Statistical Analysis of Clustering in Globally Coupled Chaotic Circuits

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Abstract: In this study, a globally coupled system of chaotic circuits with small variations in their oscillation frequencies is investigated. In our previous study, by computer simulations, the generation of clustering and chaotic change of the cluster size are confirmed for five and thirty circuits case. Further, we confirmed that phenomenon from experimental results for five circuits case. In order to understand the phenomena correctly, we make detailed investigation on the statistical information of the clustering for thirty circuits case, such as survival time of a certain size of clusters.

1. Introduction

Clustering is one of the most interesting nonlinear phenomena observed from a large number of coupled chaotic systems. A lot of studies on the clustering have been carried out for discrete-time mathematical models [1][2]. However, there have been a few studies on clustering of continuous-time real physical systems such as electrical circuits.

In this study, we analyze clustering phenomenon observed from globally coupled chaotic circuits. The chaotic circuit used in this study is a simple three-dimensional autonomous circuit proposed by Mori *et al.* [3][4]. We consider the case that the chaotic circuits have small variations in their oscillation frequencies. In our previous study [5], by computer simulations and circuit experiments, the generation of clustering and chaotic change of the cluster size were confirmed. In order to understand the phenomena correctly, we make detailed investigation on the statistical information of the clustering, such as survival time of a certain size of clusters.

2. Circuit Model

Figure 1 shows the circuit model for the case that 5 chaotic circuits are coupled. Each chaotic circuit consists of three memory elements, one linear negative resistor and one nonlinear resistor of two diodes. The chaotic circuit is three-dimensional autonomous and generates chaotic attractor shown in Fig. 2. In our coupled system, chaotic circuits with the same circuit parameters except small variations in their oscillation frequencies are globally (namely all-to-all) coupled by resistors R.

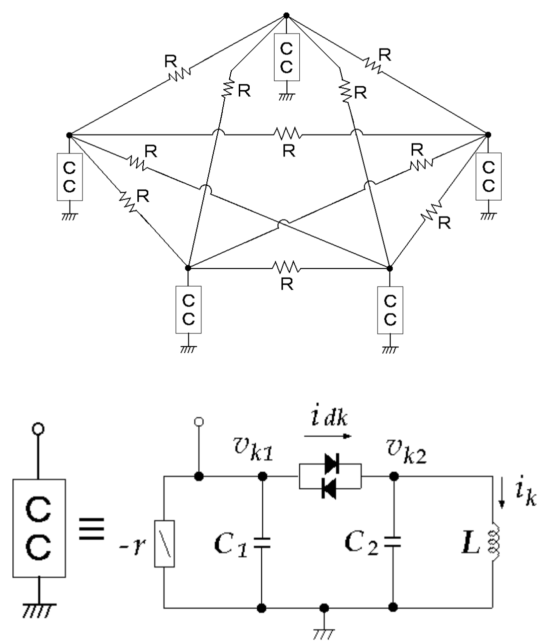


Figure 1: Circuit model.

At first, we approximate the $v-i$ characteristics of the nonlinear resistor of the diodes by the following piecewise linear function as shown in Fig. 3.

$$i_{dk} = \begin{cases} G_d(v_{k1} - v_{k2} - a) & (v_{k1} - v_{k2} > a) \\ 0 & (|v_{k1} - v_{k2}| \leq a) \\ G_d(v_{k1} - v_{k2} + a) & (v_{k1} - v_{k2} < -a). \end{cases} \quad (1)$$

By using the following variables and parameters,

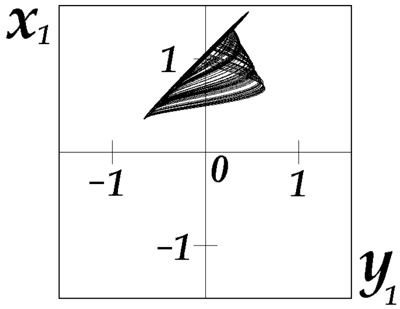


Figure 2: Example of chaotic attractors. $\alpha = 0.4$, $\beta = 0.5$ and $\gamma = 20$.

