RESPONSE OF COUPLED CHAOTIC CIRCUITS TO SINUSOIDAL INPUT SIGNAL

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ABSTRACT

In this study, synchronization state of the coupled chaotic circuits is investigated when a sinusoidal input signal is added by both of computer simulations and circuit experiments. It is confirmed that the synchronization state changes by the values of the amplitude and the angular frequency of the input signal.

1. INTRODUCTION

Studies on engineering applications of chaos, such as chaos communication systems and chaos cryptosystems, attract many researchers' attentions. Since, theoretically speaking, chaotic signals possess infinite information, it could be possible to create novel engineering systems with great advantages. However, at this moment, there have been few systems with some kinds of advantages over conventional engineering systems. Hence, basic researches on chaotic phenomena with future engineering applications in mind are still quite important.

In this study, we investigate synchronization phenomena of two chaotic circuits coupled by a resistor when an input signal is added. Because the circuit tends to minimize the current through the coupling resistor in such a coupled system [1]-[3], synchronization phenomena change to compensate the effect of the input signal. More concretely, we use a sinusoidal signal with the averaged amplitude and the averaged frequency of the chaotic signals as a standard input signal. We investigate the synchronization phenomena when the amplitude or the frequency of the sinusoidal input signal is varied. The results suggest that coupled chaotic circuits could be utilized to extract some kinds of features of unknown input signals.

2. CIRCUIT MODEL

Figure 1 shows the circuit model. The circuit in Fig. 1(a) is a three-dimensional autonomous circuit generating chaos shown in Fig. 2 and was proposed by Inaba and Mori [4]. In the circuit in Fig. 1(b), two Inaba's circuits are coupled by one coupling resistor R and an input signal is added to the coupling resistor as a current I_3.

The circuit equations governing the circuit in Fig. 1(b) are given as Eq. (1). We assume the $i-v$ characteristics of the diodes in the circuit by the two-segment piecewise linear function as Eq. (2).

$$\begin{aligned} L_1\frac{dI_k}{dt} &= ri_k + rI_k - v_k - R\sum_{j=1}^{3} I_j \\ L_2\frac{di_k}{dt} &= ri_k + rI_k - v_k - v_d(i_k) \\ C\frac{dv_k}{dt} &= i_k + I_k \\ &(k=1,2) \end{aligned} \tag{1}$$

$$v_d(i_k) = 0.5(r_d i_k + E - |r_d i_k - E|) \tag{2}$$

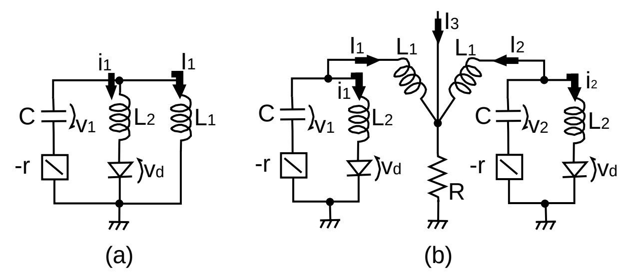

Fig. 1. Circuit Model.

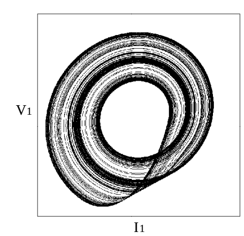

Fig. 2. An example of chaotic attractors. $\alpha = 7.0, \beta = 0.14$ and $\delta = 100$.

