Phase Synchronization in a Ring of Chaotic Circuits

Yoshifumi NISHIO and Akio USHIDA

Dept. of Electrical & Electronic Engineering, Tokushima University,
2-1 Minami-Josanjima, Tokushima 770-8506, Japan
nishio@ee.tokushima-u.ac.jp

Abstract— In this study, phase synchronization observed in a ring of chaotic circuits coupled by resistors is analyzed. For the case that an odd number ($2N$+1) of the circuits are coupled, a kind of frustration is caused and ($2N$+1)-phase synchronization of chaos is stably generated.

I. Introduction

Recently, phase synchronization of chaotic oscillators attract many researchers' attentions [1]. Because it seems to be impossible to give rigorous definition of phase for chaotic signals, theoretical analysis of the phenomenon is quite difficult and hence much still remains to be done to understand the phenomenon completely. Further, there have been few results on phase synchronization based on real physical systems such as electrical circuits. On the other hand, the authors have been working on coupled chaotic circuits and have found some of them could produce similar phenomenon [2]-[4].

In this study, we propose a ring of chaotic circuits coupled by resistors. Because the system has the coupling structure such that the energy consumed by the coupling resistors becomes minimum when the adjacent two circuits are synchronized at anti-phase, a kind of frustration is caused for the case that an odd number of the circuits are coupled. In that case, we can observe the phase synchronization. The phenomenon is observed from both of circuit experiments and computer calculations. For any odd number $2N$+1, we observe ($2N$+1)-phase synchronization of chaos. In order to analyze the phenomena, we define phase of chaotic signal using the projections of the orbits of the chaotic circuits. Although the definition cannot give exact meaning of the phase of chaotic signal, that can make it possible to give qualitative discussions on the phase synchronization.

II. Circuit Model

Figure 1 shows the chaotic subcircuit. Each subcircuit is three-dimensional autonomous one and consists of three memory elements, one linear negative resistor and one diode. We can regard the diodes as pure resistive elements, because operation frequency is not too high. Figure 2 shows typical example of chaotic attractors observed from each subcircuit.

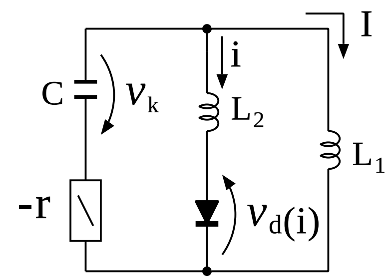

Figure 1: Chaotic Subcircuit.

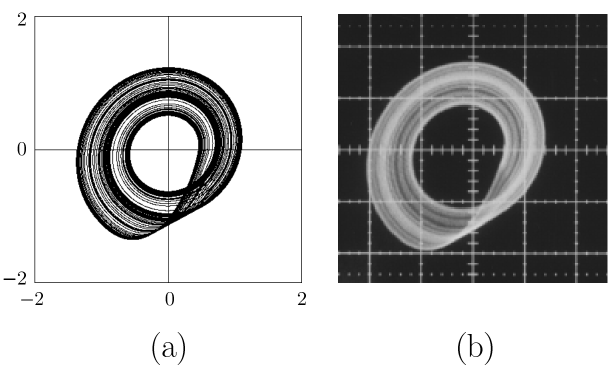

Figure 2: Typical example of chaotic attractors observed from subcircuit. (a) Computer calculated result. x vs. z. α=7.0, β=0.14 and δ=100.0. (b) Circuit experimental result. I vs. v. L_1=100.7mH, L_2=10.31mH, C=34.9nF and r=334Ω. H: 0.8mA/div. V: 1.3V/div.

