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An Approach to N-Queens Problem Using Chaos Neural Network - Challenge to Finding All Solutions of N-Queens Problem -

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Abstract

In this study, we investigate three different update methods of the chaos neural network in order to approach to finding all solutions of the N-Queens problem.

By computer simulations for various sizes of the N-Queens problem, we confirm that the method to change the weights connecting the firing neurons can successfully restrain the regeneration of the once-appeared-solutions.

1. Introduction

Combinatorial optimization problems can be solved with the Hopfield neural network [1]. If we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the network state is often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, chaos neural networks have been applied to various optimization problems successfully.

On the other hand, finding all solutions of nonlinear systems is a very important problem appearing in various fields of natural science. The N-Queens problem is a long-known combinatorial problem having a huge number of the solutions. Several researchers have also proposed the methods to solve the N-Queens problem by using the Hopfield neural networks [2][3]. However, the Hopfield neural network can find only one solution among numerous possible solutions.

In this study, we try to find many possible solutions of the N-Queens problem by using neural network. The chaos neural network could find several solutions because of its chaotic dynamical motion. However, the network state visits the once-appeared solution quite often. We investigate three different update methods of the chaos neural network in order to restrain the regeneration of the once-appeared-solutions. By computer simulations for various sizes of the N-Queens problem, we confirm that the method to change the weights connecting the firing neurons can successfully find many different solutions of the N-Queen problem.

2. N-Queens problem and chaos neural network

In the N-Queens problem, N chess queens must be placed on a squere chessboard composed of N rows and N columns, in such a way that they do not attack each other. In other words, the arrangement of the queens must satisfy the constraint that any two different queens are placed in different rows, columns and diagonals [2]. One of the possible arrangements for $N = 5$ is shown in Fig. 1. The number of the possible arrangements becomes larger exponentially as N increases. The number for $N \leq 15$ is listed in Table 1.

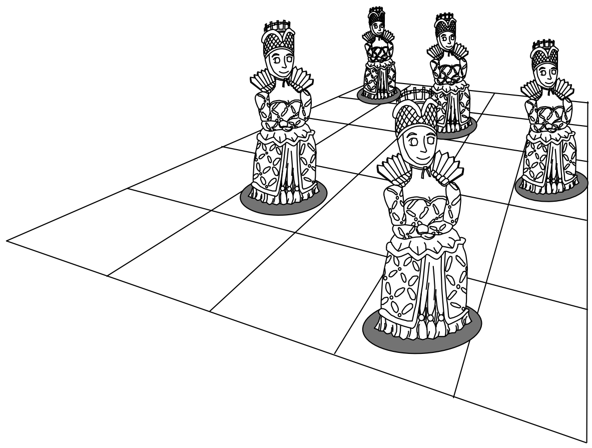

Figure 1: N-Queens problem

In order to find different solutions, as many as possible, we apply the chaos neural network to the N-Queens problem.

Table 1: Number of solutions of N-Queens problem.

N	# of Solutions
4	2
5	10
6	4
7	40
8	92
9	352
10	724
11	2680
12	14200
13	73712
14	365596
15	2279184

The equation of the chaos neural network is given as

$$\begin{aligned} x_{i,j}(t+1) &= kx_{i,j}(t) + \sum_{k=1}^{N}\sum_{l=1}^{N}\omega_{i,j,k,l} f\left(x_{k,l}(t)\right) \\ &\quad -\alpha f\left(x_{i,j}(t)\right) - \theta_{i,j}(1-k) \end{aligned} \tag{1}$$

where $f(x)$ is a sigmoid function

$$f(x) = \frac{1}{1+\exp\left(-\frac{y}{\varepsilon}\right)} \tag{2}$$

and k is time attenuation constant and α controls non-linearity.

The weights connecting neurons are defined to find solutions of the N-Queens problem as

$$\begin{aligned} \omega_{i,j,k,l} &= -\delta_{i,k}(1-\delta_{j,l}) - \delta_{j,l}(1-\delta_{i,k}) \\ &\quad -\delta_{|i-k|,|j-l|}(1-\delta_{i,k}\delta_{j,l}) \end{aligned} \tag{3}$$

where $\delta_{r,s}$ is the two-dimentional Kronecker delta, which is defined as

$$\delta_{r,s} = \begin{cases} 1, & \text{if } r = s \\ 0, & \text{if } r \neq s. \end{cases} \tag{4}$$

Also, we use the method suggested by Sato *et al.* (1.1 in [4]) to decide firing of neurons.

