Analysis of Chaotic Circuit Using Two Simple Ring Oscillators Coupled by Diodes

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Abstract— In this study, we investigate a chaotic circuit proposed by us in NOLTA'99. The circuit consists of two CMOS ring oscillators and two diodes. By using a simpler model of the original circuit, the exact solutions are derived. The exact expression of the Poincaré map and its Jacobian matrix make it possible to confirm the generation of chaos using the Lyapunov exponents and to investigate the related bifurcation phenomena.

I. INTRODUCTION

Chaos has extensive possibility of engineering applications. For instance, neural network systems, chaotic communication systems, spatio-temporal behavior in coupled chaotic systems and so on. For realizing these applications, an integration of chaotic circuits is very important subject[1]. We proposed a chaotic circuit suitable for the integration in NOLTA'99[2]. The circiut consists of two ring oscillators, two diodes, two resistors and one capacitor. A CMOS ring oscillator is used for the CMOS process performance test. This oscillator consists of a series of inverter circuits. Diodes are realized simply on IC chip. It has very simple structure. Therefore, we consider that the proposed circuit is realized very easily.

In this study, we investigate chaotic phenomena generated by the proposed circiut. The exact solutions are derived, the Lyapunov exponents are calculated and bifurcation scenario is explained.

II. CIRCUIT MODEL

A. Ring Oscillator

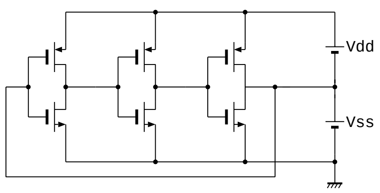


Figure 1: Three-stage ring oscillator.

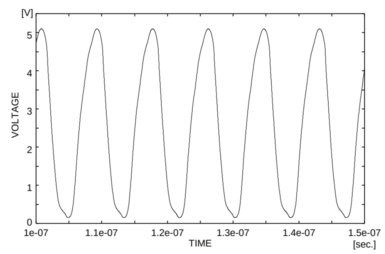


Figure 2: SPICE simulation result of Fig. 1.

Fig. 1 shows a three-stage ring oscillator. This oscillator consists of three CMOS inverters. Fig 2 shows SPICE simulation results of the ring oscillator in Fig. 1. Channel width of p-MOS is 60[μm] and length is 4 [μm]. Channel width of n-MOS is 20[μm] and length is 4 [μm]. A standard 0.5 μm layout design rule is used for the transistor model.

B. Designed Circuit

We design a chaotic circuit using this ring oscillator. Fig. 3 shows the designed circuit. In order to control amplitudes of the oscillators, $R1$ and $R2$ are connected. The frequency of the upper side oscillator are controlled by $C1$. Fig. 4 shows SPICE simulation results of the circuit in Fig. 3. Parameter values are W/L(pMOS) = 60/4, W/L(nMOS) = 20/4, R_1 = 2000[Ω], R_2 = 200[Ω], C_1 = 9.00[pF], V_{dd} = 2.70[V], V_{ss} = 2.30[V]. Maximum time step is 50.0 [ps], and observation time is 10.0-100.0 [μs]. The main oscillation frequency band width is about 100-250[MHz].

C. Circuit Model

We approximate $M1$, $M2$ and CMOS inverters in the circuit as follows. $M1$ and $M2$ have a characteristic of the same as the coupling diodes shown in Fig. 5. Therefore, $M1$ and $M2$ are approximated as the

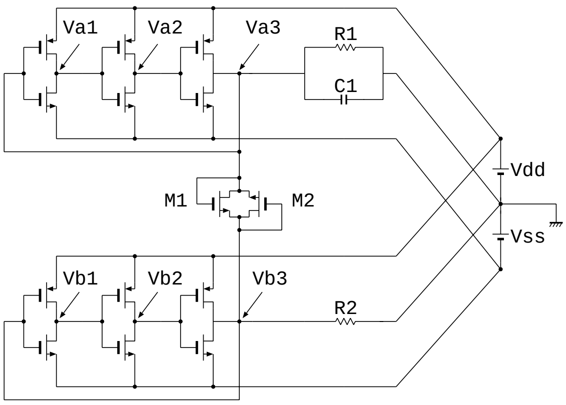


Figure 3: Designed circuit.

