ECCTD'01 - European Conference on Circuit Theory and Design, August 28-31, 2001, Espoo, Finland

Phase-Inversion-Waves in Coupled Oscillators Synchronizing at In-and-Anti-Phase

Masahiro OKUDA*, Masayuki YAMAUCHI*, Yoshifumi NISHIO* and Akio USHIDA*

Abstract - **Recently, we have discovered wave propagation phenomena; which are continuously existing waves of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in van der Pol oscillators coupled by inductors as a ladder. We named the phenomena as "phase-inversion-waves." In this study, phase-inversion-waves which exist in the state of in-and-anti-phase synchronization have been found. We observe the phenomena by circuit experiments and computer calculations, and investigate them.**

1. Introduction

A lot of studies on synchronization phenomena of coupled oscillators have been carried out up to now [1]-[13]. Endo *et al.* have reported a details of theoretical analysis and circuit experiments about some coupled oscillators as a ladder, ring and two-dimensional array [2]-[4].

Recently, the authors have discovered wave propagation phenomena that phase states between adjacent oscillators change from in-phase to anti-phase or from anti-phase to in-phase in oscillators coupled by inductors as a ladder [12]. We named the phenomena as " phase-inversion-waves." It is very important to analyze the phase-inversion-waves and make clear the mechanism of the generation, because it is similar to the propagation phenomena of electrical information in axial fiber of nervous system. In [12], we explained the mechanism of the generation of the phase-inversion-waves by using the relationship between phase states and oscillation frequencies. And we confirmed that the phenomena could be explained by a simplified mathematical model [13].

In the circuit, we can observe a synchronization phenomenon in which in-phase and anti-phase exist alternately. In this study, we name the synchronization state as "in-and-anti-phase synchronization." This is similar to the phenomenon reported in [11]. In this study, we observe phase-inversion-waves in the state of in-and-anti-phase synchronization. We carry out circuit experiments and computer calculations and investigate the phenomena.

2. Circuit Model and Phase-Inversion-Waves

Circuit model is shown in Fig. 1. N van der Pol

*Department of Electrical and Electronic Engineering, Tokushima University, 2–1 Minami-Josanjima, Tokushima, 770-8506 Japan. E-mail: okuda@ee.tokushima-u.ac.jp

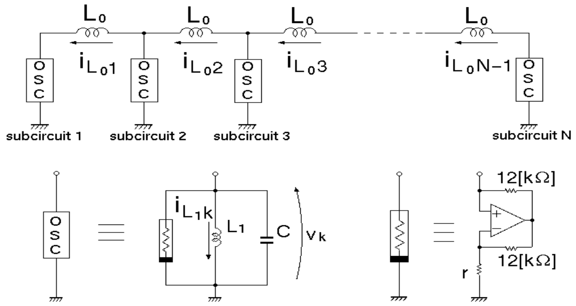


Figure 1: Coupled van der Pol oscillators as a ladder.

oscillators are coupled by coupling inductors L_0. In the computer calculations, we assume the $v-i$ characteristics of nonlinear negative resistors in the circuit by the following functions.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are expressed as

$$\begin{cases} \dot{x}_k = y_k \\ \dot{y}_k = -x_k + \alpha(x_{k+1} - 2x_k + x_{k-1}) + \varepsilon\left(y_k - \frac{1}{3}y_k^3\right) \end{cases} \tag{2}$$

$$(k = 1, 2, \cdots, N)$$

where

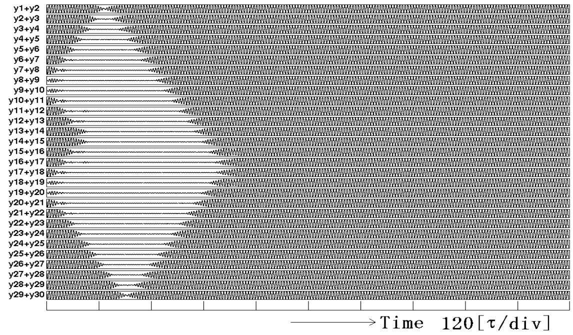
$$x_0 = x_1, \quad x_{N+1} = x_N, \quad \frac{d}{d\tau} = \text{“}\cdot\text{”},$$

$$i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_k, \quad v_k = \sqrt{\frac{g_1}{3g_3}}y_k,$$