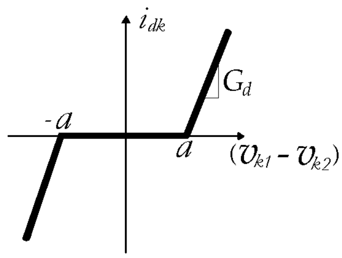


Figure 3: Approximation of the nonlinear resistor.

$$x_k = \frac{v_{k1}}{a}, \quad y_k = \frac{v_{k2}}{a}, \quad z_k = \frac{1}{a}\sqrt{\frac{L}{C_2}}i_k,$$
$$t = \sqrt{LC_2}\tau, \quad \text{“}.\text{''} = \frac{d}{d\tau}, \quad \alpha = \frac{C_2}{C_1}, \tag{2}$$
$$\beta = \frac{1}{r}\sqrt{\frac{L}{C_2}}, \quad \gamma = G_d\sqrt{\frac{L}{C_2}}, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},$$

the circuit equations for the case of N chaotic circuits are given as

$$\begin{cases} \dot{x}_k &= \alpha\delta\sum_{j=1}^{N}(x_j - x_k) + \alpha\beta x_k \\ &\quad -\alpha f(x_k - y_k) \\ \dot{y}_k &= f(x_k - y_k) - z_k \\ \dot{z}_k &= (1 + q_k)y_k \end{cases} \tag{3}$$

where $k = 1, 2, 3, \cdots N$ and q_k is the coefficients to give small variations of the oscillation frequencies. The nonlinear function $f(x_k - y_k)$ corresponds to the characteristics of the nonlinear resistor of the diodes and is described as

$$f(x_k - y_k) =$$
$$\begin{cases} \gamma(x_k - y_k - 1) & (x_k - y_k > 1) \\ 0 & (|x_k - y_k| \leq 1) \\ \gamma(x_k - y_k + 1) & (x_k - y_k < -1). \end{cases} \tag{4}$$

3. Clustering

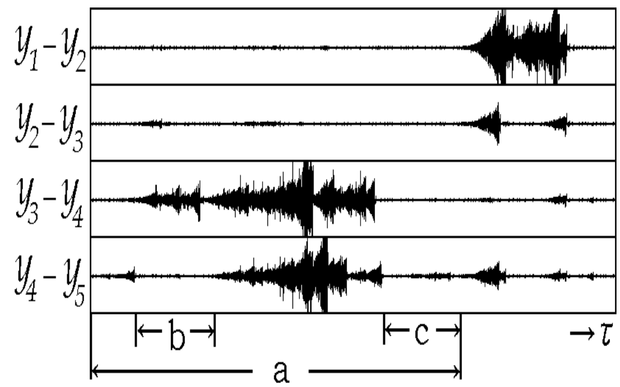


Figure 4: Computer simulated results for $N = 5$. $\alpha = 0.4$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.067$ and $q_k = 0.001(k - 1)$.

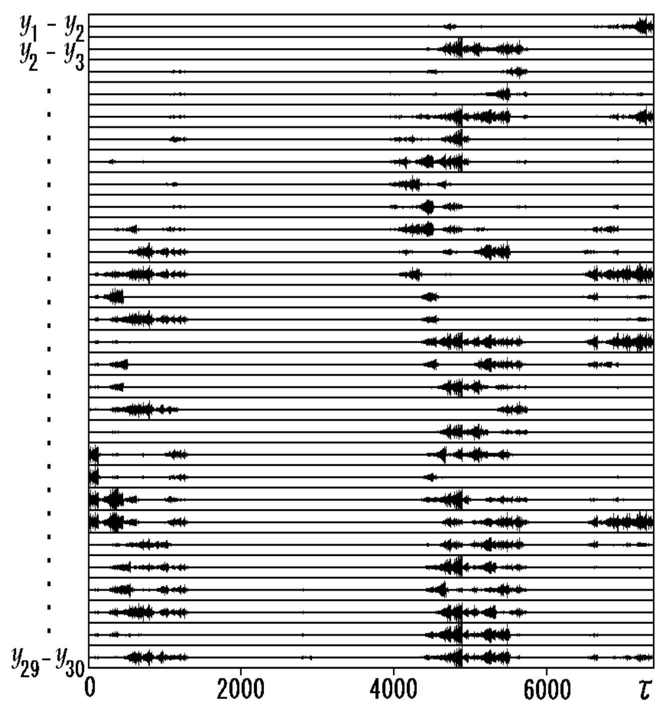


Figure 5: Computer simulated results for $N = 30$. $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0105$ and $q_k = 0.0003(k - 1)$.

