Spatio-Temporal Chaos in a Coupled System of Ring Oscillators

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1. Introduction

We have investigated a chaotic circuit utilizing CMOS ring oscillators[1]. The circuit consists of two CMOS ring oscillators and two diodes. A CMOS ring oscillator is used for CMOS process performance test. This oscillator consists of a series of inverter circuits. It has a very simple structure. Therefore, we consider that the proposed circuit is realized very easily. In the past studies, we have shown the bifurcation scenario, have calculated the Lyapunov exponents, have explained physical mechanism of chaos generation and so on.
In this study, we investigate spatio-temporal chaos in a coupled system of the ring oscillators. The system consists of many chaotic circuits utilizing CMOS ring oscillators. Because this system utilizes the circuits designed by integrated circuit technology, this study is very important for analysis and applications of large coupled chaotic systems[2]–[5]. We consider that large coupled systems should be realized as VLSI for future studies and future applications. The proposed system is very suitable for integrated circuits. It means that VLSI realization of the system is easily. In Sect. 2 and 3, the chaotic circuit and the coupled system are described. In Sect. 4, simulated results of the coupled system are shown. Some concluding remarks are presented in the Sect. 5.

2. Chaotic Circuit Using CMOS Ring Oscillators

Chaotic circuits have been realized as integrated circuits. These circuits are not complexity so far as an electric circuit are concerned. However, realized circuits on the IC chip are complexity structures. The basic problems are that the circuit is an analog circuit and included nonlinear elements. Therefore, integration techniques are needed. One of the attraction of chaos is that a simple system generate complex phenomena. We went back again to this attraction, a simple integrated circuit was proposed. We consider utilizing a ring oscillator which is a simple structure on the chip. A CMOS ring oscillator is one of the simplest circuit constructed on the chip. The oscillator consists of the odd number of CMOS inverters. We designed a chaotic circuit using ring oscillators. Figure 1 shows the designed circuit. In order to control amplitudes of oscillators, R_1 and R_2 are connected. The frequency of the upper side oscillator are controlled by C_1. Two three-dimensional autonomous oscillatory circuits are coupled by two diodes. We approximate $M1$, $M2$ and CMOS inverters in the circuit as follows. We focus attention on one inverter as shown in Fig. 2. All parasitic capacitors associated with the input node is connected parallel with all parasitic capacitors associated with the output node of the previous

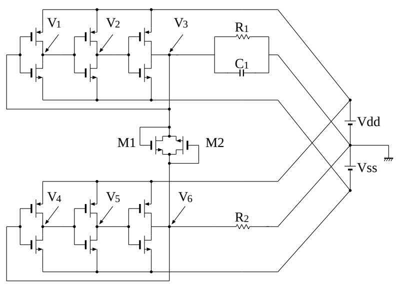


Figure 1: Chaotic circuit using ring oscillators.

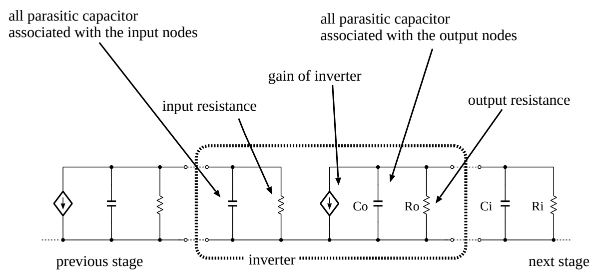


Figure 2: Simplified model of the ring oscillator's inverter.

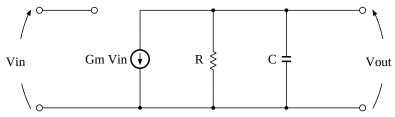


Figure 3: Simplified model of the ring oscillator's inverter.

stage inverter. The input resistor is connected parallel with the output resistor of the previous stage inverter. Correspondingly, All parasitic capacitors associated with the output node is connected parallel with all parasitic capacitors associated with the input node of the next stage inverter. The output resistor is connected parallel with the input resistor of the next stage inverter. Therefore, we can obtain a simplified inverter model as shown in Fig. 3. $M1$ and $M2$ have a characteristic of the same

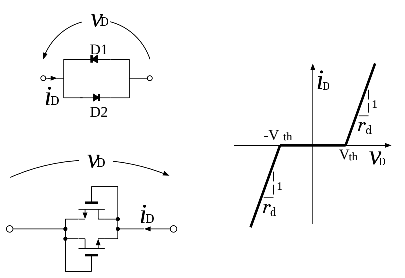


Figure 4: Diodes model.