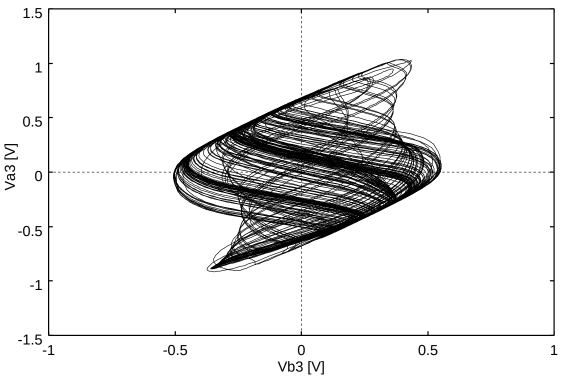


Figure 4: SPICE simulation result of Fig. 3. W/L(pMOS) = 60/4, W/L(nMOS) = 20/4, R_1 = 2000[Ω], R_2 = 200[Ω], C_1 = 9.00[pF], V_{dd} = 2.70[V] and V_{ss} = 2.30[V]. Maximum time step is 50 [ps] and analysis time is 10-100 [μs].

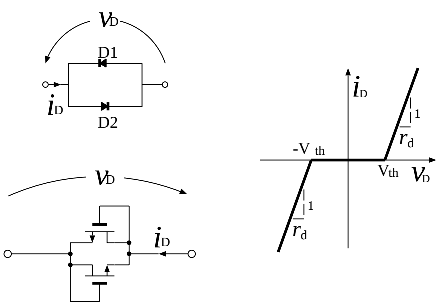


Figure 5: Coupling diodes and it's model.

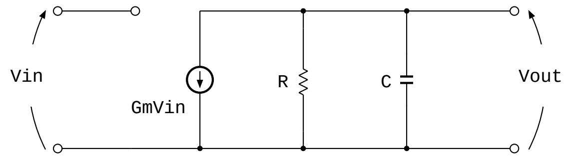


Figure 6: Inverter model.

following three-regions piecewise-linear function.

$$i_D = \begin{cases} \dfrac{1}{r_d}(v_D - V_{th}) & \text{for } v_D > V_{th}, \\ 0 & \text{for } -V_{th} \le v_D \le V_{th}, \\ \dfrac{1}{r_d}(v_D + V_{th}) & \text{for } v_D < -V_{th}. \end{cases} \quad (1)$$

Fig. 6 shows the approximation of the ring oscillator's inverter. The circuit elements C, R and Gm represent all parasitic capacitors associated with the input and the output nodes, the input and the output resistances and the inverter's gain, respectively. By the reason that the inverter is connected to the previous-stage and the next-stage inverters, the device values of the input and the output nodes are put together in this approximation.

Circuit equations are described as follows:

$$\begin{cases} \dfrac{dv_{a1}}{dt} = -\dfrac{1}{RC}v_{a1} - \dfrac{Gm}{C}v_{a3}, \\ \dfrac{dv_{a2}}{dt} = -\dfrac{1}{RC}v_{a2} - \dfrac{Gm}{C}v_{a1}, \\ \dfrac{dv_{a3}}{dt} = -\dfrac{R+R_1}{RR_1(C+C_1)}v_{a3} \\ \qquad\quad -\dfrac{Gm}{C+C_1}v_{a3} - \dfrac{i_d}{C+C_1}, \\ \dfrac{dv_{b1}}{dt} = -\dfrac{1}{RC}v_{b1} - \dfrac{Gm}{C}v_{b3}, \\ \dfrac{dv_{b2}}{dt} = -\dfrac{1}{RC}v_{b2} - \dfrac{Gm}{C}v_{b1}, \\ \dfrac{dv_{b3}}{dt} = -\dfrac{R+R_2}{RR_2C}v_{b3} - \dfrac{Gm}{C}v_{b3} + \dfrac{i_d}{C}, \end{cases} \quad (2)$$

