A Multi-Agent System and State Control of Coupled Chaotic Maps

Masahiro WADA[†], Kazumasa HIRAI[†] *and* Yoshifumi NISHIO[‡]

[†] *Dept. of Information Science and Systems Engineering, Konan University, Kobe 658-8501, Japan*
wada-m@konan-u.ac.jp, khirai@mbox.kyoto-inet.or.jp

[‡] *Dept. of Electrical and Electronic Engineering, Tokushima University, Tokushima 770-8506, Japan*
nishio@ee.tokushima-u.ac.jp

Abstract— Many types of coupled chaotic systems had been studied in the wide fields and some applications had been proposed, so several chaotic behavior are very important in the universe. Emergent behavior and construct of a multi-agent system in coupled simple chaotic maps are investigated in this study.

I. Introduction

The coupled systems existing in nature exhibit great variety of phenomena such as complex mechanisms for all of the systems in the natural fields or in the universe. These phenomena can be seen in an information network, a human society, the process of a life, self organization of neuron, a biological system, an ecological system and so many nonlinear systems. Thus the concept of the process of natural selection is very important to be realized some extended application by using nonlinear features. In fact unsolved problems in natural fields are still remained, especially the process of evolution in a human being and human intelligence. We have considered that almost phenomena are equivalently described as coupled nonlinear systems. Hitherto, many types of coupled chaotic systems have been studied as one of the most interesting phenomena in the natural fields[1]-[3]. Recently, many types of coupled chaotic systems have been investigated by many researchers and then a lot of phenomena were reported in many papers. We have studied several chaotic circuits coupled by one element for applying the engineering systems, and reported some featured phenomena. Further, cellular automata(CA)[4] and cellular neural network(CNN)[5] has been also studied in a large field scientifically. The studies of coupled map lattice(CML), globally coupled maps(GCM), generalized CA(GCA)[6] and so many studies provided us tremendous interesting phenomena and influenced some applications. Recently, a multi-agent system constructed by simple coupled chaotic maps have been reported and shown validity of the model[7][8]. The past works had been almost treated only simple coupled systems or simplified models, and the total number of systems is small. However, Ichikawa reported that chaos generates in a simple CA on any conditions[9]. It was an interesting system with emergent property. The term *emergence* leads us to the world of a novel approach for several applications. It is important to investigate several phenomena in simple coupled chaotic systems. In the past study, both CML and GCM type models were usually used as coupled chaotic systems. In this study, several coupled types are proposed and phenomena in the coupled chaotic systems are investigated. Furthermore, a multi-agent system constructed by the coupled systems are also investigated.

II. System description

Chaotic maps are generally used for several approaches to investigate chaotic phenomena on coupled chaotic systems. Especially, the logistic map and the other types of chaotic maps such as a cut map, a circle map, a tent map, a cubic map are well known and popular. CML and GCM are the special models coupled by thus chaotic maps. Let us consider a network of chaotic maps which is placed spatially on two-dimensional plane. Let a local subscript number of each cell define as

$$\begin{bmatrix} c_{i-1,j-1} & c_{i-1,j} & c_{i-1,j+1} \\ c_{i,j-1} & c_{i,j} & c_{i,j+1} \\ c_{i+1,j-1} & c_{i+1,j} & c_{i+1,j+1} \end{bmatrix}$$

Firstly we consider simple chaotic map of a subsystem in the coupled network. The subsystem considered here is very simple whose equation is presented as follows

$$x_{(t+1)} = f(x_{(t)}) = \alpha x_{(t)}(1 - x_{(t)}^2) \quad (1)$$

where α is a parameter which can determine the strength of nonlinear characteristic. We can easily confirm that chaos generates in this subsystem. It is

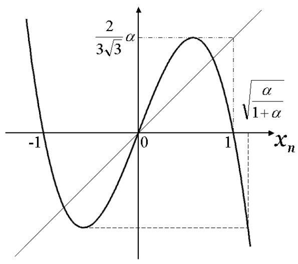

Figure 1: A mathematical cubic map model for one subsystem for $\alpha = 2.6$.