3. Three Proposed Methods

The chaos neural network can find several solutions as it is because of its chaotic dynamical motion. However, the network state visits the once-appeared solution quite often. We propose three different update methods of the chaos neural network in order to restrain the regeneration of the once-appeared-solutions.

3.1. Method 1

In the Method 1, the weights connecting the firing neurons are reduced to restain the regeneration of the firing pattern after every updating. Namely, we choose one of the firing neurons. A constant value "ΔW" is subtracted from the connecting weights from this neuron to the other firing neurons. Moreover, in order to maintain the activity of the network, "$\Delta W(N-1)/\{N(N-1)+1\}$" is added to the rest of the weights. This procedure is applied to the all firing neurons.

3.2. Method 2

In the Method 2, the threshold of the firing neurons are reduced to restain the regeneration of the firing pattern after every updating. A constant value "$\Delta\theta$" is subtracted from the thresholds of the all firing neurons. Moreover, in order to maintain the activity of the network, "$\Delta\theta/(N-1)$" is added to the non-firing neurons.

3.3. Method 3

The Method 3 adds another term to the definition of the weights connecting the neurons. The weights are re-calculated after every updating according to Eq. (5).

$$\begin{aligned} \omega_{i,j,k,l} &= -\delta_{i,k}(1-\delta_{j,l}) - \delta_{j,l}(1-\delta_{i,k}) \\ &\quad -\delta_{|i-k|,|j-l|}(1-\delta_{i,k}\delta_{j,l}) \\ &\quad -D\sum_{m=1}^{s}(S_{i,j,m}S_{k,l,m})(1-\delta_{i,k}\delta_{j,l}) \end{aligned} \tag{5}$$

where s is the number of the solutions before the updating and $S_{i,j,m}$ is the constants memorizing the positions of the firing neurons for the m-th solution, namely, $S_{i,j,m}$ is 1 if the m-th solution has a queen at the postion (i, j), otherwise zero. The term $1-\delta_{i,k}\delta_{j,l}$ is introduced to neglect the connection of neurons to themselves.

4. Simulated results

Computer simulations of N-Queens problem are performed for the cases of N=10 and 15. The number of the update of the network state is fixed as 10,000 and this trial is iterated 10 times for different initial conditions. The average of the number of the obtained solutions is evaluated. The simulated results using the three proposed methods are compared with the results obtained from the normal chaos neural network.

The parameter α is varied from 1.0 to 4.0. Moreover, k and $\theta_{i,j}$ are fixed as $k = 0.8$ and $\theta_{i,j} = 0.1$. We carried out computer simulations for various values of the parameters of the three methods ΔW, $\Delta\theta$ and D. The following results are the best we obtained.

Figure 2 shows the simulated results for $N = 10$.

Next, the simulated results for $N = 15$ are shown in Fig. 3.

From these figures, we can say that the proposed three methods can find more solutions than the ordinary chaos neural networks, especially for the case of N=15.

5. Conclusions

In this study, we have tried to find many possible solutions of the N-Queens problem by using the chaos neural network. We investigated three different update methods of the chaos neural network in order to restrain the regeneration of the once-appeared-solutions. By computer simulations for various sizes of the N-Queens problem, we confirm that the method to change the weights connecting the firing neurons can successfully find many different solutions of the N-Queen problem.

However, the number of the found solutions is far away from the number of the all solutions. Finding new methods better than the three methods is our important future research.

