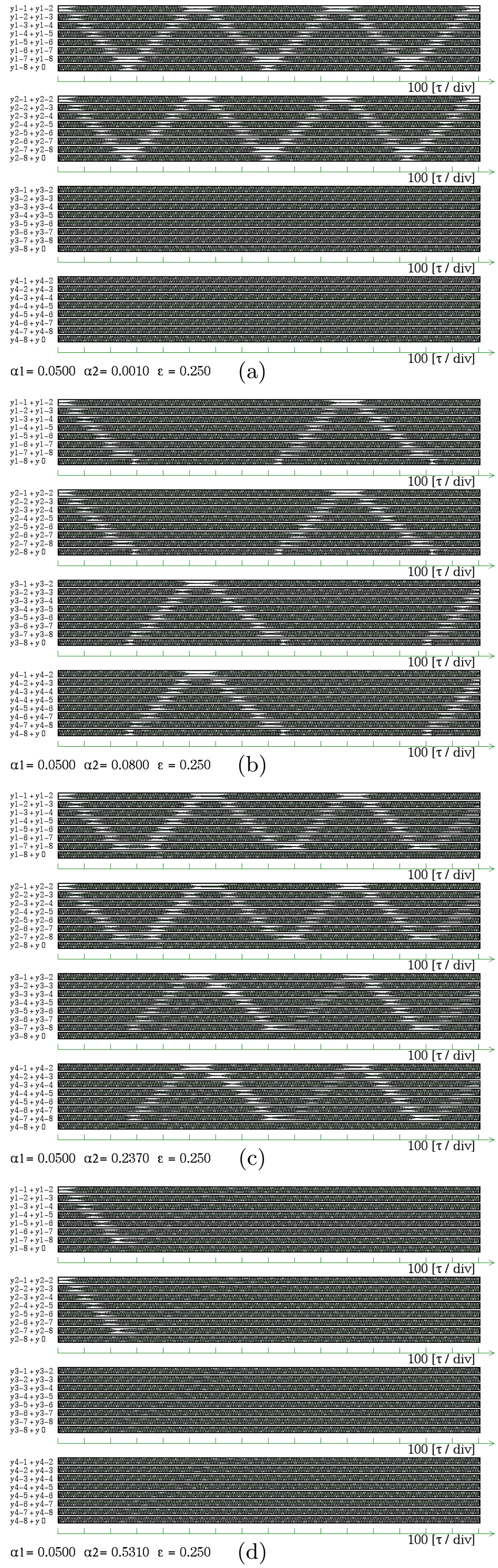
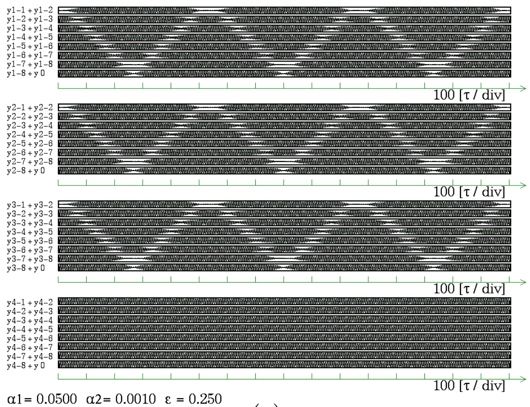
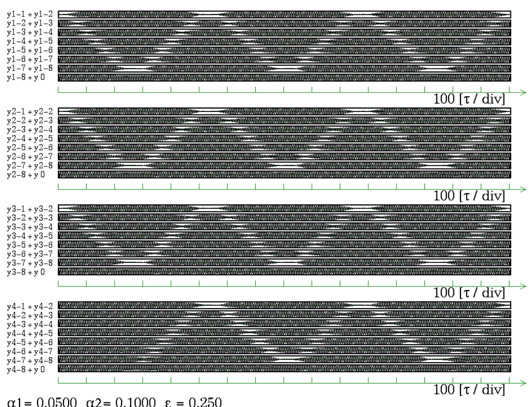


Figure 3: Phase-inversion-waves generated at $OSC_{1,1}$ and $OSC_{2,1}$. (a) $\alpha_2 = 0.001$. (b) $\alpha_2 = 0.080$. (c) $\alpha_2 = 0.237$. (d) $\alpha_2 = 0.531$.

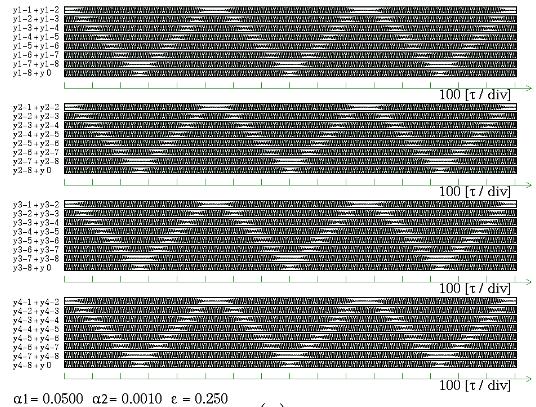


(a)

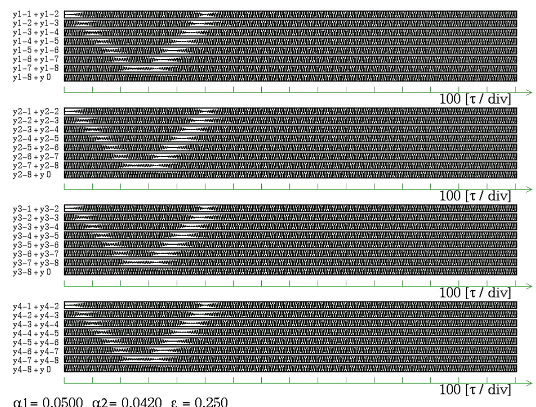


(b)

Figure 4: Phase-inversion-waves generated at $OSC_{1,1}$, $OSC_{2,1}$ and $OSC_{3,1}$. (a) $\alpha_2 = 0.001$. (b) $\alpha_2 = 0.100$.



(a)



(b)

Figure 5: Phase-inversion-waves generated at all edge oscillators. (a) $\alpha_2 = 0.001$. (b) $\alpha_2 = 0.042$.

Table 4: Phase-inversion-waves generated at all edge oscillators

domain number	α_2	sample figure
4−(a)	∼0.011	5(a)
4−(b)	0.014∼0.016 0.042∼0.045	5(b)
4−(a)	0.046∼	5(a)

4. Conclusions

In this study, four ladders of van der Pol oscillators were coupled by an oscillator and four inductors as a cross. We investigated various phenomena of the phase-inversion-waves such as “reflection,” “disappearance” and “penetration” at a coupling point of four ladders, by changing initial values and parameters. When the phase-inversion-waves were generated at an oscillator of an edge, the observed phenomena were classified into 3 pattern. When the phase-inversion-waves were generated at each oscillators of two edge, the observed phenomena were classified into 4 pattern. When the phase-inversion-waves were generated at each oscillators of three edge, the observed phenomena were classified into 2 pattern. When the phase-inversion-waves were generated at each oscillators of all edge, the observed phenomena were classified into 2 pattern.

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