By using the variables and the parameters in Eq. (3), the circuit equations in Eq. (1) are normalized as Eq. (4).

$$\begin{aligned} &I_k = \sqrt{\frac{C}{L_1}}Ex_k, \quad i_k = \sqrt{\frac{C}{L_1}}Ey_k, \\ &v_k = Ez_k, \quad t = \sqrt{L_1C}\tau, \quad \alpha = \frac{L_1}{L_2}, \\ &\beta = r\sqrt{\frac{C}{L_1}}, \quad \gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}} \end{aligned} \tag{3}$$

$$\frac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k - \gamma \sum_{j=1}^{3} x_j$$

$$\frac{dy_k}{d\tau} = \alpha \{\beta(x_k + y_k) - z_k - f_d(y_k)\} \quad (4)$$

$$\frac{dz_k}{d\tau} = x_k + y_k$$

$$(k = 1, 2)$$

It is possible to consider various kinds of signals as input signal x_3 in future, for example, simple signals represented by basic mathematical functions, chaotic signals obtained from other chaotic circuits, audio/visual signals in real engineering systems and so on. In this study, as the first step toward the total understanding of the response of the coupled circuits to various input signals, we consider the case that a simple sinusoidal signal is input. Namely, we use the following sinusoidal function with the normalized amplitude A_m and the normalized angular frequency ω as the input x_3.

$$x_3(\tau) = A_m \sin \omega\tau \quad (5)$$

3. RESPONSE TO SINUSOIDAL INPUT

Figure 3 shows the computer simulated results of Eq. (4) using the fourth-order Runge-Kutta method. At first, in order to understand the typical response of the coupled system, we fix the angular frequency of the input signal as $\omega = 1.037$ which is close to the value of the averaged angular frequency of the chaotic signals obtained from the chaotic circuits. Further, the amplitude of the input signal A_m is chosen using the averaged amplitude of the chaotic signals as a standard value.

Figure 3(a) shows the result for the case of $A_m = 0$, namely no input signal is added. The two chaotic signals almost synchronize at anti-phase. Figure 3(b) shows the result for the case of $A_m = 1.17$, namely the sinusoidal signal with the averaged amplitude and the averaged angular frequency of the chaotic signal is input. In this case, the two chaotic signal x_1 and x_2 and the input signal x_3 almost synchronize at three-phase. Figure 3(c) shows the result for the case of $A_m = 2.34$, namely the sinusoidal signal with about doubled amplitude of the chaotic signal is input. Even in this case, the shapes of the chaotic attractors observed in the two chaotic circuits are not influenced by the input signal. And the two chaotic signals almost synchronize at in-phase and they almost synchronize to the input signal at anti-phase.

Next, we show the circuit experimental results in Fig. 4. We can observe similar results to the computer simulated results.

4. DETAILED INVESTIGATION

In the previous section, we fixed the angular frequency of the input signal as the averaged angular frequency of the chaotic signal and investigated the response of the coupled

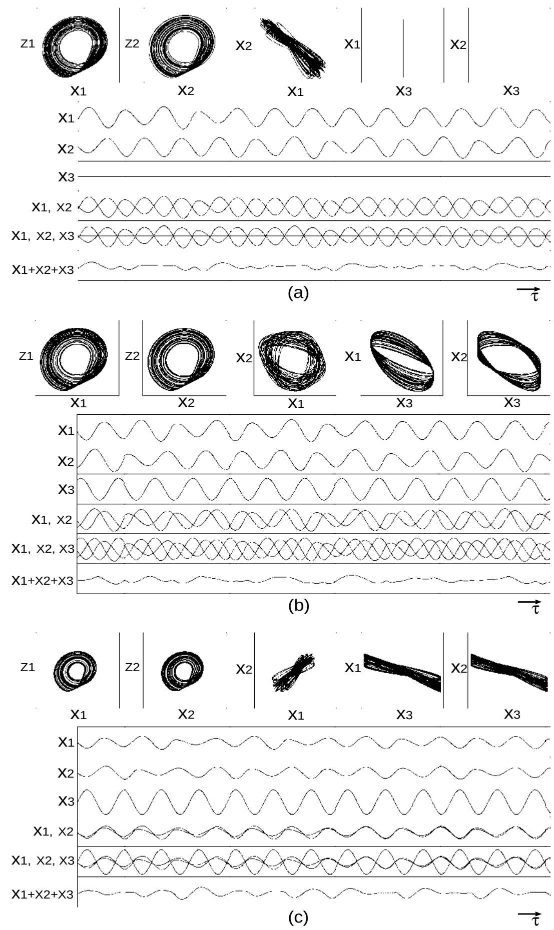

Fig. 3. Computer simulated results. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$ and $\omega = 1.037$. (a) $A_m = 0$. (b) $A_m = 1.17$. (c) $A_m = 2.34$.

