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Comparison between Chaotic Noise and Iterative Simulated Annealing for Combinatorial Optimization Problems

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Abstract
In this paper, we investigate the solving ability of Hopfield neural network with iterative simulated annealing noise for traveling salesman problems (TSPs) and quadratic assignment problems (QAPs) by comparing with chaotic noise. From several numerical experiments, we can confirm that the solving ability of iterative simulated annealing noise is almost same as chaotic one.

Key words
Hopfield neural network, Iterative simulated annealing noise, Chaotic noise, TSP, QAP

1. Introduction

One of the key application of Hopfield neural network (H-NN)[1] is combinatorial problems, such as traveling salesman problems (TSPs) and quadratic assignment problems (QAPs). Since there are generally many local minima in such problems, the state of the H-NN is often trapped into these local minima. It remains, therefore, an unsettled question how to escape easily from them and reach global minimum. Many methods to try to overcome this problem have been proposed, for examples, Boltzmann machine[2], chaotic neural network[3], simulated annealing (SA)[4], neural network with noise[5] and so on. These methods could be classified into two categories from their dynamical behaviors[8]. First category is 'autonomous method'. In this method, by modifying the characteristic of dymanics of each neuron to stochastic or chaotic one, then the network can search every candidate of global minimum itself. Boltzmann machine and chaos neural network are included in this category. The other category is 'nonautonomous method'. By adding noise to each neuron, the network can avoid to trap the local minima. The amplitude of noise should be controlled appropriately. Exmaples in this category are SA, neural network with noise and so on. Recently, for second category, chaotic time series which is generated from the logistic map is paid great attention as a good noise candidates [6, 7]. Hayakawa et al. [6] pointed out that H-NN with the intermittent chaos noise near the period-3 window of the logistic map is more effective for traveling salesman problem(TSP)s. Ueta et al. [8] mentioned that period-5 and 7 are also effective for TSPs. Uwate et al. [9] tried to find the cause of the good performance of chaotic noise for QAPs. They also indicated that the chaotic noise could be replaced by the stochastic one whose statistical characteristic is similar to the chaotic one[6, 7, 8, 9].

On the other hand, there is a general agreement that SA is effective for escaping local minima. But, in our previous research, the solving ability of H-NN with simple SA noise is inferior to H-NN with chaotic one[10]. Then we have developed the H-NN with iterative SA noise and confirmed that the searching ability of global minimum of iterative SA noise is almost same as chaotic one, and searching performance for detecting local minima is superior to chaotic one in the case of some small TSPs[11]. In this paper, therefore, we investigate the solving ability of iterative SA noise for larger TSPs and some QAPs by comparing with the chaotic one. By computing simulations we confirm that the iterative SA noise is effective and can pick up larger number of local minima than chaotic one in both problems. Thus, we can say that this is superior property of iterative SA noise for practical use.

2. H-NN with Noise

The dynamics of the H-NN is given as:

$$x_{ik}(t+1) = f\left(\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ikjl}x_{jl}(t) + \theta_{ik} + \varepsilon\right) \quad (1)$$

where ε is an additional noise. And f is a sigmoidal

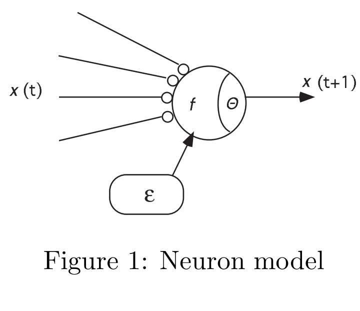

Figure 1: Neuron model

function defined as follows:

$$f(x) = \frac{1}{2}\left(1 + \tanh\frac{x}{\mu}\right) \qquad (2)$$

where μ is a constant in chaotic noise, but it's a variable parameter in SA noise. Fig. 1 shows a conceptual neuron model for this H-NN. The decision method of neuron's state, fire or not, is used the method in [6, 8].