In this study we consider a ring of the circuits as shown in Fig. 3. In the circuit adjacent two subcircuits are coupled by one resistor R. Because such coupling systems tend to minimize the energy consumed by the coupling resistors [2][4], every two adjacent subcircuits

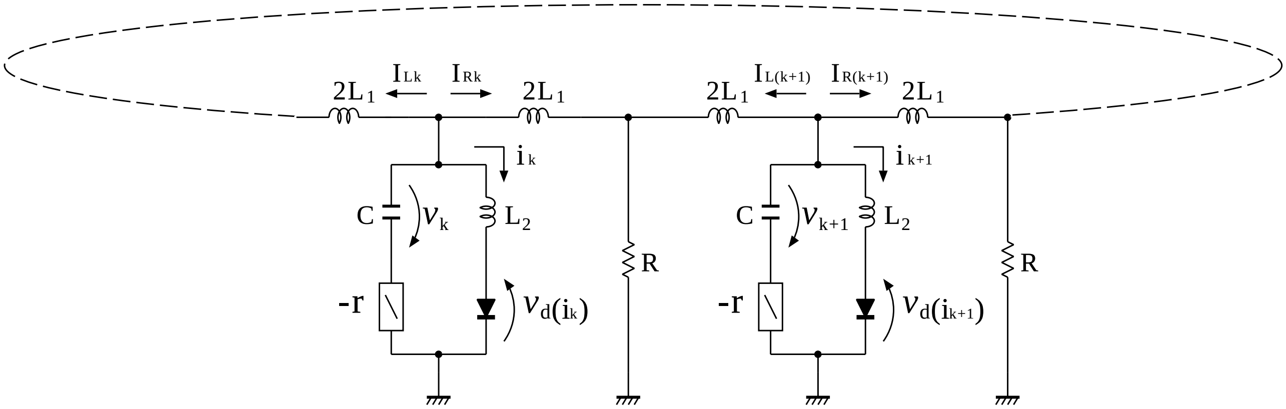

Figure 3: Ring of chaotic circuits.

tend to synchronize with anti-phase. For the case that the total number of the subcircuits is even, the above-mentioned phase state does not cause any frustration. However, for the case that the total number is odd, such a apparently stable phase state does not exist and the system gets frustrated.

At first, the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = \frac{1}{2}\left(r_d\, i_k + E - |\, r_d\, i_k - E\,|\right). \tag{1}$$

By changing the variables and the parameters,

$$\begin{cases} I_{Rk} = \sqrt{\dfrac{C}{L_1}}E\, x_{Rk}, \quad I_{Lk} = \sqrt{\dfrac{C}{L_1}}E\, x_{Lk}, \\ i_k = \sqrt{\dfrac{C}{L_1}}E\, y_k, \quad v_k = E\, z_k, \\ t = \sqrt{L_1 C}\,\tau, \quad \alpha = \dfrac{L_1}{L_2}, \quad \beta = r\sqrt{\dfrac{C}{L_1}}, \\ \gamma = R\sqrt{\dfrac{C}{L_1}}, \quad \delta = r_d\sqrt{\dfrac{C}{L_1}}, \end{cases} \tag{2}$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_{Rk}}{d\tau} = \dfrac{1}{2}\{\beta(x_{Rk}+x_{Lk}+y_k) - z_k \\ \qquad\qquad -\gamma(x_{Rk}+x_{L(k+1)})\} \\ \dfrac{dx_{Lk}}{d\tau} = \dfrac{1}{2}\{\beta(x_{Rk}+x_{Lk}+y_k) - z_k \\ \qquad\qquad -\gamma(x_{Lk}+x_{R(k-1)})\} \\ \dfrac{dy_k}{d\tau} = \alpha\{\beta(x_{Rk}+x_{Lk}+y_k) - z_k \\ \qquad\qquad -f(y_k)\} \\ \dfrac{dz_k}{d\tau} = x_{Rk}+x_{Lk}+y_k \end{cases} \tag{3}$$

$$(\ k=1,\ 2,\ 3,\ \cdots,\ N\ )$$

where

$$f(y_k) = \frac{1}{2}\left(\delta\, y_k + 1 - |\,\delta\, y_k - 1\,|\right) \tag{4}$$

and

$$x_{L(N+1)} = x_{L1}, \quad x_{R0} = x_{RN}. \tag{5}$$

Note that when the coupling parameter γ, which is in proportion to R, is equal to zero, the coupling term in (3) vanishes. For all of computer calculations, the fourth-order Runge-Kutta method is used with step size $h = 0.005$.