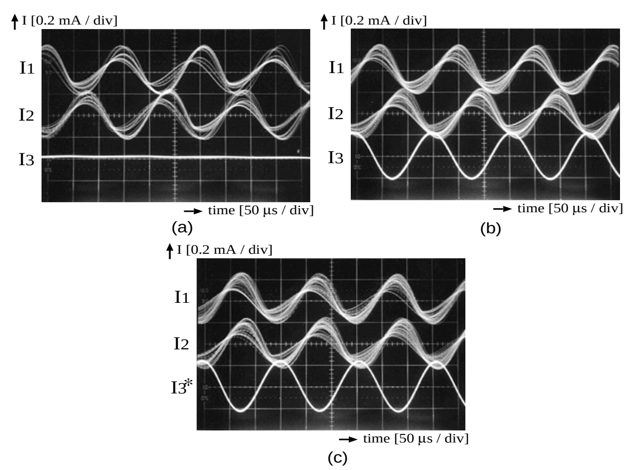

Fig. 4. Circuit experimental results. L_1=100mH, L_2=10mH, C=33nF and R=320Ω. (a) $A_m = 0$ (no input signal). (b) $A_m = 1.17$ (averaged amplitude of the chaotic signals). (c) $A_m = 2.34$ (doubled amplitude of the chaotic signals). *In (c) vertical scale of I_3 is doubled.

system only for the cases of the three typical values of the amplitude of the input signal. In this section these values are varied continuously.

In order to investigate the synchronization in detail, we define the phase difference of the two chaotic signals. By using the time when the chaotic signals take extrema as shown in Fig. 5, the phase difference θ[deg] is defined as follows.

$$\theta = \frac{\tau_{20} - \tau_{10}}{\tau_{11} - \tau_{10}} \times 360 \tag{6}$$

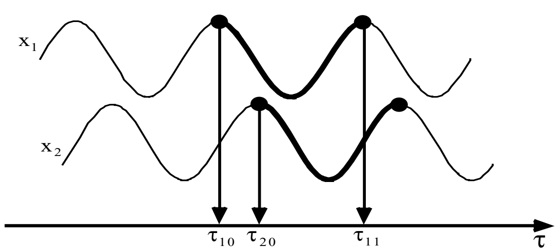

Fig. 5. Definition of the phase difference θ.

4.1. Variation of Amplitude

Figure 6(a) shows the results for the case that the angular frequency of the input is fixed as $\omega = 1.037$ and the amplitude A_m is varied continuously. In the figure, 1500 data of θ are plotted after the system settles to the steady state for each A_m. As observed in the previous section, θ is almost around 180° (anti-phase) for $A_m = 0$, θ is almost around 120° (three-phase) for $A_m = 1.17$ and θ is almost around 0° (in-phase) for $A_m = 2.34$. Further, between these typical values of the amplitude, the phase difference continuously varies.

Figure 6(b) shows the average of θ in Fig. 6(a). Because the fluctuation caused by chaotic feature is averaged, we can see the variation of θ more clearly.

The results in Fig. 6 suggest that it is possible to distinguish the amplitude of the input sinusoidal signal by the phase difference between the two chaotic signals.

4.2. Variation of Angular Frequency

Next, we fix the amplitude of the input signal as $A_m = 1.17$ and vary the angular frequency ω. Figure 7(a) shows 1500 data of θ and Fig. 7(b) shows their average. In this case the coupled system responds to the input clearly only for very limited values of angular frequency. Namely, as we can see from Fig. 7(b), the two chaotic signals almost synchronize at anti-phase for wide parameter region and they synchronize to the input signal at three-phase only around $\omega = 1.037$.

