Solving Ability of Hopfield Neural Network with Chaotic Noise and Burst Noise for Quadratic Assignment Problem

Yoko UWATE†, Yoshifumi NISHIO†, Tetsushi UETA†
Tohru KAWABE‡ and Tohru IKEGUCHI§

† Tokushima University, Japan
‡ Tsukuba University, Japan
§ Saitama University, Japan

Abstract— Solving combinatorial optimization problems is one of the important applications of the neural network. Many researchers have reported that exploiting chaos achieves good solving ability. However, the reason of the good effect of chaos has not been clarified yet.

In this article, intermittent chaos noise near three-periodic window and burst noise generated by the Gilbert model are applied to the Hopfield neural network for quadratic assignment problem. By computer simulations we confirm that the burst noise generated by the Gilbert model is effective to solve the quadratic assignment problem and we can say that the existence of the laminar part and the burst part is one reason of the good performance of the Hopfield NN with chaos noise.

I. Introduction

Solving combinatorial optimization problem is one of the important applications of the neural network (abbr. NN). Recently many researchers suggested that chaotic noise is more effective than stochastic one for solving the traveling salesman problem (abbr. TSP) with the Hopfield NN [1]. Hayakawa and Sawada pointed out that intermittent chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded that the good result might be obtained by some properties of the chaotic noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noise, and they concluded that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with similar autocorrelation [3]. In order to understand the reason of the good performance of the Hopfield NN with chaotic noise, we investigated their solving abilities with the burst noise generated by the Gilbert model [4] for several kinds of the traveling salesman problem [5]-[7].

In this article, intermittent chaos noise near three-periodic window and burst noise generated by the Gilbert model are applied to the Hopfield NN for quadratic assignment problem (abbr. QAP) said to be much more difficult to solve than TSP. By computer simulations we confirm that the burst noise generated by the Gilbert model is also effective to solve QAP and we can say that the existence of the laminar part and the burst part is one reason of the good performance of the Hopfield NN with chaotic noise.

II. Solving QAP with Hopfield NN

Various methods are proposed for solving the QAP which is one of the NP-hard combinatorial optimization problems. The QAP is expressed as follow: given two matrices, distance matrix C and flow matrix D, and find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in Eq. (1).

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \tag{1}$$

where C_{ij} and D_{ij} are the (i,j)-th elements of C and D, respectively, $\mathbf{p}(i)$ is the i th element of vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is to find an arrangement of the factories to make a cost the minimum. The cost is given by the distance between the cities and the flow of the products between the factories (Fig. 1). Other examples are the placement of logical modules in a IC chip, the distribution of medical services in large hospital.

Because the QAP is very difficult, it is almost impossible to solve the optimum solutions in larger problems. The largest problem which is solved by deterministic methods may be only 20 in recent study. Further, computation time is very long to obtain the exact

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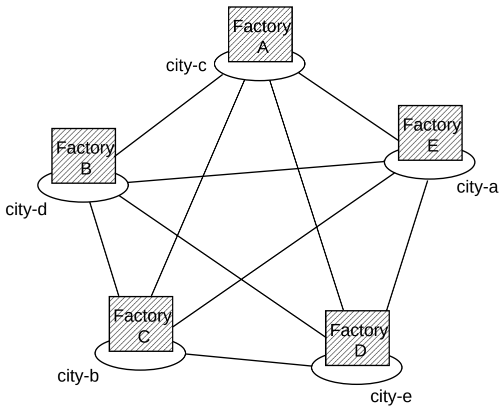

Figure 1: QAP model.

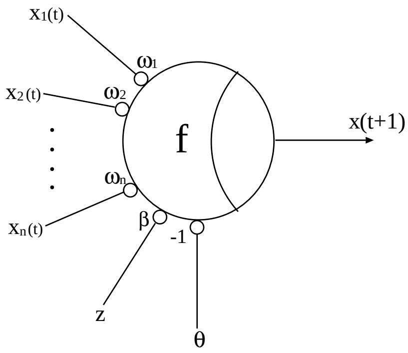

Figure 2: A neuron model.

optimum solutions. Therefore, it is usual to develop heuristic methods which search near optimal solutions in reasonable time.

For solving N-element QAP by Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i,j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \tag{2}$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by:

$$\begin{aligned} w_{im;jn} &= -2\Big\{ A(1-\delta_{mn})\delta_{ij} \\ &\quad + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q} \Big\} \qquad (3) \\ \theta_{im} &= A + B \end{aligned}$$

where A and B are positive constant, and δ_{ij} is Kronecker's delta. The state of $N \times N$ neurons are unsynchronously updated due to the following difference equation:

$$\begin{aligned} x_{im}(t+1) = f\Bigg(&\sum_{j,n=1}^{N} w_{im;jn} x_{im}(t) x_{jn}(t) \\ &- \theta_{im} + \beta z_{im}(t) \Bigg) \end{aligned} \tag{4}$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{1 + \exp\left(-\frac{x}{\varepsilon}\right)} \tag{5}$$

z_{im} is additional noise, and β limits amplitude of the noise. Figure 2 shows a conceptual neuron model for this NN.

