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Performance of Chaos Noise Injected to Hopfield NN for Quadratic Assignment Problems

Yoko Uwate†, Yoshifumi Nishio†, Tetsushi Ueta‡, Tohru Kawabe§ and Tohru Ikeguchi¶

† Dept. of Electrical and Electronics Engineering, Tokushima University
Email: {uwate,nishio}@ee.tokushima-u.ac.jp
‡ Dept. of Information Science and Intelligent Systems, Tokushima University
§ Institute of Information Sciences and Electronics, Tsukuba University
¶ Dept. fo Information and Computer Sciences, Saitama University

1. Introduction

Combinatorial optimization problems can be solved with the Hopfield Neural Network. If we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the solutions are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kinds of noise for solving traveling salesman problems (TSP) with the Hopfield NN [1]. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded that the good result might be obtained by a property of the chaos noise; short time correlations of the time-sequence. Hasegawa *et al.* investigated solving abilities of the Hopfield NN with various surrogate noises, and they concluded that the effects of the chaos sequence for solving optimization problems can be replaced by stochastic noise with similar autocorrelation [3]. In order to make clear the reason why intermittency chaos is better than fully developed chaos, we have investigated the performance with the burst noise generated by the Gilbert model [4] for TSP [5]-[7] and quadratic assignment problems (QAP) [6]-[8]. We confirmed that the burst noise gained similar performance to the intermittent chaos noise. We consider that one reason of the good performance of the chaos noise can be explained by the existence of the laminar part and the burst part. However, we do not consider that ability of chaos noise has been clarified completely.

In this study, we use various evaluation methods to find outstanding ability of chaos noise shrouded in mystery. Control parameter α of the logistic map is changed to generate various chaos noises. We chose several problems from the benchmark site QAPLIB [9]; "Nug12," "Nug25" and "Nug30." We carried out computer simulations for these problems and confirmed various characteristics of chaos noises.

2. Solving QAP with Hopfield NN

Various methods are proposed for solving QAP which is one of NP-hard combinatorial optimization problems. QAP is expressed as follow: given two matrices, distance matrix C and flow matrix D, and find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in Eq. (1) .

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \tag{1}$$

where C_{ij} and D_{ij} are the (i, j)-th elements of C and D, respectively, $\mathbf{p}(i)$ is the i-th element of vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is to find the arrangement of factories to make a cost the minimum. The cost is given by the distance between the cities and the flow of the products between the factories (Fig. 1). Other examples are the placement of logical modules in an IC chip, the distribution of medical services in a large hospital.

Because QAP is very difficult, it is almost impossible to solve the optimum solutions in larger problems. The largest problem which is solved by deterministic methods may be only 20 in recent study. Further, computation time is very long to obtain the exact optimum solutions. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time. For solving N-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i, j)-th neuron at the

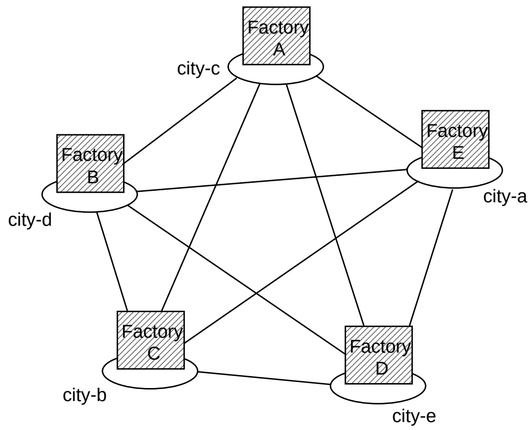


Figure 1: QAP model.

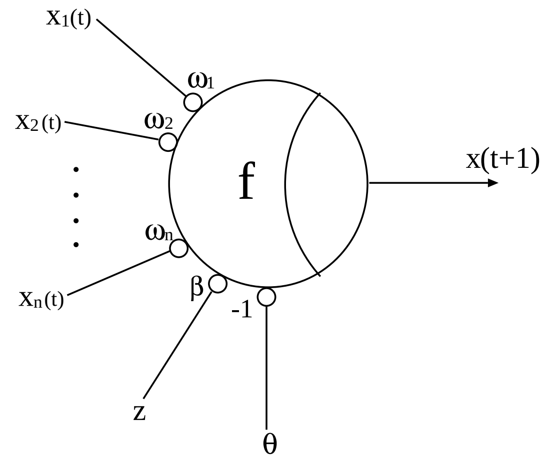


Figure 2: A neuron model.

optimal position:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \qquad (2)$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between (i, m)-th neuron and (j, n)-th neuron and the threshold of the (i, m)-th neuron are described by:

$$\begin{aligned} w_{im;jn} &= -2\Big\{A(1-\delta_{mn})\delta_{ij} \\ &\quad + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q}\Big\} \qquad (3) \\ \theta_{im} &= A + B \end{aligned}$$

where A and B are positive constants, and δ_{ij} is Kronecker's delta. The states of $N \times N$ neurons are unsynchronously updated due to the following difference equation:

$$\begin{aligned} x_{im}(t+1) = g\Bigg(&\sum_{j,n=1}^{N} w_{im;jn} x_{im}(t) x_{jn}(t) \\ &- \theta_{im} + \beta z_{im}(t)\Bigg) \qquad (4) \end{aligned}$$

where g is sigmoidal function defined as follows:

$$g(x) = \frac{1}{1 + \exp\left(-\frac{x}{\varepsilon}\right)} \qquad (5)$$

$z_{im}(t)$ is additional noise, and β limits amplitude of the noise. Figure 2 shows a conceptual neuron model for this NN.