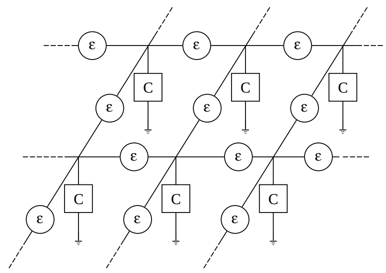

Figure 2: A model of coupled network subsystems by 4 neighbors.

well known that we can confirm a crisis of chaos if α is greater than around 2.6, which it is obtained from Fig. 1. Hence the chaotic map can obtain both positive and negative value while α is over 2.6. If $\alpha > 3$, i.e. $\frac{2\alpha}{3\sqrt{3}} > \sqrt{\frac{\alpha}{1+\alpha}}$, we can also confirm that the system diverges.

Secondly, we propose some types of structure of coupled chaotic systems, in which a chaotic subsystem are coupled each other in a local group. Generally the coupled types are roughly divided into two categories, a locally coupled network and a globally coupled network. A schematic model of the coupled systems is shown in Fig. 2 in the case of locally coupled network. Now we consider the locally coupled network because of the reason for state-control of each subsystem in the later part. The adopted structures in this study are divided into three connected types. In the normal case, subsystems placed on the edge of the coupled network are connected only to the local subsystems, that is, the edge of network is free, not connected. In the other cases, subsystems placed on the edge are connected to the subsystem on the opposite side, just like as the structure of a tube or a pipe, and subsystems placed on the edge are all connected to the opposite side of the coupled network, just like as the surface of a ring or a doughnut shape. Throughout this paper, we usually treat above three types of locally coupled systems. Then, the subsystems are distributed on the two-dimensional place, that is, the subsystems are placed on a plane and then all the subsystems are connected to 4, 8 or more local subsystems. Generally, the case of coupling for 4 neighbors(Neumann type) and 8 neighbors(Moore type) can be considered. For example, while the subsystems are coupled locally in the case that 4 neighbors subsystems are coupled to one by the weight ε as shown in Fig. 2, then the state of a subsystem at any placed on the total system is described as follows

$$x_{i,j(t+1)} = k_{i,j}\ \alpha\ x_{i,j(t)} \left(1 - x^2_{i,j(t)}\right) + \sum_{m,n} k_{m,n}\ x_{m,n(t)} \qquad (2)$$

we assume

$$k_{i,j} + \sum_{m,n} k_{m,n} = 1, \qquad (3)$$

where if it is coupled by 4 neighbors then $k_{m,n} = \varepsilon/4$, hence $k_{i,j} = 1-\varepsilon$, the first and second subscripted letters $\{i, j\}$ of the variable x mean identical numbers at each position, the third letter t means iteration time respectively, and ε corresponds to the strength of a coupling gain for neighborhoods. The subscripted number follows the cyclic rule. It is to be noted that if $\varepsilon = 0$, it means that the systems are not coupled each other, the fixed points of each subsystem can calculate easily.

A characteristic of this coupled system is of using the state in the time one step before. Therefore the parameter α can be chosen larger than 3, provided that ε is enough large(< 1), because the value of $\alpha(1-\varepsilon)$ in the equation of the total system works to be suppressed its divergence.

III. Computer simulation

The computer simulation in those systems is shown. The case of the normal type coupled system which the subsystems are placed on 2-dimensional plane is mainly treated hereafter. Now we choose coupled chaotic maps in case of 100 times 100 scale network. Some illustrated figures are shown. Some waveforms of the subsystems $(C_{i,i}, 1 \leqq i \leqq 100)$ with respect to the iteration of the time step are shown in Fig. 3. It is shown that each subsystem oscillates randomly in the first half of the time step while ε is set to be 0 in this interval and they converge to the fixed point when ε is set to be 0.3 after 300 time steps. We can confirm that all or almost subsystems are stabilized and they converge to the fixed point or periodic oscillation. The initial conditions for all subsystems are set as an uniform random number between -1 and 1. Figure 4 (a) and (b) are illustrated the spatial pattern of each state with black ■ and white □ in the same parameter of Fig. 3 at 100 and 500 time steps respectively. The black(white) painted square means that the state is positive(negative). Figure 5 is also illustrated several spatial pattern results in which (a) and (b) are the case of $\varepsilon = 0.1$ and 0.4 for $\alpha = 2.6$, (c) is the case of $\varepsilon = 0.1$