as the coupling diodes shown in Fig. 4. Therefore, $M1$ and $M2$ are approximated as the following three-regions piecewise-linear function.

$$i_D = \begin{cases} \frac{1}{r_d}(v_D - V_{th}) & \text{for } v_D > V_{th}, \\ 0 & \text{for } -V_{th} \le v_D \le V_{th}, \\ \frac{1}{r_d}(v_D + V_{th}) & \text{for } v_D < -V_{th}. \end{cases} \quad (1)$$

Circuit equations are described as follows:

$$\begin{cases} \frac{dv_1}{dt} &= -\frac{1}{RC}v_1 - \frac{Gm}{C}v_3, \\ \frac{dv_2}{dt} &= -\frac{1}{RC}v_2 - \frac{Gm}{C}v_1, \\ \frac{dv_3}{dt} &= -\frac{R+R_1}{RR_1(C+C_1)}v_3 \\ & \quad -\frac{Gm}{C+C_1}v_3 - \frac{i_d}{C+C_1}, \\ \frac{dv_4}{dt} &= -\frac{1}{RC}v_4 - \frac{Gm}{C}v_6, \\ \frac{dv_5}{dt} &= -\frac{1}{RC}v_5 - \frac{Gm}{C}v_4, \\ \frac{dv_6}{dt} &= -\frac{R+R_2}{RR_2C}v_6 - \frac{Gm}{C}v_6 + \frac{i_d}{C}, \end{cases} \quad (2)$$

where

$$i_d = \begin{cases} \frac{1}{r_d}(v_3 - v_6 - V_{th}) & \text{for } v_3 - v_6 > V_{th}, \\ 0 \quad \text{for } -V_{th} \le v_3 - v_6 \le V_{th}, \\ \frac{1}{r_d}(v_3 - v_6 + V_{th}) & \text{for } v_3 - v_6 < -V_{th}. \end{cases} \quad (3)$$

By substituting the variables and the parameters,

$$\begin{aligned} & x_n = \frac{v_n}{V_{th}}, \quad y_d = \frac{R_d}{V_{th}}i_d, \quad \tau = \frac{1}{RC}t, \\ & \alpha = GmR, \quad \beta = \frac{C}{C+C_1}, \quad \gamma = \frac{R}{R_1}, \\ & \delta = \frac{R}{R_d}, \quad \varepsilon = \frac{R}{R_2}. \end{aligned} \quad (4)$$

(2) and (3) are normalized as

$$\begin{cases} \dot{x}_1 &= -x_1 - \alpha x_3, \\ \dot{x}_2 &= -x_2 - \alpha x_1, \\ \dot{x}_3 &= -\beta(\gamma+1)x_3 - \alpha\beta x_2 - \beta\delta y_d, \\ \dot{x}_4 &= -x_4 - \alpha x_6, \\ \dot{x}_5 &= -x_5 - \alpha x_4, \\ \dot{x}_6 &= -(\varepsilon+1)x_6 - \alpha x_5 + \delta y_d, \end{cases} \quad (5)$$

where

$$y_d = \begin{cases} x_3 - x_6 - 1 & \text{for } x_3 - x_6 > 1, \\ 0 & \text{for } -1 \le x_3 - x_6 \le 1, \\ x_3 - x_6 + 1 & \text{for } x_3 - x_6 < -1. \end{cases} \quad (6)$$

3. Coupled system of the chaotic circuits

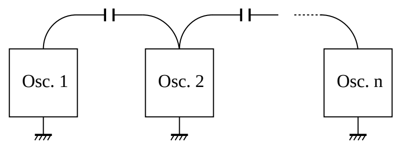


Figure 5: Coupled system of the chaotic circuits

The circuits are coupled by capacitors as shown in Fig. 5. The coupling points are V_3 of Fig. 1. The value of all coupling capacitors is as C_c. Coupled system equations are same as Eq. (2) except the third equation.