Also, we use the method suggested by Sato *et al.* (1.1 in [10]) to decide firing of neurons.

3. Chaos Noises

In this section, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate chaos noise:

$$\hat{z}_{im}(t+1) = \alpha \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \qquad (6)$$

Varying parameter α, Eq. (6) behaves chaotically via a period-doubling cascade. When we inject chaos noise to the Hopfield NN, we normalize $\hat{z}_{im}$ by Eq. (7).

$$z_{im}(t+1) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \qquad (7)$$

Where $\bar{z}$ is the average of $\hat{z}(t)$, and σ_z is the standard deviation of $\hat{z}(t)$. Figure 3 shows an example of the time series of the chaos noise.

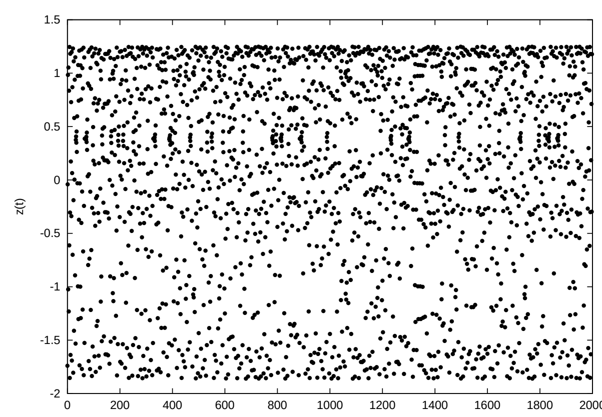


Figure 3: Chaos noise. α=3.8000.

4. Simulated Results

The problem used here was chosen from the site QAPLIB which collects the bench mark problems. We carried out computer simulation for three problems; "Nug12," "Nug25" and "Nug30." In this article, we show the result of "Nug30" only. The global minimum of this problem is known as 6124. The parameters of the Hopfield NN are fixed as $A = 1.0$, $B = 1.0$, $q = 640$ and $\varepsilon = 0.02$ and the amplitude of the injected chaos noise is fixed as $\beta = 0.6$. The number of updating the Network N_{it} is 10000.

At first, we explain how to accept solutions. The Hopfield NN with the chaos noise searches various solutions. However, the state of the Hopfield NN sometimes stays around a group of several solutions. We consider that such a behavior is not useful to find the optimal or nearly optimal solutions. So, we take the only-different-solutions method. Namely, we take into account the solutions which have not found ever.

Next, we propose two methods evaluating the solutions obtained by the Hopfield NN with the chaos noise.

4.1. Depth_1

Until now, we have evaluated the performance of the Hopfield NN with chaos noise by using only the optimal solution; time to find the optimal solution, frequency of finding the optimal solution during a given time, and so on. However, when the Network can not find the optimal solution and can find a lot of nearly optimal solutions, such methods do not give a correct evaluation of the Network. We consider that it is very important for the Network to find a lot of nearly optimal solutions. In order to evaluate the ability, we propose an evaluation method *Depth_1* which is defined as

$$Depth\_1 = \sum_{k=0}^{n} \{f(\mathbf{P}_k) - D_\infty\}^2 \qquad (8)$$

where D_∞ is a constant which is large enough to include the energies of all solutions, n is the number of the accepted solutions and the energy $f(\mathbf{P}_k)$ is calculated by Eq. (1) using the permutation $\mathbf{P}_k$ corresponding to the k-th solution.

The calculated result of *Depth_1* is shown in Fig. 4. The horizontal axis is α and the vertical axis is *Depth_1*. On the contrary to our expectation, the value of *Depth_1* around $\alpha = 3.70$ is very large.

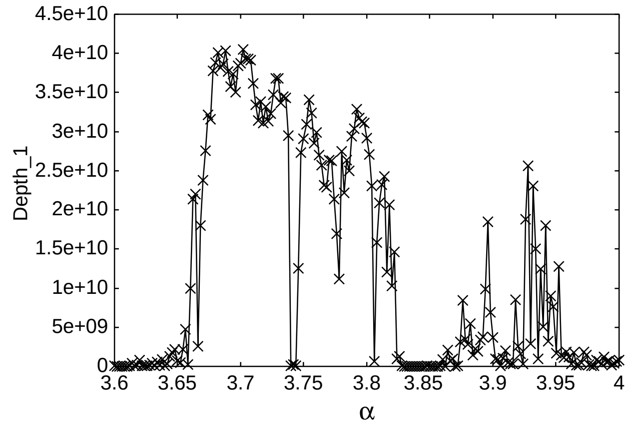
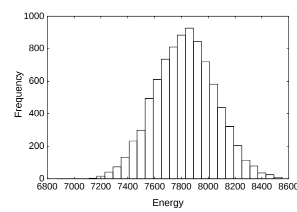
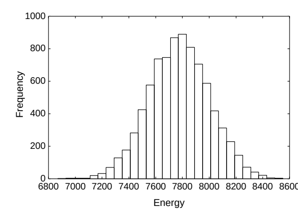