where

$$i_d = \begin{cases} \dfrac{1}{r_d}(v_{a3} - v_{b3} - V_{th}) \\ \qquad \text{for } v_{a3} - v_{b3} > V_{th}, \\ 0 \\ \qquad \text{for } -V_{th} \le v_{a3} - v_{b3} \le V_{th}, \\ \dfrac{1}{r_d}(v_{a3} - v_{b3} + V_{th}) \\ \qquad \text{for } v_{a3} - v_{b3} < -V_{th}. \end{cases} \quad (3)$$

By substituting the variables and the parameters,

$$
\begin{aligned}
&x_n = \frac{v_n}{V_{th}}, \quad y_d = \frac{R_d}{V_{th}} i_d, \quad \tau = \frac{1}{RC} t, \\
&\alpha = GmR, \quad \beta = \frac{C}{C + C_1}, \quad \gamma = \frac{R}{R_1}, \\
&\delta = \frac{R}{R_d}, \quad \varepsilon = \frac{R}{R_2},
\end{aligned}
\tag{4}
$$

(2) and (3) are normalized as

$$
\left\{
\begin{aligned}
\dot{x}_{a1} &= -x_{a1} - \alpha x_{a3}, \\
\dot{x}_{a2} &= -x_{a2} - \alpha x_{a1}, \\
\dot{x}_{a3} &= -\beta(\gamma + 1) x_{a3} - \alpha\beta x_{a2} - \beta\delta y_d, \\
\dot{x}_{b1} &= -x_{b1} - \alpha x_{b3}, \\
\dot{x}_{b2} &= -x_{b2} - \alpha x_{b1}, \\
\dot{x}_{b3} &= -(\varepsilon + 1) x_{b3} - \alpha x_{b2} + \delta y_d,
\end{aligned}
\right.
\tag{5}
$$

where

$$
y_d = \begin{cases} x_{a3} - x_{b3} - 1 & \text{for } x_{a3} - x_{b3} > 1, \\ 0 & \text{for } -1 \le x_{a3} - x_{b3} \le 1, \\ x_{a3} - x_{b3} + 1 & \text{for } x_{a3} - x_{b3} > -1. \end{cases}
\tag{6}
$$

By using the exact solutions of Eq. (5), we can calculate attractors and represent the Poincaré map and its Jacobian matrix for calculating the Lyapnov exponents. We define three piecewise linear regions as Table 1.

	State of Diodes	
Region	D1	D2
D_+	ON	OFF
D_0	OFF	OFF
D_-	OFF	ON

Table 1: Three piecewise linear regions

We calculate the eigenvalues in each region and the equilibrium points in $\mathbf{D}_+$ and $\mathbf{D}_-$. The equilibrium points are calculated by putting the right side of Eq. (5) to be equal to zero. Then, we can describe the exact solutions in each linear region.

III. ANALYSIS

A. Poincaré Map

In order to confirm the generation of chaos and to investigate bifurcation scenario, we derive the Poincaré map.

Let us define the following five-dimensional subspace

$$
\mathbf{S} = \mathbf{S_1} \cap \mathbf{S_2} \tag{7}
$$

where

$$
\begin{aligned}
\mathbf{S_1}:&\quad x_{a3} - x_{b3} = 1 \\
\mathbf{S_2}:&\quad \alpha\beta x_{a2} + \beta(\gamma + 1) x_{a3} - \alpha x_{b2} - (\varepsilon + 1) x_{b3} < 0
\end{aligned}
\tag{8}
$$

The subspace $\mathbf{S_1}$ corresponds to the boundary condition between $\mathbf{D}_+$ and $\mathbf{D_0}$, while the subspace $\mathbf{S_2}$ corresponds to the condition $\dot{x}_{a3} - \dot{x}_{b3} > 0$. Namely, $\mathbf{S}$ corresponds to the transitional condition from $\mathbf{D_0}$ to $\mathbf{D}_+$. Let us consider the solution starting form an initial point on $\mathbf{S}$. The solution returns back to $\mathbf{S}$ again after wandering as shown in Fig. 7.