$$\begin{cases} \dot{x}_{k1} &= -x_{k1} - \alpha x_{k3}, \\ \dot{x}_{k2} &= -x_{k2} - \alpha x_{k1}, \\ \dot{x}_{k4} &= -x_{k4} - \alpha x_{k6}, \\ \dot{x}_{k5} &= -x_{k5} - \alpha x_{k4}, \\ \dot{x}_{k6} &= -(\varepsilon+1)x_{k6} - \alpha x_{k5} + \delta y_k, \\ & \quad for 1 \le k \le n, \end{cases} \quad (7)$$

$$\left\{ \begin{array}{lll} \dot{x}_{13} & = & \frac{\beta}{\zeta} F_{(n)} G_{(n)} \\ \dot{x}_{k3} & = & F_{(n-k+1)} \left\{ \frac{\beta}{\zeta} H_{(n-k+1)} + \dot{x}_{(k-1)3} \right\} \end{array} \right. \tag{8}$$

where

$$y_d = \left\{ \begin{array}{ll} x_3 - x_6 - 1 & \text{for } x_3 - x_6 > 1, \\ 0 & \text{for } -1 \leq x_3 - x_6 \leq 1, \\ x_3 - x_6 + 1 & \text{for } x_3 - x_6 > -1. \end{array} \right. \tag{9}$$

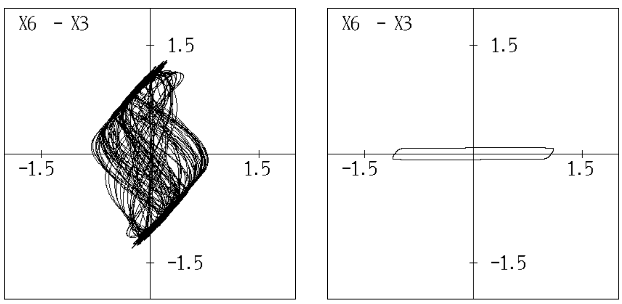
$$\zeta = \frac{C_c}{C + C_1}. \tag{10}$$

$$\left\{ \begin{array}{ll} F_{(1)} = & \frac{1}{\zeta + 1}, \\ F_{(k)} = & \frac{1}{\zeta + 2 - F_{(k-1)}}, \quad for \;\; 2 \leq k \leq n-1, \\ F_{(n)} = & \frac{1}{\zeta + 1 - F_{(n-1)}}. \end{array} \right. \tag{11}$$

$$\begin{array}{rl} G_{(k)} = & -\alpha x_{k2} - (\gamma + 1) x_{k3} - \delta y_k, \\ & \qquad for \;\; 1 \leq k \leq n. \end{array} \tag{12}$$

$$\left\{ \begin{array}{ll} H_{(1)} = & G_{(n)} \\ H_{(k)} = & G_{(n-k+1)} + F_{(n-k+1)} H_{(k-1)} \\ & \qquad for \;\; 2 \leq k \leq n. \end{array} \right. \tag{13}$$

4. Spatio-Temporal Chaos



(a) (b)

Figure 6: Attractor of the chaotic circuit using ring oscillators. $\alpha = 4$, $\beta = 0.1$, $\gamma = 4.5$, $\delta = 70$ and $\varepsilon = 2.4$

Figures 6 shows attractors of the single chaotic circuit using ring oscillators. Chaotic attractor is observed in Fig. 6 (a). Periodic orbit is observed in Fig. 6 (b). Same parameters are used, different initial values are used. The coupled system is simulated using the same parameters of Fig 6. The number of coupled oscillators is 25. We investigate time series of the amplitude x_3, about some kinds of initial values. Figures 7–10 show simulated results of the coupled system. Horizontal axis shows time. Vertical axes shows amplitudes $x_{k3}(k = 1, 2, \cdots 25)$ of each oscillators. Attractors of small amplitude parts are similar of Fig. 6 (b). One small amplitude part is observed in Fig. 7. The position of this part is changing irregularly. In Fig. 8, all oscillators generate small amplitudes like Fig. 6 (b). Three small amplitude parts are observed in Fig. 9. These parts do not move. In the case that the number of coupled oscillators is 25, four or more small amplitude parts are not observed. This result means that about seven oscillators are needed in order to make existing one small amplitude part. Two small amplitude parts are observed in Fig 10. Basically, this state is quasi-stable. However, the position of these parts sometimes move irregularly as shown in Fig. 11. In the case that initial values of all oscillators are the same, one small amplitude parts is observed as a Fig. 7 via no small amplitude part state. Therefore, it is considered that the system wishes to make existing small amplitude parts. From these results, we can consider that each small amplitude part hates closing.

5. Conclusions

In this study, we have proposed coupled chaotic systems utiling CMOS ring oscillators. It is the physical feature that the system is constructed by simple CMOS elements. This system was simulated using many kinds of initial values. As a result, the characteristics of this system are shown as follows: The system wishes to make existing small amplitude parts. The existence of one small amplitude part needs about seven oscillators. Each small part hate closing. We consider that the chaotic characteristic makes changing the position of a small amplitude part. However, by the reason that one small amplitude part needs about seven oscillators and each part hates closing, in the case of many small amplitude parts we could not observed changing the position of a part. These results make studies of large coupled chaotic systems wide. Namely, these results contribute studies on VLSI realization and applications of large coupled chaotic systems.