Figures 7(c) and (d) are the magnifications of the part of Figs. 7(a) and (b), respectively. From the figures we can see that the phase difference between the two chaotic signals varies from about 90° to about 120° when ω is varied around 1.037.

The results in Fig. 7 suggest that the coupled system could be useful as a filter to distinguish the frequency of the input sinusoidal signal.

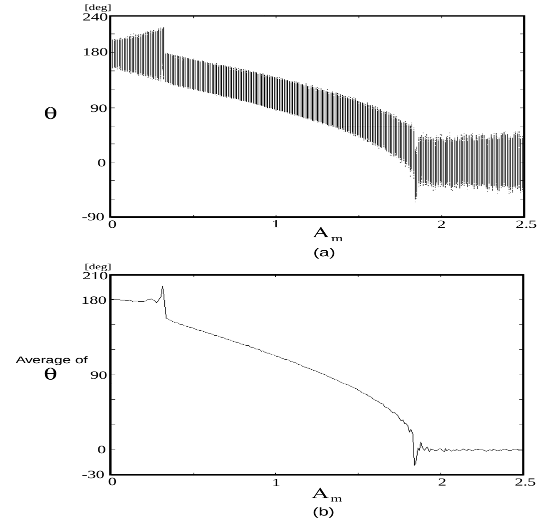

Fig. 6. Phase difference θ as varying A_m. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$ and $\omega = 1.037$. (a) θ. (b) Average of θ.

4.3. Variation of Amplitude and Angular Frequency

At last, Fig. 8 shows the results obtained by varying both of the amplitude A_m and the angular frequency ω. Figure 8(a) and (b) shows the results for the cases of $\gamma = 0.03$ and $\gamma = 0.06$, respectively. We can see that the parameter region where the coupled system respond to the input signal is changed by the value of the coupling strength.

5. RESPONSE TO RECTANGULAR INPUT

Additionally, we confirm the response of the circuits to rectangular input. Figure 9 shows the computer simulated results when the rectangular signal with the same frequency and amplitude as the sinusoidal signal in Fig. 3, is added to the circuits as I_3. The two chaotic signals almost synchronize similarly. We show the circuit experimental results in Fig. 10.

6. CONCLUSIONS

In this study, we have investigated the synchronization state of the coupled chaotic circuits when an sinusoidal input signal was added by both of computer simulations and circuit experiments. We confirmed that the synchronization state changed by the values of the amplitude and the angular frequency of the input signal.

The results of this study suggest that the coupled chaotic circuits could be useful to distinguish the sinusoidal input signals. Our future researches are to investigate the response of the coupled system to more complicated input signals and to utilize the system to extract some kinds of important features of unknown input signals.