III. Chaotic and Burst Noises

In this section, we describe two kinds of noise injected to the Hopfield NN.

At first, the logistic map is used to generate chaotic noise:

$$z_{im}(t+1) = \alpha z_{im}(t)(1 - z_{im}(t)). \tag{6}$$

Varying parameter α, Eq. (6) behaves chaotically via a period-doubling cascade. We use intermittent chaos near the three-periodic window obtained from Eq. (6), because in some literatures such chaotic bursts are confirmed to be more effective than fully developed chaotic sequence.

Figure 3 shows the time series of the intermittent chaos noise near the three-periodic window. In this figure, the amplitude of this sequence is normalized by the average and the standard deviation from the original chaotic burst obtained from Eq. (6) with α=3.8268.

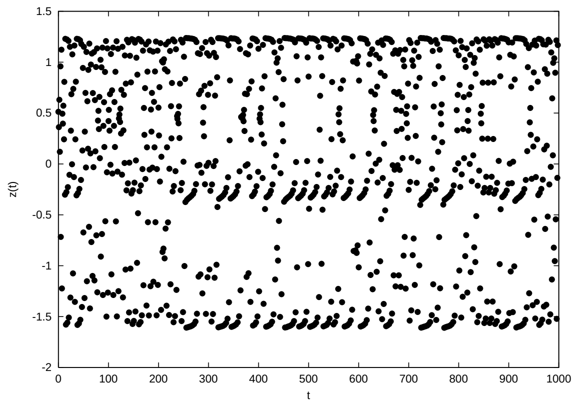

Figure 3: Intermittent chaos noise near three-periodic window. α=3.8268.

A. Burst Noise

In order to simulate the chaotic bursts by stochastic noise, we use the two-state Gilbert model as shown in Fig. 4. Gilbert model is sometimes used for characterizing error-generating mechanisms in digital communication channels. One state corresponds to the burst part and generates uniformly distributed noise, while the other corresponds to the laminar part and generates three-periodic sequence imitating the three-periodic window of the logistic map. We denote the states belonging each part as S_1 and S_2, respectively. Then the transition probabilities are given by $P(S_i|S_j)$, $i, j = 1, 2$, where $P(S_i|S_i) = 1 - P(S_i|S_j)$.

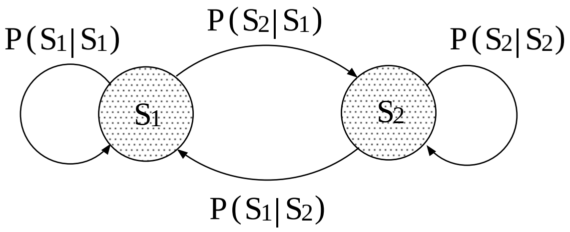

Figure 4: Two-state Gilbert model.

Figure 5 shows the time series of the burst noise obtained by the two-state Gilbert model with $P(S_1|S_1) = P(S_2|S_2) = 0.88$.

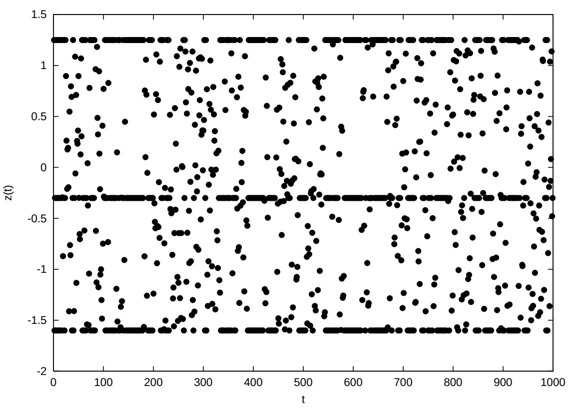

Figure 5: Burst noise imitating intermittent chaos noise. $P(S_1|S_1) = P(S_2|S_2) = 0.88$.

IV. Simulated Results

In this section, the simulated results of the Hopfield NN with the chaotic noise and the burst noise for 12-element QAP are shown. For the comparison, we also carried out computer simulation for the case that simple uniform noise in Fig. 6 is injected to the Hopfield NN.

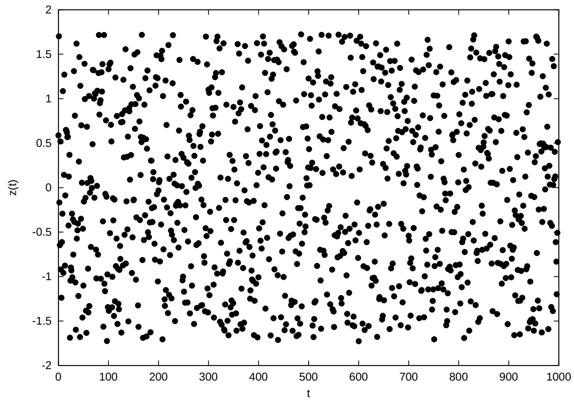

Figure 6: Uniform noise.