2.1. H-NN with Chaotic Noise

For chaotic noise, we generate a time series by the logistic map:

$$z_{ik}(t+1) = az_{ik}(t)(1 - z_{ik}(t)) \qquad (3)$$

where a is a bifurcating parameter of the logistic map. Then the dynamics of H-NN with chaotic noise is given as :

$$x_{ik}(t+1) = f\left(\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ikjl}x_{jl}(t) + \theta_{ik} + \beta z_{ik}(t)\right) \qquad (4)$$

where β is amplitude of noise which is design parameter. Fig. 2 shows an example of the periodic-3 intermittent chaotic noise ($t = 1 \sim 1000$).

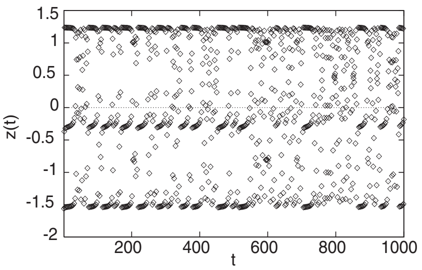

Figure 2: Time series of periodic-3 intermittent chaos

2.2. H-NN with iterative SA

H-NN with SA is also based on sigmoidal function and has a noise term (ε) depends on Gaussian distribution of which mean is 0 and standard deviation is $\sqrt{\delta/\pi}T$. Where δ is a empirical parameter and T is a control parameter, called temperature, which tends to zero according to a diterministic "annealing schedule".

The value of μ which is a parameter of sigmoidal function is determined by "sharpening schedule". As the value of ε is closer to 1, neuron's input-output function is closer to step function. We employ annealing and sharpening schedules as follows:

$$T = T_0 \exp\left[\frac{-c}{\rho}\right], \quad \mu = \mu_0 \exp\left[\frac{-c}{\rho}\right] \qquad (5)$$

Table 1: Scheduling parameters for SA noise

Iteration	T_0	μ_0	ρ
1	100.0	100.0	38.0
5	100.0	100.0	26.7
10	100.0	100.0	16.0

where T_0 is the initial value of temperature and μ_0 is the initial value of μ. And where c is a loop counter and ρ is a descent parameter. From former results [10], we know the solving ability of H-NN with simple SA noise is not good and inferior to H-NN with chaotic noise. We consider one of the reasons for this fact is that chaotic noise is always injected all the simulation time but the amplitude of SA noise is decreased as the temperature is reducing. We, therefore, propose the iterative SA-noise for improving normal SA noise. In the iterative SA noise, when the temperature becomes zero, it is initialized and repeated procedure of simulated annealing till terminal condition is satisfied. Hence, annealing is done more than twice per one trial. Thus, we call this noise iterative SA noise. Fig. 3 indicates time serieses of iterative-10 SA noise, and scheduling parameters of iterative SA noise are shown in Table 1.

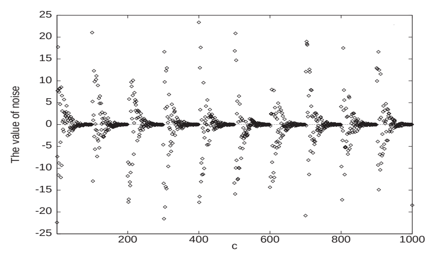

Figure 3: Time serieses of iterative-10 SA noises

3. Simulation results and discussion

3.1. Results of TSP

For solving N-city TSP, $N \times N$ neurons are required and the following energy function is defined to fire (i, j) neuron at the optimal position:

$$E = \sum_{i,j}^{N}\sum_{k,l}^{N} w_{i,k,j,l}x_{i.j}x_{k,l} - \sum_{i,j}^{N}\theta_{i,j}x_{i,j} \qquad (6)$$

where $w_{i,k,j,l}$ is a weight coefficient between (i, j) and (k, l) neurons. They are defined as

$$\begin{aligned} w_{i,k,j,l} &= -A\{\delta_{i,j}(1-\delta_{k,l}) + \delta_{k,l}(1-\delta_{i,j})\} \\ &\quad -B\delta_{i,j}(\delta_{l,k+1} + \delta_{l,k-1}) \end{aligned} \qquad (7)$$

where $A > 0$, $B > 0$, $\delta_{i,j}$ is Kroneker's delta and $\theta_{i,j} := A + B$ is the threshold value.

