Clustering in Globally Coupled System of Chaotic Circuits

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Abstract: In this study, a globally coupled system of chaotic circuits with small variations in their oscillation frequencies is proposed. By computer simulations and circuit experiments, the generation of clustering and chaotic change of the cluster size are confirmed for five circuits case. Further, statistical analysis is carried out for thirty circuits case.

1. Introduction

Clustering is one of the most interesting nonlinear phenomena observed from a large number of coupled chaotic systems. A lot of studies on the clustering have been carried out for discrete-time mathematical models [1][2]. However, there have been a few studies on clustering of continuous-time real physical systems such as electrical circuits.

In this study we propose a globally coupled system of chaotic circuits and investigate clustering phenomenon observed from the circuits. The chaotic circuit used in this study is a simple three-dimensional autonomous circuit proposed by Shinriki and Mori [3][4]. We consider the case that the chaotic circuits have small variations in their oscillation frequencies. By computer simulations and circuit experiments, the generation of clustering and chaotic change of the cluster size are confirmed for five circuits case. Further, statistical analysis is carried out for thirty circuits case.

2. Circuit Model

Figure 1 shows the circuit model for the case that 5 chaotic circuits are coupled. Each chaotic circuit consists of three memory elements, one linear negative resistor and one nonlinear resistor of two diodes. The chaotic circuit is three-dimensional autonomous and generates chaotic attractor shown in Fig. 2. In our coupled system, chaotic circuits with the same circuit parameters except small variations in their oscillation frequencies are globally (namely all-to-all) coupled by resistors R.

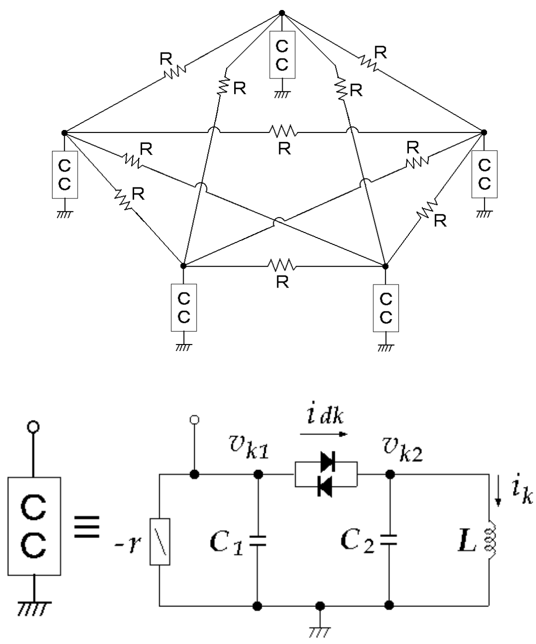

Figure 1: Circuit model.

At first, we approximate the $v-i$ characteristics of the nonlinear resistor of the diodes by the following piecewise linear function as shown in Fig. 3.

$$i_{dk} = \begin{cases} G_d(v_{k1} - v_{k2} - a) & (v_{k1} - v_{k2} > a) \\ 0 & (|v_{k1} - v_{k2}| \leq a) \\ G_d(v_{k1} - v_{k2} + a) & (v_{k1} - v_{k2} < -a). \end{cases} \quad (1)$$

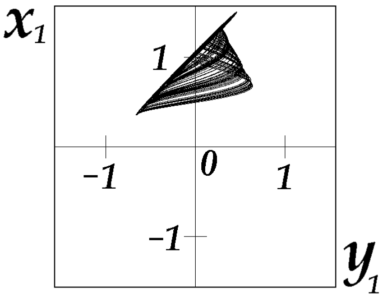

Figure 2: Example of chaotic attractors. $\alpha = 0.4$, $\beta = 0.5$ and $\gamma = 20$.

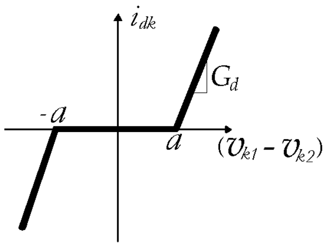

Figure 3: Approximation of the nonlinear resistor.