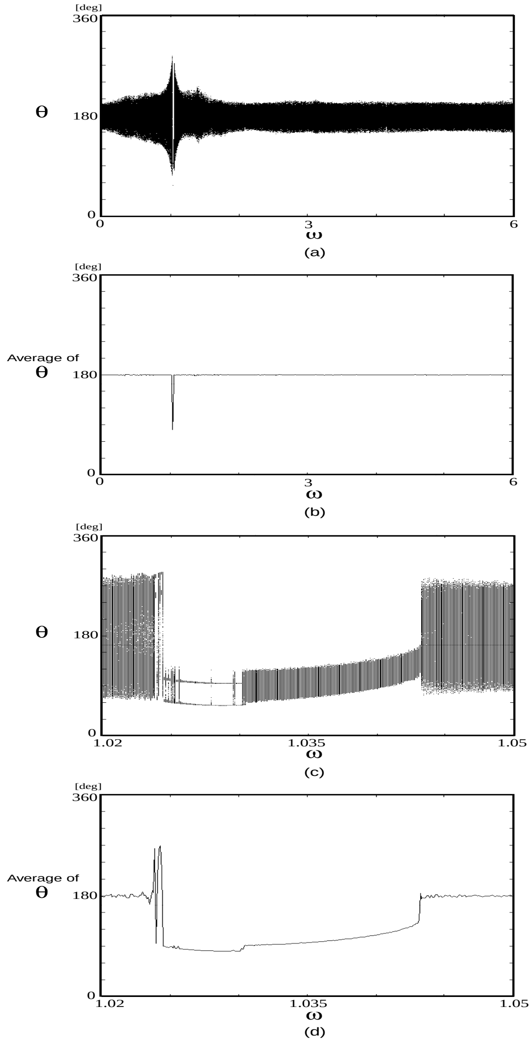

Fig. 7. Phase difference θ as varying ω. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$ and $A_m = 1.17$. (a) θ. (b) Average of θ. (c) Magnification of (a). (d) Magnification of (b).

7. REFERENCES

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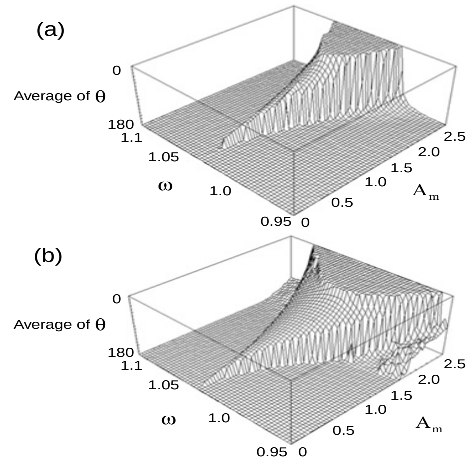

Fig. 8. Phase difference θ as varying ω and A_m. $\alpha = 7.0$, $\beta = 0.14$ and $\delta = 100$. (a) $\gamma = 0.03$. (b) $\gamma = 0.06$.

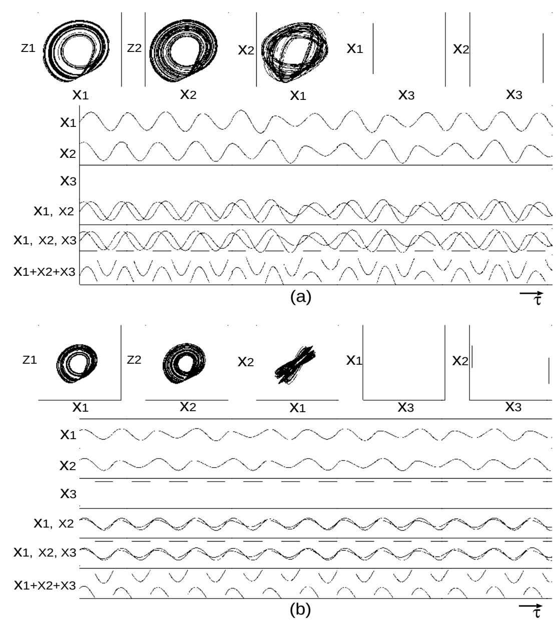

Fig. 9. Computer simulated results. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, and $\delta = 100$. (a) $A_m = 1.17$. (b) $A_m = 2.34$.

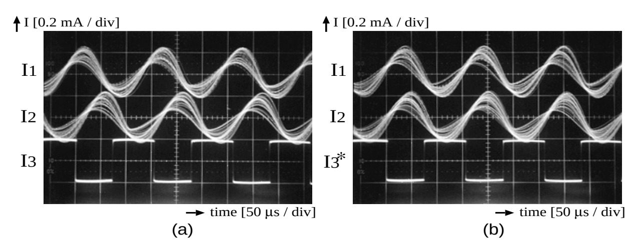

Fig. 10. Circuit experimental results. L_1=100mH, L_2=10mH, C=33nF and R=320Ω. (a) $A_m = 1.17$ (averaged amplitude of the chaotic signals). (b) $A_m = 2.34$ (doubled amplitude of the chaotic signals). *In (c) vertical scale of I_3 is doubled.