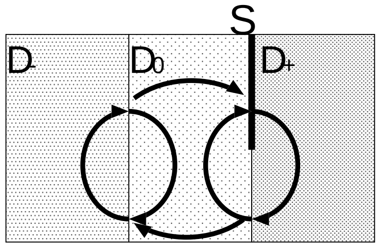


Figure 7: Route map

Hence, we can derive the Poincaré map as a five-Dimensional map as follows.

$$
\mathbf{T}: \ \mathbf{S} \to \mathbf{S}, \ \mathbf{x_0} \mapsto \mathbf{T}(\mathbf{x_0}) \tag{9}
$$

where $\mathbf{x_0}$ is an initial point on $\mathbf{S}$, while $\mathbf{T}(\mathbf{x_0})$ is the point at which the solution starting from $\mathbf{x_0}$ hits $\mathbf{S}$ again. $\mathbf{T}(\mathbf{x_0})$ is given using the exact solutions of Eq. (5). The Jacobian matrix $\mathbf{DT}$ of the Poincaré map can be also derived rigorously from the exact solutions.

We can calculate the largest Lyapunov exponent by

$$
\mu = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} \log |\mathbf{DT}_j \cdot e_j| \tag{10}
$$

where e_j is a normalized base.

B. Computer Calculation

One-parameter bifurcation diagram of the Poincaré map $\mathbf{T}$ and the calculated largest Lyapunov exponent are shown in Figs. 8 (a)(b).

By using these results, we can say the generation of chaos is confirmed numerically and we can describe detailed bifurcation scenario as follows.

As ε decreases, one-periodic orbit bifurcates to torus around $\varepsilon = 4.00$. For $3.13 < \varepsilon < 4.00$, we can observe several phase-locked states in torus region. For 1.81 $< \varepsilon <$ 3.13, the largest Lyapunov exponent becomes positive . Namely, chaos is generated.

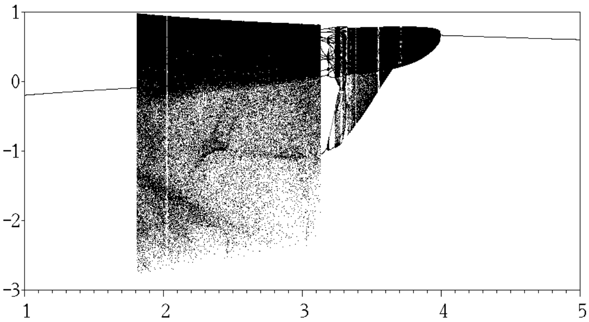
IV. CONCLUSIONS

In this paper, we investigated the chaotic circuit using two simple ring oscillators coupled by diodes in detail. By using a simpler model of the original circuit, chaos has been explained. Further, we have confirmed the generation of chaos by calculating the Lyapunov exponents and have investigated the related bifurcation phenomena.

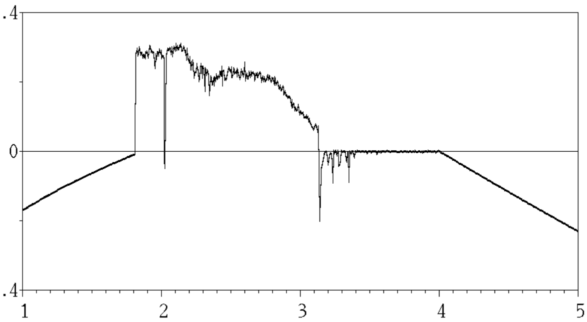
References

[1] J. M. Cruz and L. O. Chua, "An IC Chip of Chua's Circuit, " *IEEE Trans. Circuits Syst. II*, vol. 40, no. 10, pp. 614–625, Oct. 1993.