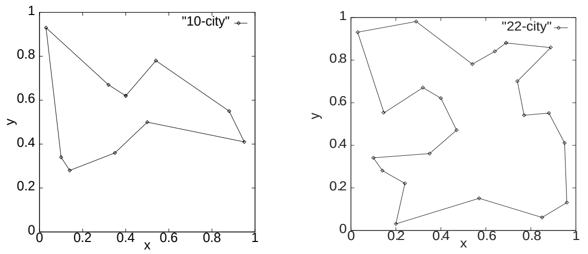

Figure 4: 10-city and 22-city TSPs

Although detection for the global mimimum is important for solving TSP, it should be added that how many minima are detected within a reasonable trial time is also important, because the dynamics of this H-NN with noise is nonautonomous and the network is not converge to the local minima and global minimum. We, therefore, use two performance criterion for investigating the solving abilities of H-NN with iterative SA noise and choatic one as follows:

$$\gamma \;:=\; \frac{\text{number of success trials}}{\text{total trial number}} \times 100 \;\; (\%) \tag{8}$$

$$\sigma \;:=\; \text{average number of detected local minima} \tag{9}$$

where "success trials" means the trial that could detect global minimum. One trial is defined as 1000 times of transfer of every neurons' input-output signal and total trial number is 100. We investigate solving abilities of above mentiond two type of noise for TSPs indicated in fig. 4 from the points of the two performance criteria mentiond above eqs. (8) and (9). The values of parameters for chaotic noise, a and β in eqs. (3) and (4), are tuned as shown in table 2. The initial values of all neurons and noises are chosen at random, and the value of parameters of energy function are tuned adequetly according to simulation conditions.

Table 2: Values of parameters for chaotic noise

City number	Periodic number	Parameters
10	7	$a = 3.7001, \beta = 0.47$
	5	$a = 3.7379, \beta = 0.47$
	3	$a = 3.8250, \beta = 0.47$
22	7	$a = 3.7001, \beta = 0.47$
	5	$a = 3.7379, \beta = 0.45$
	3	$a = 3.8252, \beta = 0.42$

Table 3 shows the best solving abilities of H-NN with iterative SA noise and H-NN with chaotic noise respectively. From table 3, we can see that solving abilities of iterative-5 and 10 SA noise are more effective than iterative-1 SA noise which is a normal SA method in both performace criteria, γ and σ. When we compare the performance between SA noise and chaotic one, we see that performance about γ between iterative SA noise and chaotic one are almost same. On the other hand, values of σ are different between chaotic and iterative SA noises. Iterative SA noise can pick up local minima more than chaotic one. We can say that this is superior property of iterative SA noise for practical use.

Table 3: Results of TSP

City No.	iterative SA noise			Chaotic noise		
	Itera.	γ (%)	σ	Period.	γ (%)	σ
10	10	89	14.0	7	84	7.5
	5	86	10.2	5	81	8.8
	1	62	6.4	3	91	13.1
22	10	47	8.2	7	60	8.3
	5	49	8.6	5	53	8.8
	1	20	4.3	3	50	10.3

3.2. Results of QAP

Results of QAP In this section, simulation results of QAP are shown. The QAP is one of the most difficult combinatorial problem and it is to find the permutation vector p which minimize the objective function $f(p)$ under given distance matrix C and flow matrix D. The objective function of QAP is defined as

$$f(p) \;:=\; \sum_{i=1}^{N}\sum_{j=1}^{N} C_{i,j} D_{p(i).p(j)} \tag{10}$$

where $C_{i,j}$ and $D_{i,j}$ are the (i,j)-th elements of C and D respectively, and $p(i)$ is the i th element of p.

The QAP's energy function for H-NN could be expressed as TSP's, eq. (6), the weight, $w_{i,k,j,l}$, is a bit different. It's described as

$$w_{i,k,j,l} = -2\left[A\delta_{i,j}(1-\delta_{k,l}) + B\delta_{k,l}(1-\delta_{i,j}) + \frac{C_{i,j}D_{k,l}}{q}\right] \tag{11}$$

where $A > 0$, $B > 0$ and $q > 0$ are constants, and $\delta_{i,j}$ is Kroneker's delta. And where $\theta_{i,j} := A + B$ is the threshold value. Two problems, "Nug5" and "Nug12", are chosen here from QAPLIB[12].