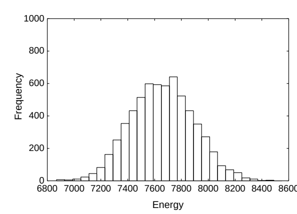
Figure 4: *Depth_1* for Nug30 ($D_\infty = 10000$).



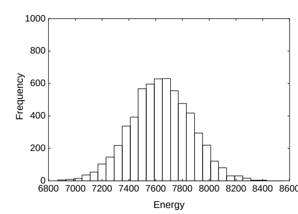
(a) $\alpha = 3.6820$.



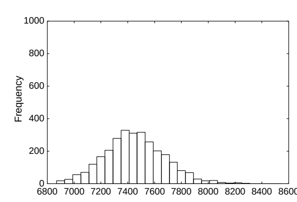
(b) $\alpha = 3.7020$.



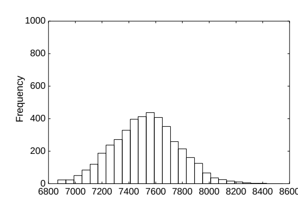
(c) $\alpha = 3.7540$.



(d) $\alpha = 3.7920$.



(e) $\alpha = 3.8960$.



(f) $\alpha = 3.9280$.

Figure 5: Frequency distribution of solutions for Nug30.

In order to make clear the reason of the large values of *Depth_1* around $\alpha = 3.70$, we examined frequency distribution of the solutions for several α which get large values of *Depth_1*. The results of the frequency distribution are shown in Fig. 5. We can see that a lot of solutions are found for (a) and (b). However, the distribution is similar to that obtained by random search. On the other hand, for (c), (d), (e) and (f), the averages of the accepted solutions are better than those of (a) and (b). Although finding a lot of solutions is important, solutions with energies more than the average obtained by random search should not be taken into account for the evaluation.

4.2. Depth_2

We propose another evaluation method to obtain chaos noises with better performance for finding a lot of GOOD solutions. *Depth_2* is defined as follows:

$$Depth\_2 = \begin{cases} \sum_{k=0}^{n} \{f(\mathbf{P}_k) - D_{th}\}^2 & (f(\mathbf{P}_k) \leq D_{th}) \\ -\sum_{k=0}^{n} \{f(\mathbf{P}_k) - D_{th}\}^2 & (f(\mathbf{P}_k) > D_{th}) \end{cases} \quad (9)$$

In order to eliminate the effect of bad solutions, we not only set up a threshold but give a penalty according to the energy.

The calculated result of *Depth_2* is shown in Fig. 6. Let us note around $\alpha = 3.70$. We can say that *Depth_2* evaluates the accepted solutions as we expect. Namely, *Depth_2* is a good method to evaluate the solutions solved by the Hopfield NN with chaos noise.

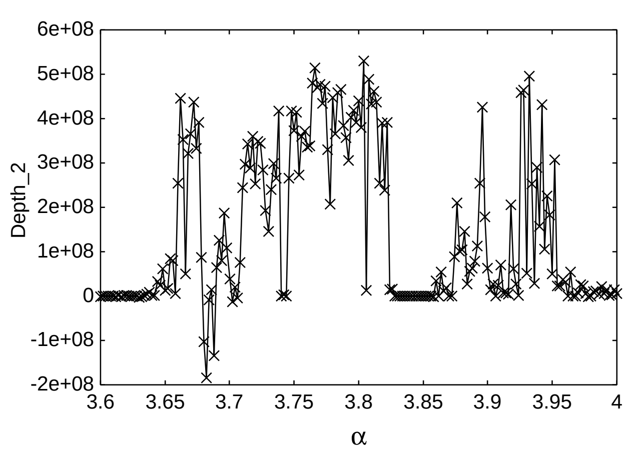


Figure 6: *Depth_2* for Nug30 ($D_{th} = 7800$).

From Fig. 6 we found several characteristics of chaos noises. Namely, as we have reported in our previous studies, intermittency chaos near the three-periodic window (around $\alpha = 3.81$) gains a good performance.

Further, as several researchers have pointed out, fully-developed chaos near $\alpha = 4.00$ can not gain a good performance. We also found that chaos noises around $\alpha = 3.66$, 3.76 and 3.93 gains good performances. We consider that the reason of the good performances is not the same for all of these regions.

5. Conclusions

In this study, we have researched the characteristic of the chaos noise when control parameter α is changed. We proposed the two methods to evaluate the performance of the chaos noises. As a result, we found several characteristics of chaos noises. In the future, we make clear the reason of the good performances of chaos noises obtained for various α.

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