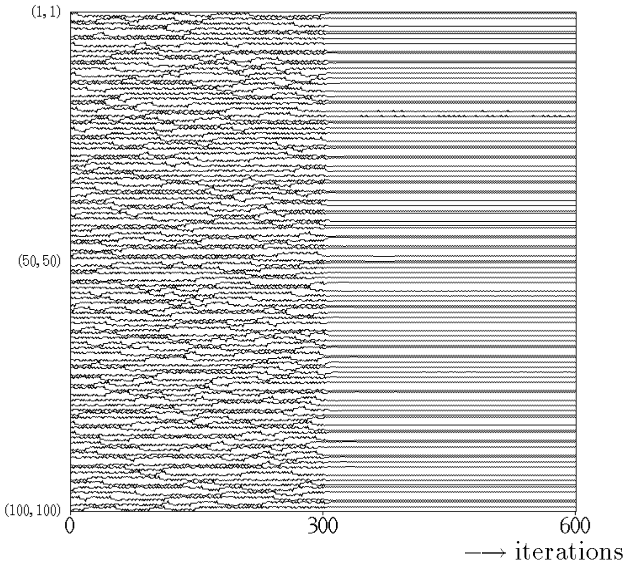

Figure 3: Several waveforms of the subsystems ($C_{i,i}$; $1 \leqq i \leqq 100$) in the total system without/with coupling for $\alpha = 2.6$ when the all subsystems are coupled at 300 time steps after.

(a) (b)

Figure 4: Simulation results illustrated to 2-dimensional plane. These figures correspond to Fig. 3 at 100 and 500 time steps after respectively. (a) without coupling, the all subsystems oscillate separately, (b) with coupling, the all subsystems are coupled to 4 neighbors by ε.

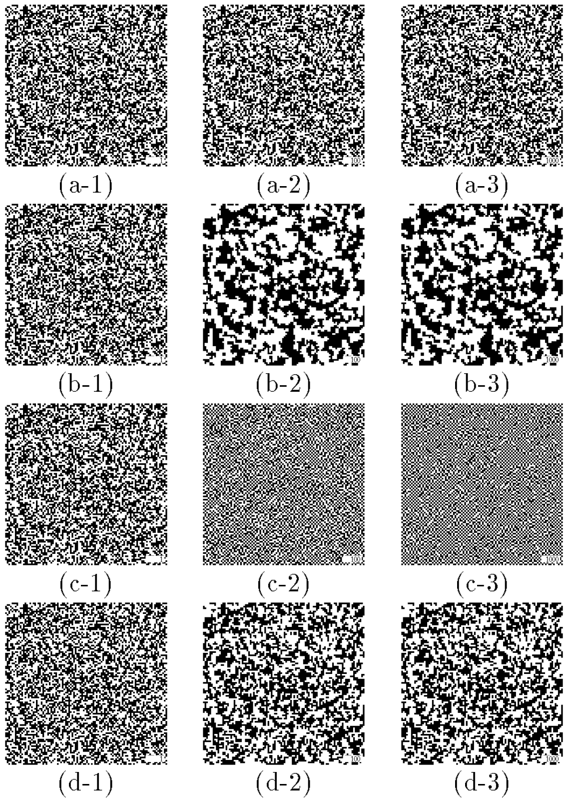

Figure 5: Several computer simulation results in the case of 100×100 subsystems. (a) for $\alpha = 2.6$, $\varepsilon = 0.1$, (b) for $\alpha = 2.6$, $\varepsilon = 0.4$, (c) for $\alpha = 2.8$, $\varepsilon = 0.1$, and (d) for $\alpha = 3.0$, $\varepsilon = 0.4$. The figures from the left of each line, the time steps 1, 100 and 1000 are shown respectively. The initial conditions are set randomly, but all simulations are started at the same condition.

for $\alpha = 2.8$, (d) is the case of $\varepsilon = 0.4$ for $\alpha = 3.0$ respectively. In these cases, we can confirm easily several phenomena that stabilization of chaos, bifurcation to chaos via periodic oscillation, chaotic itinerancy and spatio-temporal chaos occur in this system. If ε is chosen for a small value, the spatio-time transition of the chaotic behavior can be observed. Several interesting chaotic behavior can be observed in these coupled chaotic maps, like a "game of life" as time growth, hence it has a transition of the fixed point.