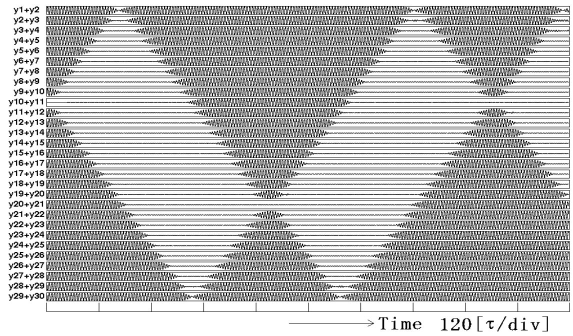
$$t = \sqrt{L_1 C}\tau, \quad \alpha = \frac{L_1}{L_0}, \quad \varepsilon = g_1\sqrt{\frac{L_1}{C}}. \tag{3}$$

It should be noted that α corresponds to the coupling and that ε corresponds to the nonlinearity. Equation (2) are calculated by using the fourth-order Runge-Kutta method.

Figure 2 shows typical examples of the phase-inversion-waves reported previously. They are computer calculated results from the circuit with the size of $N = 30$. Vertical axes are sum of two voltages of adjacent oscillators and horizontal axes are time. White regions in the diagram correspond to the states that sum of voltages of the two oscillators are close to zero, namely adjacent two oscillators synchronized at anti-phase. While, black regions correspond to the states that sum of voltages

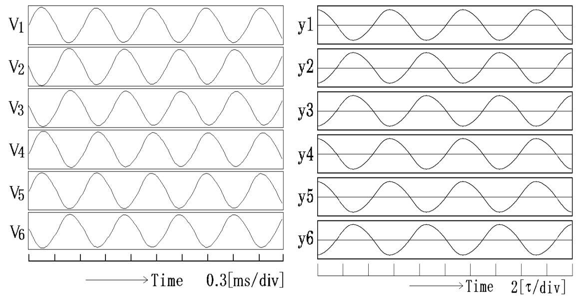


(a) Extinction by collision of two waves.



(b) Reflection by collision of two waves.

Figure 2: Examples of phase-inversion-waves. $\alpha = 0.10$, $\varepsilon = 0.30$, $\Delta\tau = 0.001$.



(a) (b)

Figure 3: Example of in-and-anti-phase synchronization. (a) Circuit experimental result, L_0=1170mH, L_1=200mH, C=68nF, r=1kΩ. (b) Computer calculated result, $\alpha = 0.10$, $\varepsilon = 0.30$, $\Delta\tau = 0.001$.

of the two oscillators oscillates with large amplitude, namely adjacent two oscillators synchronized at in-phase. In the Figures, we can see the wave propagation, reflection and extinction.

It has been known that two van der Pol oscillators coupled by inductor have two stable phase states, namely in-phase synchronization and anti-phase synchronization. Further, the oscillation frequency for the in-phase synchronization is smaller than the oscillation frequency for the anti-phase synchronization. By using these features, we could explain the mechanism by using the relationship between phase states and oscillation frequencies. (See [12] for the details.)

3. Phase-Inversion-Waves in the State of In-and-Anti-Phase Synchronization

3.1 In-and-Anti-Phase Synchronization

In the circuit model, we can observe another type of synchronization as shown in Fig. 3. Vertical axes are the amplitudes of voltages and horizontal axes are time. We name this as "in-and-anti-phase synchronization" because in-phase and anti-phase are existing alternately. Also, in this synchronization state, the edge of the array and its adjacent oscillator can not synchronize at in-phase. Hence, this synchronization state can be observed only when N is even number. While the phase-inversion-waves are observed when N is both even number and odd number. And, the oscillation frequency for the in-and-anti-phase synchronization f_{mid} is almost average of the oscillation frequency for the in-phase synchronization f_{low} and the oscillation frequency for the anti-phase synchronization f_{high}.

3.2 Experimental Results

Figures 4 and 5 show examples of phase-inversion-waves in the state of in-and-anti-phase synchronization. Figure 4 is the array with the size of $N \leq 9$ by circuit experiments and computer calculations.