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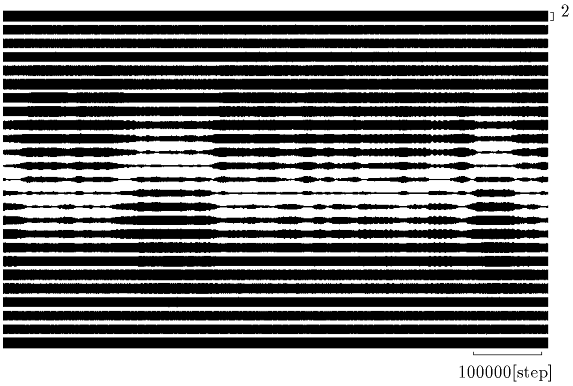

Figure 7: Simulated result in the case that one small amplitude part is observed.

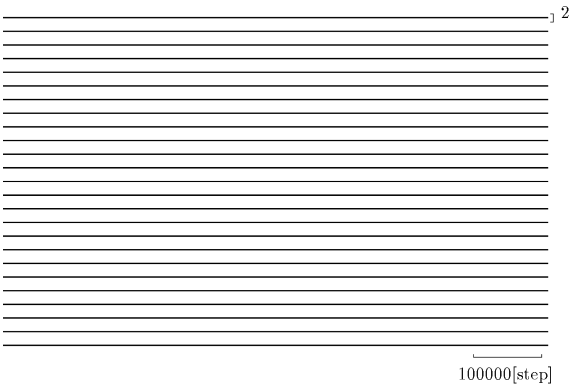

Figure 8: Simulated result in the case that all oscillators generate small amplitudes like Fig. 6 (b).

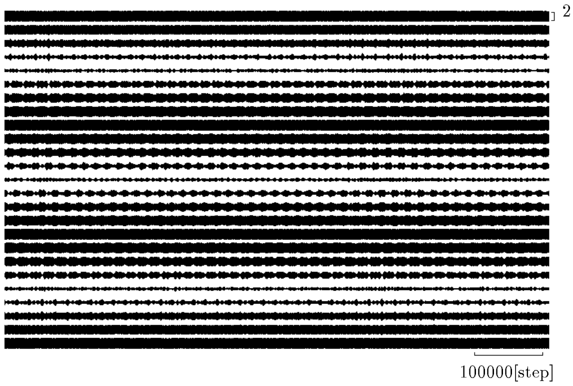

Figure 9: Simulated result in the case that three small amplitude parts are observed.

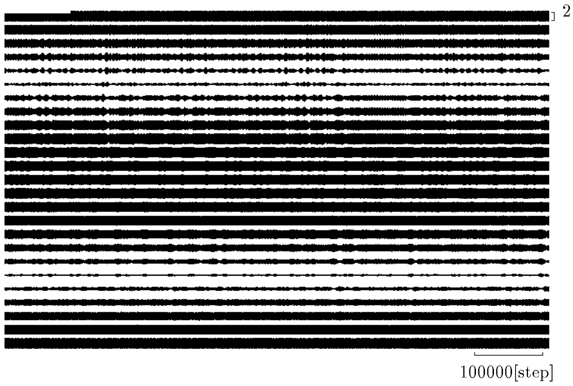

Figure 10: Simulated result in the case that two small amplitude parts moves irregulary.

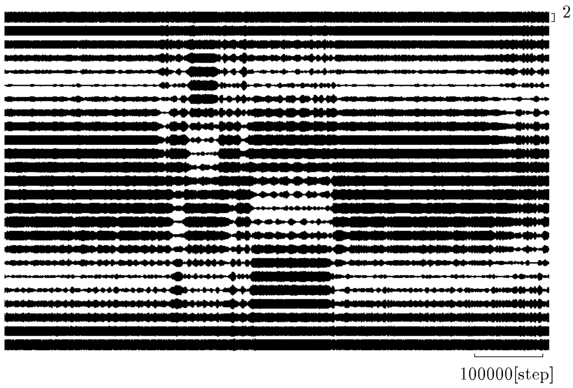

Figure 11: Simulated result in the case of observed two small amplitude parts.(continued of Fig. 10)