III. Simulation Results

We carried out computer simulations for the case of $N = 3 \sim 15$ and circuit experiments for $N = 3 \sim 5$.

Figure 4 shows the computer simulated result for the case of $N = 7$. We can see the adjacent subcircuits are almost synchronized with anti-phase. However, because of the boundary condition of the ring structure, the phase difference between the adjacent subcircuits is not around π but around $\pi - \pi/7$. The margin $\pi/7$ is accumulated along the ring and makes π phase difference to compensate the frustration. Namely, in this case 7-phase synchronization of chaos appears in the ring.

Figure 5 indicates the changes of the amplitudes by 20 levels of gray scale. The phase synchronization can be observed more clearly.

In order to investigate the observed phase synchronization, let us define the Poincaré section as $z_1 = 0$ and $x_{R1}+x_{L1} < 0$ and plot the solutions of the subcircuits on $(x_{Rk}+x_{Lk}) - z_k$ plane. The data are shown in Fig. 6. Because the attractor observed from each subcircuit is strongly constrained onto the plane $y_k = 0$ when the diode is off, we can see the phase difference from the data. The data of the subcircuit 1 are not visible, because the points $(x_{R1}(n)+x_{L1}(n), z_1(n))$ are always on the Poincaré section.

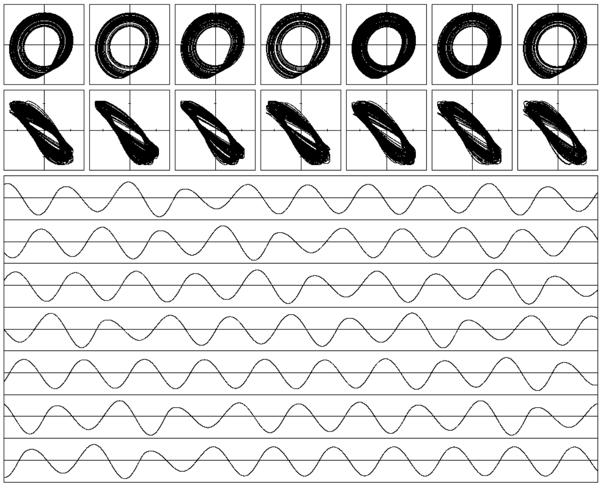

Figure 4: Computer simulated result for $N = 7$. α=7.0, β=0.14, γ=0.1 and δ=50.0. Upper figures: $x_{Rk} + x_{Lk}$ vs. z_k. Middle figures: $x_{Rk} + x_{Lk}$ vs. $x_{R(k+1)} + x_{L(k+1)}$. Lower figures: τ vs. $x_{Rk} + x_{Lk}$. k=1, 2, 3, $\cdots$, 7.

Further, we introduce the following independent variables from the discrete data of $x_{Rk}(n) + x_{Lk}(n)$ and $z_k(n)$ on the Poincaré map.

$$\varphi_k(n) = \begin{cases} \pi - \tan^{-1} \dfrac{z_{k+1}(n)}{x_{R(k+1)}(n) + x_{L(k+1)}(n)} \\ \quad \cdots \;\; x_{R(k+1)}(n) + x_{L(k+1)}(n) \geq 0 \\ -\tan^{-1} \dfrac{z_{k+1}(n)}{x_{R(k+1)}(n) + x_{L(k+1)}(n)} \\ \quad \cdots \;\; x_{R(k+1)}(n) + x_{L(k+1)}(n) < 0 \\ \quad \text{and } z_{k+1}(n) \geq 0 \\ 2\pi - \tan^{-1} \dfrac{z_{k+1}(n)}{x_{R(k+1)}(n) + x_{L(k+1)}(n)} \\ \quad \cdots \;\; x_{R(k+1)}(n) + x_{L(k+1)}(n) < 0 \\ \quad \text{and } z_{k+1}(n) < 0 \end{cases} \tag{6}$$