By using the following variables and parameters,

$$x_k = \frac{v_{k1}}{a}, \quad y_k = \frac{v_{k2}}{a}, \quad z_k = \frac{1}{a}\sqrt{\frac{L}{C_2}}i_k,$$
$$t = \sqrt{LC_2}\tau, \quad \text{“}\cdot\text{''} = \frac{d}{d\tau}, \quad \alpha = \frac{C_2}{C_1}, \tag{2}$$
$$\beta = \frac{1}{r}\sqrt{\frac{L}{C_2}}, \quad \gamma = G_d\sqrt{\frac{L}{C_2}}, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},$$

the circuit equations for the case of N chaotic circuits are given as

$$\left\{ \begin{array}{rcl} \dot{x}_k &=& \alpha\delta \sum_{j=1}^{N}(x_j - x_k) + \alpha\beta x_k \\ && -\alpha f(x_k - y_k) \\ \dot{y}_k &=& f(x_k - y_k) - z_k \\ \dot{z}_k &=& (1 + q_k)y_k \end{array} \right. \tag{3}$$

where $k = 1, 2, 3, \cdots N$ and q_k is the coefficients to give small variations of the oscillation frequencies. The nonlinear function $f(x_k - y_k)$ corresponds to the characteristics of the nonlinear resistor of the diodes and is described as

$$f(x_k - y_k) =$$
$$\left\{ \begin{array}{ll} \gamma(x_k - y_k - 1) & (x_k - y_k > 1) \\ 0 & (|x_k - y_k| \leq 1) \\ \gamma(x_k - y_k + 1) & (x_k - y_k < -1). \end{array} \right. \tag{4}$$

3. Clustering

Figure 4 shows computer simulated results for the case of $N = 5$. In the figures, the vertical axes are the differences between the voltages of the two chaotic circuits. Namely, if the two chaotic circuits synchronize, the value of the graph should be almost zero. During the time interval $\leftarrow c \rightarrow$ in the figure, all of the values are almost zero. This means that all of the 5 chaotic circuits almost synchronize, namely the number of the clusters is 1. While, during $\leftarrow b \rightarrow$, only $y_3 - y_4$ oscillates with certain amplitude. This means that y_1, y_2 and y_3 almost synchronize and that y_4 and y_5 almost synchronize. In this case, the number of the clusters is 2. As time goes, the number of the clusters changes chaotically as $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ during $\leftarrow a \rightarrow$ in the figure. Namely, we confirm the generation of the clustering phenomenon and chaotic change of the cluster size.

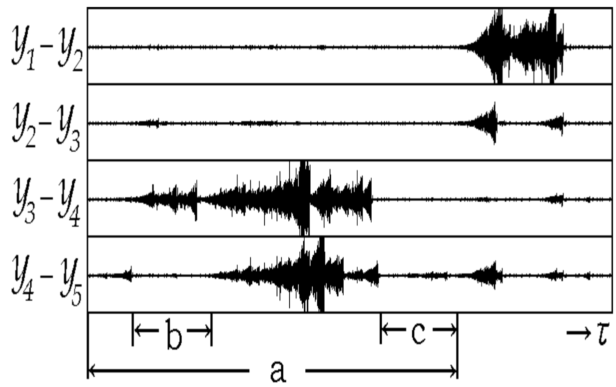

Figure 4: Computer simulated results for $N = 5$. $\alpha = 0.4$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.067$ and $q_k = 0.001(k - 1)$.

Figure 5 shows circuit experimental results for the case of $N = 5$. In the figures, the vertical axes are the differences between the voltages of the two chaotic circuits and the horizontal axis is the time. we confirm the generation of the clustering phenomenon and chaotic change of the cluster size as well as Fig. 4. In the circuit experiments, we use an operational amplifier to make the linear negative resistor.