IV. Multi-agent system

In this part, we consider that state around the fixed point will be controlled by using a characteristic of the subsystem. The construction of a multi-agent system by the supposed coupled chaotic systems are investigated. For the purpose of some applications, we can control the sequence of chaotic itinerancy for each subsystem in a large scale coupled chaotic systems by using state-control. Therefore it is difficult to know state of all the subsystems and it is not for practical use in the large scale coupled chaotic networks, we consider that a subsystem can be controlled by itself as an autonomic control. On the other hand, the subsystems can have the state either positive or negative values for the origin. We investigate how to control the two state which we suppose each state as an agent. Let us consider one subcomponent which is constructed by an agent and coupled 4 neighbors, the parameter α and/or ε of the agent are controlled by the state of neighborhoods with response. As a feature of this proposed system, we can know the state of the neighborhoods at one step before and we can put the state of the other agents near to one agent when the coupling gain changes. By changing the coupling gain ε or the strength of nonlinear characteristic α, the state of each subsystem can be controlled as a agent. In this study, we only control the parameter ε hereafter, because the parameter α is related with system characteristics closely. The system is divided roughly into two state, that is randomly chaotic and two positive-negative state coexist stably as like shown in Fig. 3. Namely, an agent controls the coupling gain ε by itself in spite of the other agents, because the agent knows the state one step before. The concept of controlling parameter shows in the following.

1. it should be set α as the agent behaves like the transition of chaos when the systems are uncoupled.
2. set appropriate ε to get each agent to be stable.
3. make the coupling gain ε of some agents to be week, then let an agent be the transition of chaos.
4. intensify ε again when the system is to be expected state.

Repeat its procedure until the total system attain to the expected state. Now the purpose of the all subsystems is to be the positive state. Here the coupling gain is set to $\varepsilon = 0.4$ when we want it to be stabilized, it is set to $\varepsilon = 0.05$ when we want it to be generated the transition. One of the simulation results is shown in Fig. 6 when the system is controlled by the proposed method. In this result, we added control sequence at 100 time steps after. The upper part of this figure shows waveform of the subsystems from $C_{1,1}$ to $C_{100,100}$. The lower part shows a number of agent which attained to positive state in the total system. We can confirm that agents almost attain to the purpose, but not all. If ε is large, the probability into the positive attractors increases. It is especially shown that agents almost accomplish the purpose quickly. The proposed systems which behave those phenomena are considered as one of models of a multi-agent system. If we can control the number of clusters of the state, we respect of these features to be utilized for several engineering applications.

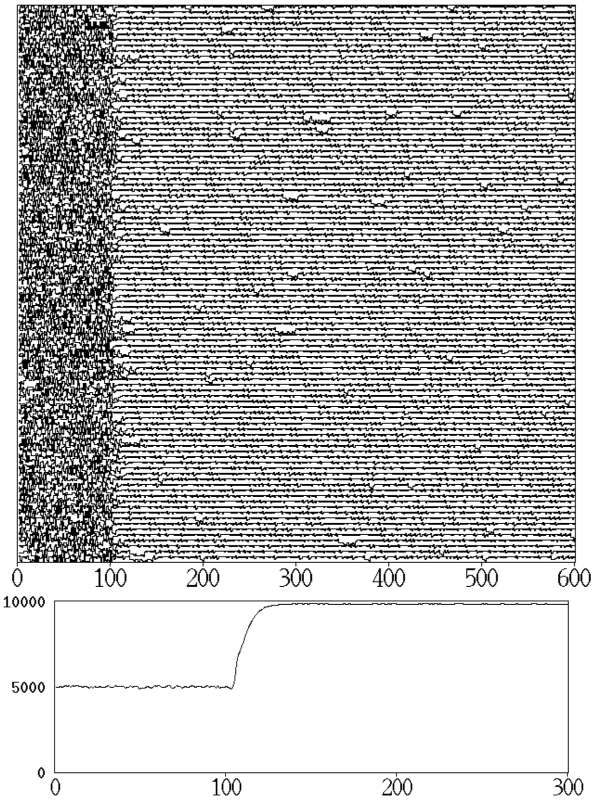

Figure 6: Transition of chaos and time evolution of the total number of agent as a multi-agent system controlled by $\varepsilon = 0.4$ and 0.05 for $\alpha = 3.0$. Transition of the total number of agent in the positive state is also shown.

V. Conclusions

In this study, chaotic phenomena in several types of coupled chaotic systems have been investigated and considered how to construct a multi-agent system. Some illustrated computer simulation results have been shown. The construct of a multi-agent system with state-control for each subsystem has also been shown. We conclude that the supposed coupled systems whose several chaotic phenomena and multi-agent systems are expected to be utilized for realization of a new type application, although study of the mechanism of such phenomena and many works have been left.

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