$$(\;\; k=1,\ 2,\ 3,\ \cdots,\ N-1. \;\;)$$

These variables can correspond to the phase differences between the subcircuit 1 and the others. (Note that the argument of the point $(x_{R1}(n) + x_{L1}(n), z_1(n))$ is always π, because of the definition of the Poincaré map.) The distribution of the $\varphi_k(n)$ for $N = 7$ is shown in Fig. 7.

Finally, the computer simulated results for the case of $N = 15$ are shown in Figs. 8 and 9. We can see 15-phase synchronization of chaos is stably generated.

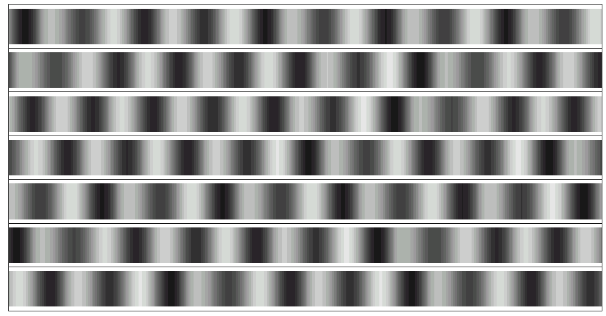

Figure 5: Phase synchronization for $N = 7$. α=7.0, β=0.14, γ=0.1 and δ=50.0. τ vs. $x_{Rk} + x_{Lk}$. k=1, 2, 3, $\cdots$, 7.

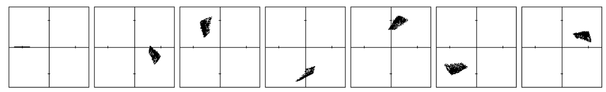

Figure 6: Poincaré map for $N = 7$. α=7.0, β=0.14, γ=0.1 and δ=50.0. $x_{Rk}(n) + x_{Lk}(n)$ vs. $z_k(n)$. k=1, 2, 3, $\cdots$, 7.

IV. Conclusions

In this study, a ring of chaotic circuits coupled by resistors has been proposed and the phase synchronization caused by a kind of frustration of the coupled system has been analyzed.

We consider that it is interesting to investigate the effect of the frustration of coupled systems.

References

[1] "Phase Synchronization in Regular and Chaotic Systems," A. Pikovsky, M. Rosenblum and J. Kurths, *International Journal of Bifurcation and Chaos*, vol. 10, no. 10, pp. 2291-2305, 2000.

[2] "Synchronization in Mutually Coupled Chaotic Circuits," Y. Nishio and S. Mori, *Technical Report of IEICE*, no. NLP92-100, pp. 176–181, Mar. 1993.

[3] "On a Ring of Chaotic Circuits Coupled by Inductors," Y. Nishio and A. Ushida, *IEICE Transactions on Fundamentals*, vol. E78-A, no. 5, pp. 608–617, May 1995.

[4] "Quasi-Synchronization Phenomena in Chaotic Circuits Coupled by One Resistor," Y. Nishio and A. Ushida, *IEEE Transactions on Circuits and Systems I*, vol. 43, no. 6, pp. 491-496, June 1996.