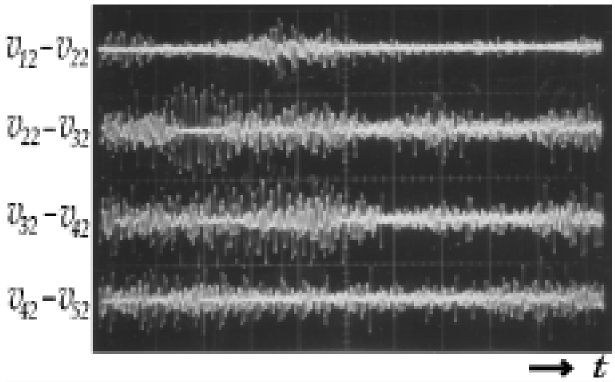

Figure 5: Circuit experimental results for $N = 5$. vertical: 0.5V/div, holizontal: 0.5msec/div, $C_1 =$ 22nF, $C_2 = 4.7$nF, $L = 5$mH and $R = 8.2$kΩ.

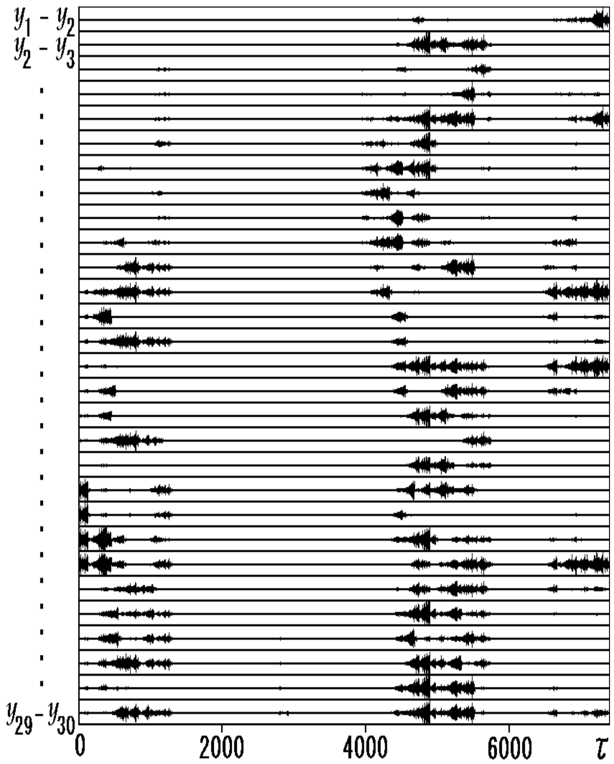

Figure 6: Computer simulated results for $N = 30$. $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0105$ and $q_k = 0.0003(k-1)$.

Next, we show the computer simulated result for the case of $N = 30$ in Fig. 6. We can also confirm the generation of the clustering phenomenon and chaotic change of the cluster size.

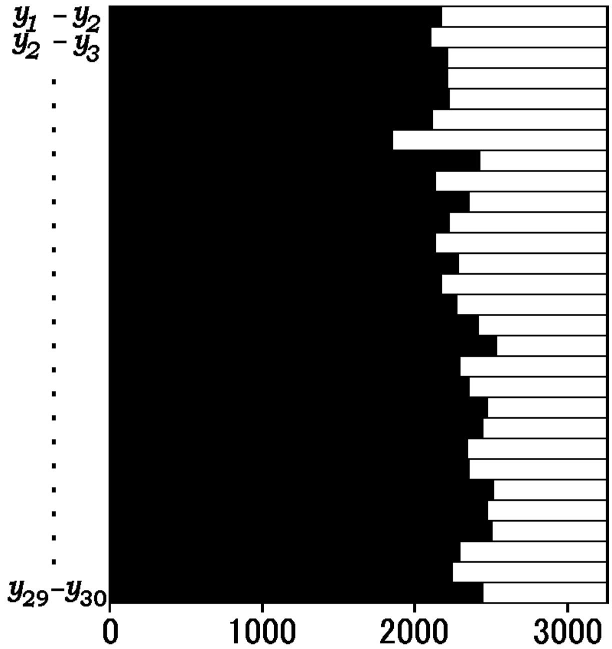

Figure 7: Number of out-of-phase state between two circuits during 50000. $N = 30$, $D_{th} = 0.06$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0105$ and $q_k = 0.0003(k-1)$.

4. Statistical Analysis

In this section, we analyze the clustering phenomenon observed from the 30 coupled chaotic circuits.