In our previous study, we confirmed the generation of the clustering phenomenon and chaotic change of the cluster size such as Figs. 4 and 5. Figure 4 shows computer simulated results for the case of $N = 5$. In the figures, the vertical axes are the differences between the voltages of the two chaotic circuits. Namely, if the two chaotic circuits synchronize, the value of the graph should be almost zero. During the time interval $\leftarrow c \rightarrow$ in the figure, all of the values are almost zero. This means that all of the 5 chaotic circuits almost synchronize, namely the number of the clusters is 1. While, during $\leftarrow b \rightarrow$, only $y_3 - y_4$ oscillates with certain amplitude. This means that y_1, y_2 and y_3 almost synchronize and that y_4 and y_5 almost synchronize. In this case, the number of the clusters is 2. As time goes, the number of the clusters changes chaotically as $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ during $\leftarrow a \rightarrow$ in the figure.

As well as Fig. 4, we confirmed the clustering phenomenon from circuit experiments for the case of $N = 5$ and from computer simulations for the case of $N = 30$ which is shown in Fig. 5.

4. Statistical Analysis

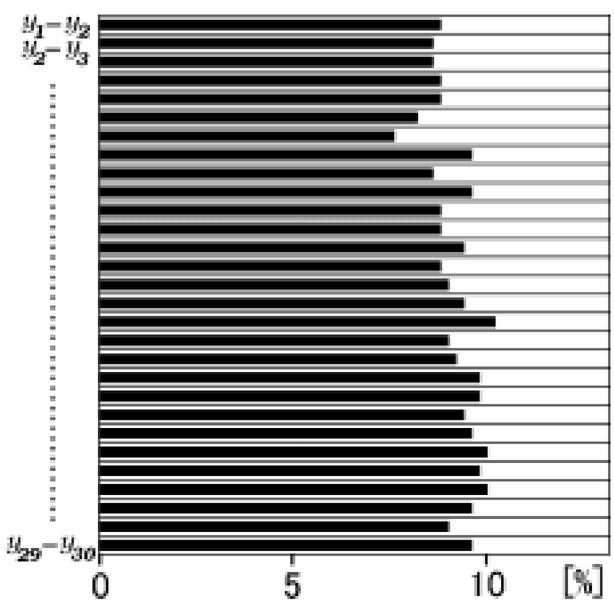


Figure 6: Probability of out-of-phase state between two circuits during $0 \leq \tau \leq 50{,}000$. $N = 30$, $D_{th} = 0.06$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0105$ and $q_k = 0.0003(k-1)$.

We confirmed from Fig. 6, in our previous study, that during any two chaotic circuits, the rate of out-of-phase is almost the same. Figure 6 shows how many times two chaotic circuits are out-of-phase during a certain time interval. We checked whether if the values of $|y_k - y_{k+1}|$ are larger than D_{th} or not for $0 \leq \tau \leq 50{,}000$. The horizontal axis is the percentage that the condition $|y_k - y_{k+1}| > D_{th}$ is satisfied.

Now, in this study, in order to understand the phenomena correctly, we make detailed investigation on the statistical information of the clustering for N=30. In statistical analysis, we regard out-of-phase as the case $|y_k - y_{k+1}| > 0.06$ as shown in Fig. 7. Furthermore, we choose parameters as follows: $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0105$.

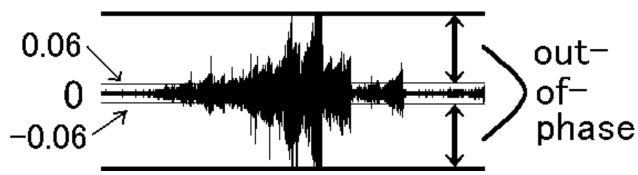


Figure 7: Decision of synchronization.

Figure 8 shows the distribution of the number of the clusters during a certain time interval. In this case, we checked in-phase or out-of-phase every τ=1 and total 1,000,000 times. Then, we counted the number of the cluster. The holizontal axis is the number of the cluster and vertical axis is the probability of the generation. In this parameter, we could not observe the clusters with the size of 27 or more.