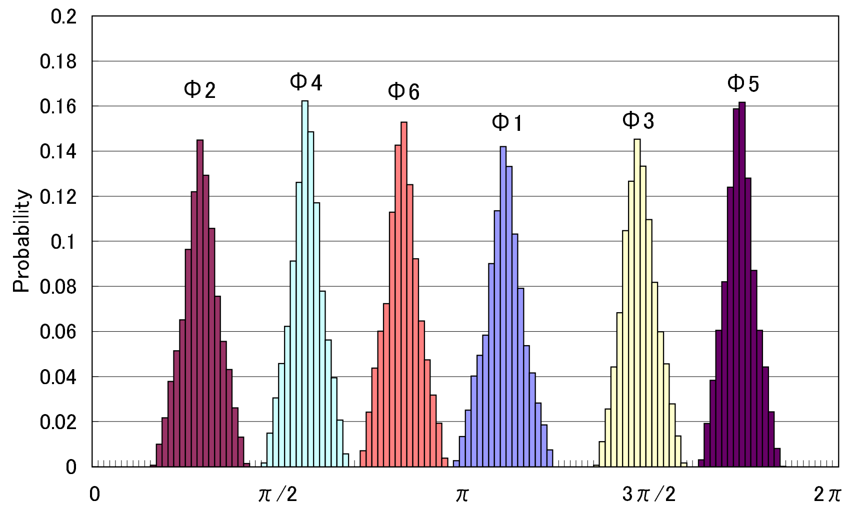

Figure 7: Distribution of $\varphi_k(n)$ for $N = 7$. α=7.0, β=0.14, γ=0.1 and δ=50.0.

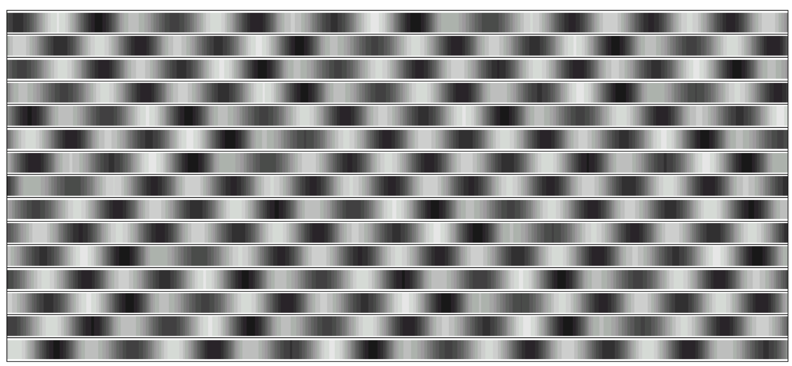

Figure 8: Phase synchronization for $N = 15$. α=7.0, β=0.14, γ=0.1 and δ=50.0. τ vs. $x_{Rk} + x_{Lk}$. k=1, 2, 3, $\cdots$, 15.

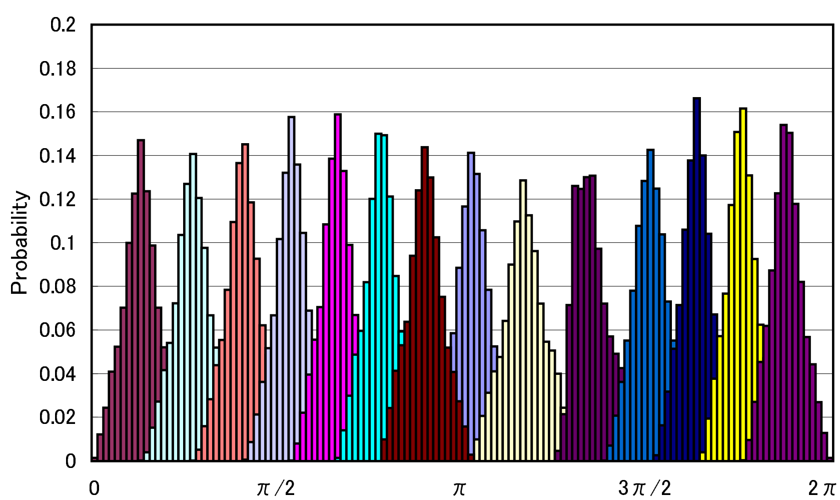

Figure 9: Distribution of $\varphi_k(n)$ for $N = 15$. α=7.0, β=0.14, γ=0.1 and δ=50.0.