Figure 7 shows how many times two chaotic circuits are out-of-phase during a certain time interval. We checked whether if the values of $y_k - y_{k+1}$ are larger than D_{th} or not for 50000 different time instant. The horizontal axis is the number that the condition $y_k - y_{k+1} > D_{th}$ is satisfied. As shown in the figure, the distribution seems to be uniform if the time interval goes infinity. This means that there are no special boundaries to separate the circuits to clusters.

Figure 8 shows the distribution of the number of the clusters during a certain time interval. We decide that the two circuits are in the same cluster if the value of $y_k - y_{k+1}$ is smaller than D_{th}. The horizontal axis is the number of the cluster and the vertical axis is the distribution density of 50000 checks in total. As increasing the coupling strength δ, the distribution changes and the probability that the number of the cluster is 1, namely full synchronization, becomes quite large.

5. Conclusions

In this study, we have proposed a globally coupled system of chaotic circuits with small variations in their oscillation frequencies. We confirmed that the generation of clustering and chaotic change of the cluster size for five and thirty circuits cases. In order to understand the phenomena correctly, more detailed investigation should be carried out.

References

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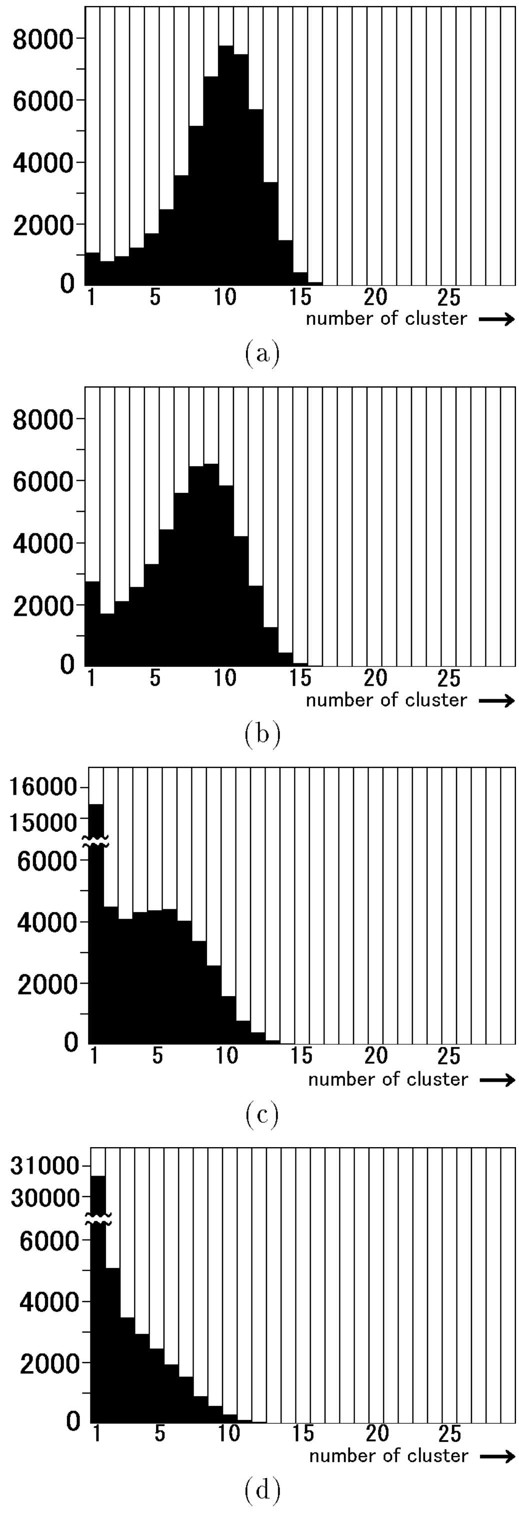

Figure 8: Distribution of the number of the clusters for different coupling strength. $N = 30$, $D_{th} = 0.06$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 20$ and $q_k = 0.0003(k-1)$. (a) $\delta = 0.0080$. (b) $\delta = 0.0090$. (c) $\delta = 0.0100$. (d) $\delta = 0.0105$.