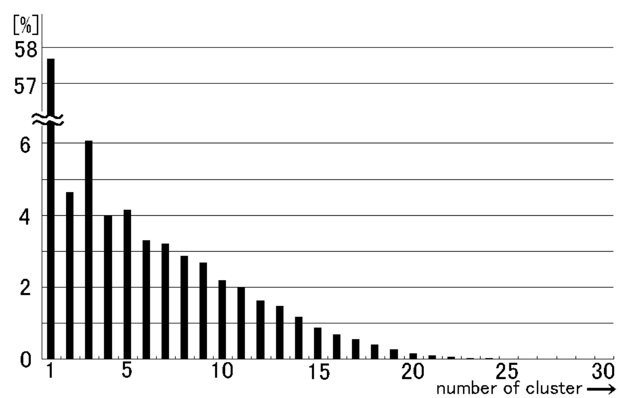


Figure 8: Distribution of the number of the clusters during a certain time interval.

Figure 9 shows the distribution of the survival time of the state such that all of the chaotic circuits are in-phase. We checked 10,000 all-in-phase synchronization states. In this figure, the numbers 1, 2, $\cdots$ indicate the survival time. Namely, 65% of all-in-phase synchronization disappear immediately. A part of "more than 5" is 5%, and the maximum survival time is $\tau = 4518$.

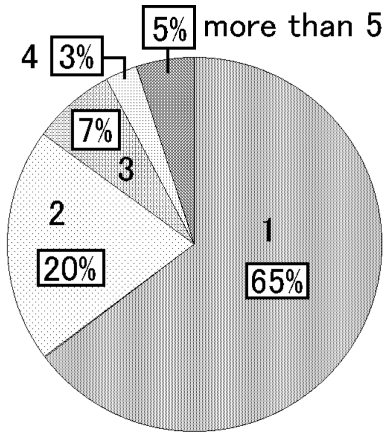
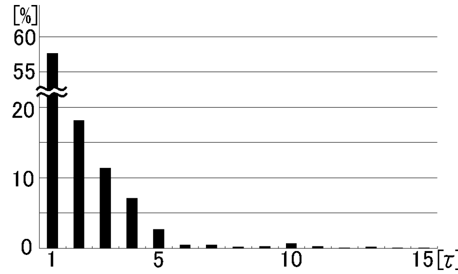


Figure 9: Distribution of the survival time of the state such that all of the chaotic circuits are in-phase.

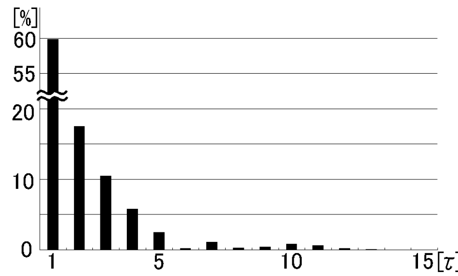
Finally, Fig. 10 shows the survival time of various sizes of the clusters. We confirmed that the larger the cluster size is, the shoter survival time is.

5. Conclusions

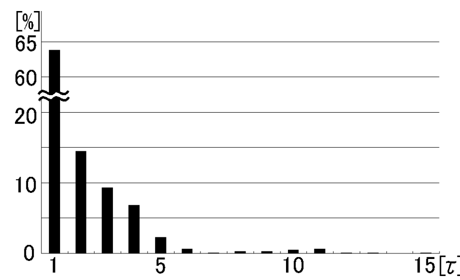
In this study, we analyzed clustering phenomenon observed from a large number of coupled chaotic systems. We made detailed investigation on the statistical information of the clustering.



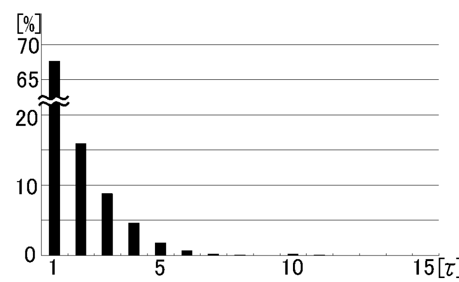
(a) Cluster size = 5.



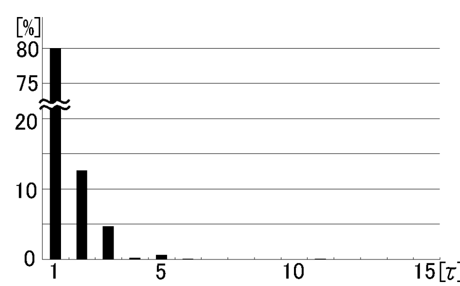
(b) Cluster size = 10.



(c) Cluster size = 15.



(d) Cluster size = 20.



(e) Cluster size = 25.

Figure 10: Distribution of the survival time of various sizes of the clusters.

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