The problem used here was chosen from the site QAPLIB [7] named "Nug12." The global minimum of this target problem is known as 578. The distance and the flow matrices are given as

Distance:

$$
\begin{pmatrix}
0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 5 \\
1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 & 3 & 4 \\
2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 & 5 & 4 & 3 & 2 \\
1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 \\
2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 \\
3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 \\
4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 \\
2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 \\
3 & 2 & 3 & 4 & 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 \\
4 & 3 & 2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\
5 & 4 & 3 & 2 & 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0
\end{pmatrix}
$$

Flow:

$$
\begin{pmatrix}
0 & 5 & 2 & 4 & 1 & 0 & 0 & 6 & 2 & 1 & 1 & 1 \\
5 & 0 & 3 & 0 & 2 & 2 & 2 & 0 & 4 & 5 & 0 & 0 \\
2 & 3 & 0 & 0 & 0 & 0 & 0 & 5 & 5 & 2 & 2 & 2 \\
4 & 0 & 0 & 0 & 5 & 2 & 2 & 10 & 0 & 0 & 5 & 5 \\
1 & 2 & 0 & 5 & 0 & 10 & 0 & 0 & 0 & 5 & 1 & 1 \\
0 & 2 & 0 & 2 & 10 & 0 & 5 & 1 & 1 & 5 & 4 & 0 \\
0 & 2 & 0 & 2 & 0 & 5 & 0 & 10 & 5 & 2 & 3 & 3 \\
6 & 0 & 5 & 10 & 0 & 1 & 10 & 0 & 0 & 0 & 5 & 0 \\
2 & 4 & 5 & 0 & 0 & 1 & 5 & 0 & 0 & 0 & 10 & 10 \\
1 & 5 & 2 & 0 & 5 & 5 & 2 & 0 & 0 & 0 & 5 & 0 \\
1 & 0 & 2 & 5 & 1 & 4 & 3 & 5 & 10 & 5 & 0 & 2 \\
1 & 0 & 2 & 5 & 1 & 0 & 3 & 0 & 10 & 0 & 2 & 0
\end{pmatrix}
$$

Since we know the global minimum in advance, we define the **Success** as that the NN finds the global minimum at least once during the defined iteration number $N_{iteration}$. We repeat this trial 100 times and count the number of the **Success** for the solving ability

Table 1: Solving abilities for 12-element QAP.

	Chaos			Burst			Uniform		
$N_{iteration}$	SA[%]	Average	$Error$[%]	SA[%]	Average	$Error$[%]	SA[%]	Average	$Error$[%]
1000	1	613.10	6.073	1	622.74	7.740	0	625.58	8.232
2000	2	605.44	4.747	1	613.86	6.204	0	617.56	6.844
3000	3	601.74	4.107	2	608.24	5.232	0	613.96	6.221
4000	3	599.84	3.779	2	605.30	4.723	0	611.02	5.713
5000	5	598.06	3.471	3	603.62	4.433	0	608.58	5.291
6000	6	596.98	3.284	3	602.84	4.298	0	607.52	5.107
7000	7	595.66	3.055	5	601.08	3.993	1	605.30	4.723
8000	7	595.18	2.972	6	599.96	3.799	1	604.62	4.606
9000	7	594.62	2.875	6	599.84	3.779	1	602.38	4.218
10000	8	594.08	2.782	7	598.92	3.619	1	602.12	4.173

SA defined as

$$SA = \frac{\text{Number of Success}}{\text{Number of Trials}} \times 100[\%]. \tag{7}$$

We also record the minimum cost found during the trial and use the **Average** of the minima for the evaluation. The **Error** is defined as

$$Error = \frac{\text{Average} - \text{Optimal Solution}}{\text{Optimal Solution}} \times 100[\%]. \tag{8}$$

The results are summarized in Table 1. The parameters of the Hopfield NN are fixed as $A = 0.9$, $B = 0.9$, $q = 140$, $\varepsilon = 0.02$ and the amplitude of the injected noise is fixed as $\beta = 0.6$. The results show that the chaotic noise and the burst noise have much better performance than the uniform noise. Furthermore, it is interesting to note that the burst noise generated by the Gilbert model has the similar performance to the chaotic noise. We consider that one reason of the good performance of the chaotic noise can be explained by the existence of the laminar part and the burst part.

V. Conclusions

We have investigated the solving abilities of the Hopfield NN with noises for QAP. We confirmed by the computer simulations that the burst noise achieved the good performance as the chaotic noise for QAP. Hence, we can say that the existence of the laminar part and the burst part is one reason of the good performance of the Hopfield NN with chaotic burst.

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