[2] Y. Hosokawa, Y. Nishio and A. Ushida, "Chaotic Circuit Using Two Simple Ring Oscillators Coupled by Diodes," *Proc. of NOLTA'99*, vol. 1, pp. 107–110, Nov. 1999.



(a)



(b)

Figure 8: (a) Bifurcation diagram. Horizontal: ε. Vertical: x_{a1}. (b) Largest Lyapnov exponent. Horizontal: ε. Vertical: μ. $\alpha = 3.70$, $\beta = 0.10$, $\gamma = 1.00$ and $\delta = 100$.

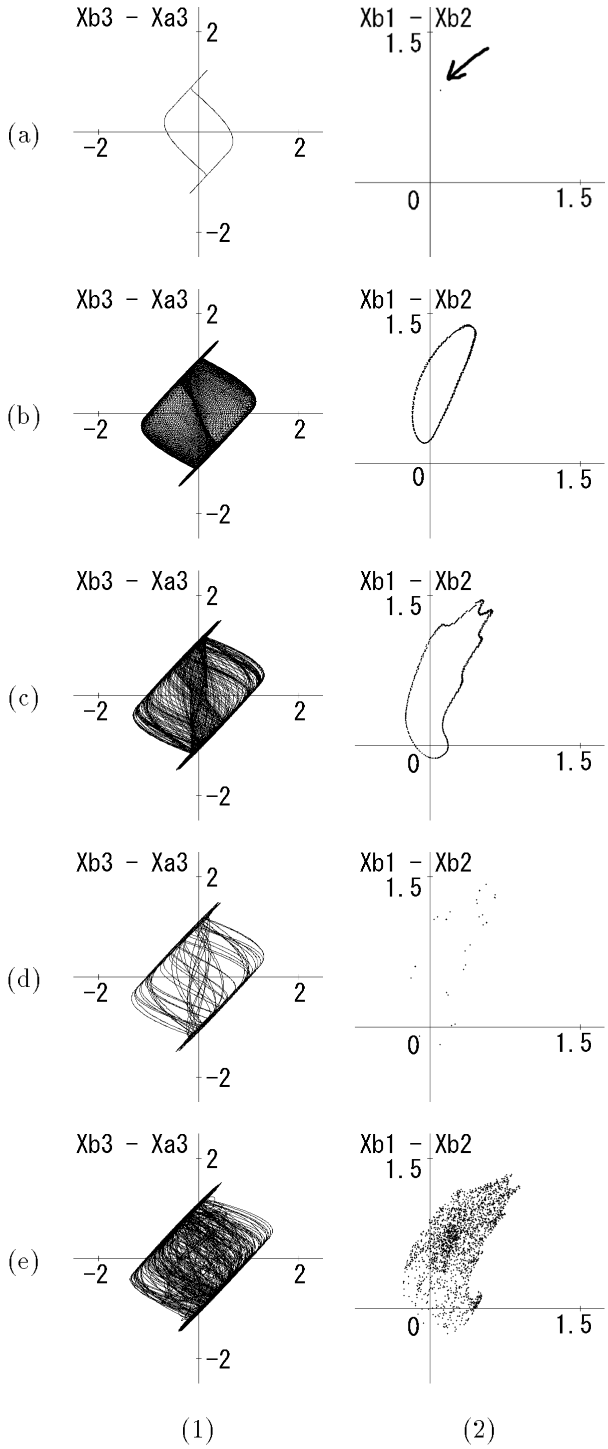


Figure 9: Projections of attractors onto x_{b3} and x_{a3}(1) and their Poincaré map(2). $\alpha = 3.70$, $\beta = 0.10$, $\gamma = 1.00$, $\delta = 100$. (a)$\varepsilon = 4.20$, (b)$\varepsilon = 3.80$, (c)$\varepsilon = 3.40$, (d)$\varepsilon = 3.20$,(e)$\varepsilon = 2.50$ and (f)$\varepsilon = 1.50$.