References

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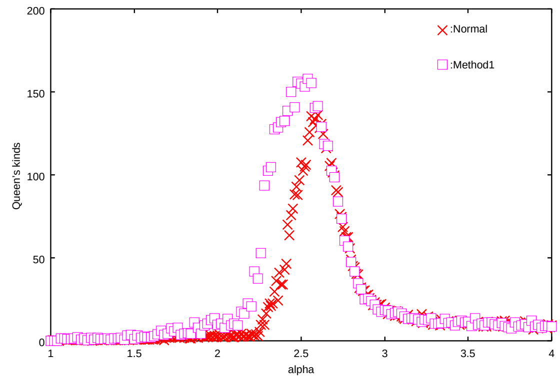
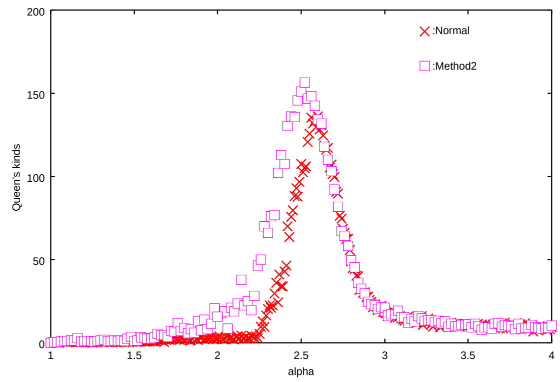
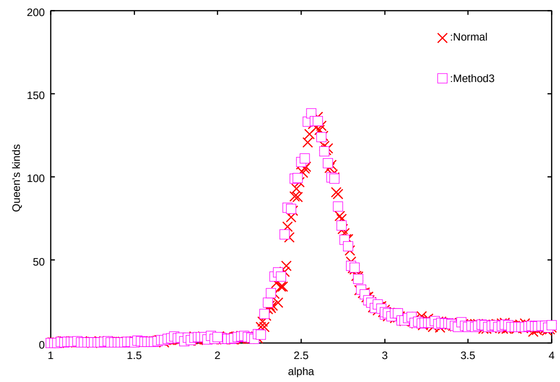


(a) Method 1. $\triangle W = 0.002$.

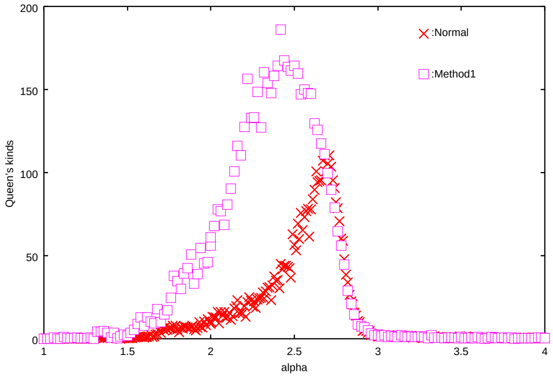


(b) Method 2. $\triangle\theta = 0.0004$.

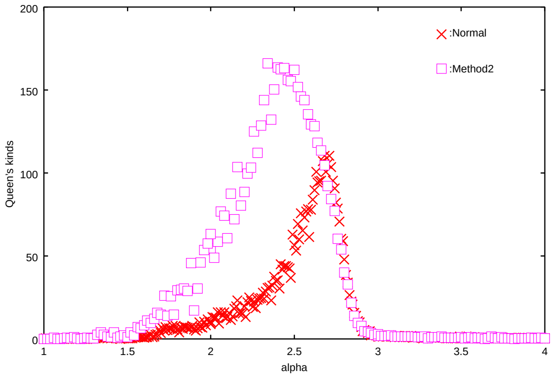


(c) Method 3. $D = 0.0004$.

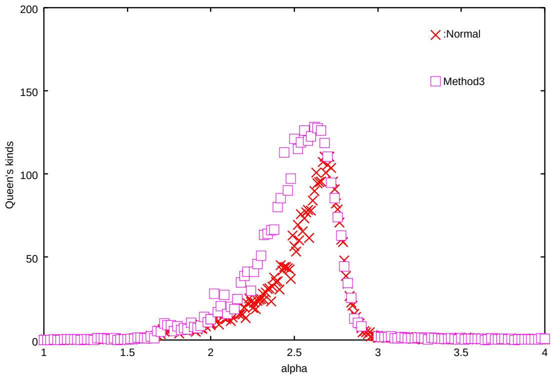
Figure 2: Simulated results for N=10.



(a) Method 1. $\triangle W = 0.002$.



(b) Method 2. $\triangle\theta = 0.0004$.



(c) Method 3. $D = 0.0004$.

Figure 3: Simulated results for N=15.