1. [Nug 5]

$$C = \begin{pmatrix} 0 & 4 & 3 & 5 & 2 \\ 4 & 0 & 6 & 2 & 1 \\ 3 & 6 & 0 & 5 & 7 \\ 5 & 2 & 5 & 0 & 3 \\ 2 & 1 & 7 & 3 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 3 & 1 & 2 & 4 \\ 3 & 0 & 2 & 1 & 4 \\ 1 & 2 & 0 & 5 & 3 \\ 2 & 1 & 5 & 0 & 1 \\ 4 & 4 & 3 & 1 & 0 \end{pmatrix}$$

The global optimum is known as 158.

2. [Nug 12]

$$
C = \begin{pmatrix}
0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 5 \\
1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 & 3 & 4 \\
2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 & 5 & 4 & 3 & 2 \\
1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 \\
2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 \\
3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 \\
4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 \\
2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 \\
3 & 2 & 3 & 4 & 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 \\
4 & 3 & 2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\
5 & 4 & 3 & 2 & 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0
\end{pmatrix}
$$

$$
D = \begin{pmatrix}
0 & 5 & 2 & 4 & 1 & 0 & 0 & 6 & 2 & 1 & 1 & 1 \\
5 & 0 & 3 & 0 & 2 & 2 & 2 & 0 & 4 & 5 & 0 & 0 \\
2 & 3 & 0 & 0 & 0 & 0 & 0 & 5 & 5 & 2 & 2 & 2 \\
4 & 0 & 0 & 0 & 5 & 2 & 2 & 10 & 0 & 0 & 5 & 5 \\
1 & 2 & 0 & 5 & 0 & 10 & 0 & 0 & 0 & 5 & 1 & 1 \\
0 & 2 & 0 & 2 & 10 & 0 & 5 & 1 & 1 & 5 & 4 & 0 \\
0 & 2 & 0 & 2 & 0 & 5 & 0 & 10 & 5 & 2 & 3 & 3 \\
6 & 0 & 5 & 10 & 0 & 1 & 10 & 0 & 0 & 0 & 5 & 0 \\
2 & 4 & 5 & 0 & 0 & 1 & 5 & 0 & 0 & 0 & 5 & 0 \\
1 & 5 & 2 & 0 & 5 & 5 & 2 & 0 & 0 & 0 & 5 & 0 \\
1 & 0 & 2 & 5 & 1 & 4 & 3 & 5 & 10 & 5 & 0 & 2 \\
1 & 0 & 2 & 5 & 1 & 0 & 3 & 0 & 10 & 0 & 2 & 0
\end{pmatrix}
$$

The global optimum is known as 578.

In these problems, we do not only use γ but Av which means the average value of every minimum costs found in trials[9]. The results of H-NN with iterative SA noise are summarized in Table 4. The results of H-NN with the periodic-3 intermittent chaos noise ($a = 3.8276$) are also shown for comparison.

Table 4: Results of QAP

Nug No.	iterative SA noise			Chaotic noise	
	Itera.	γ (%)	Av	γ (%)	Av
5	10	70	159.2	72	158.9
	5	62	160.6		
	1	8	198.3		
12	10	6	597.3	3	612.0
	5	3	609.9		
	1	0	–		

From table 4, we can also say that solving abilities of iterative-5 and 10 SA noise are more effective than iterative-1. From comparison with the chaotic noise, the performance of iterative SA noise and chaotic one are almost same. But, it is remains the room for improvement of solving ability of iterative SA noise. We need to find a suitable parameter set of energy function for iterative SA noise. To settle this difficulty is a future problem.

4. Conclusion

In this paper, we propose the H-NN with iterative SA noise and investigate the solving abilities of this H-NN for TSPs and QAPs. Then we confirm that H-NN with iterative SA noise is as effective as H-NN with chaotic noise in these combinatorial problems. As future works, suitable parameter set of energy function for iterative SA noise is required to find. In addition to this we need to analyze the solving abilities of iterative SA noise and chaotic one with a theoretical approach.

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