In Fig. 4(a), phase-inversion-waves do not exist. A phase-inversion-wave is existing continuously in Fig. 4(b). The change of phase states from in-phase to anti-phase or from anti-phase to in-phase is propagated. In Fig. 4(c), two waves are existing and a wave is propagating on the heels of another wave.

Figure 5 shows computer calculated results obtained for the case of $N = 29$ and 30. We can observe phase-inversion-waves similar to Fig. 4.

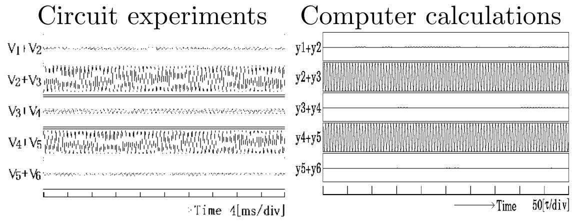
On the system, we could not observe wave extinction by collision of two waves like Fig. 2(a).

3.3 Single Phase-Inversion-Wave

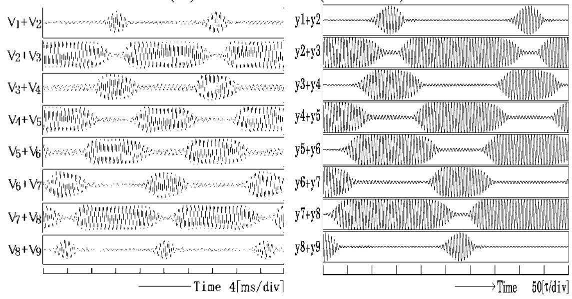
In phase-inversion-waves reported previously [12], single wave could not exist. However, single wave can exist on the system of coupled oscillators of odd number. (See Fig. 4(b).) Figure 6 shows outline chart of single wave generated at the edge of the array with the size of $N = 9$. Generation of the wave is explained as follows:

1. Set initial conditions that in-phase and anti-phase exist alternately, OSC_1 and OSC_2 synchronize at anti-phase, OSC_8 and OSC_9 synchronize at in-phase. (① in Fig. 6.)

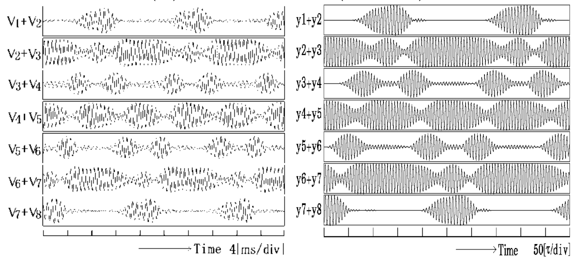
2. A phase-inversion-wave is generated because phase state between OSC_8 and OSC_9 changes from in-phase to anti-phase. (② in Fig. 6.)



(a) No wave ($N = 6$).



(b) Single wave ($N = 9$).



(c) Double waves ($N = 8$).

Figure 4: Experimental results. Circuit experiments, L_0 = 1170mH, L_1 = 200mH, C = 68nF, r = 1kΩ. Computer calculations, $\alpha = 0.10$, $\varepsilon = 0.30$, $\Delta\tau = 0.01$.

3. Phase state between OSC_7 and OSC_8 starts to change from anti-phase to in-phase. (③ in Fig. 6.)

4. Phase state between OSC_6 and OSC_7 changes from in-phase to anti-phase, wave is propagated toward OSC_1. (④ in Fig. 6.) Single wave exists because OSC_8 and OSC_9 synchronize at anti-phase.

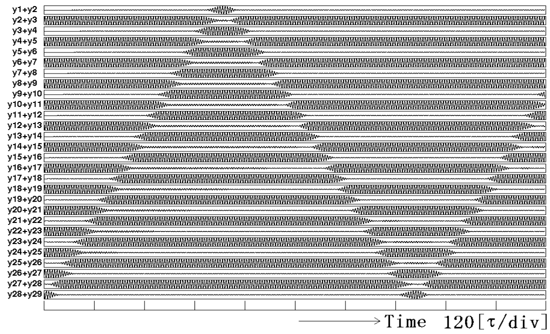
The single wave has been observed for the first time in this study.

3.4 Mechanism of Wave Propagation

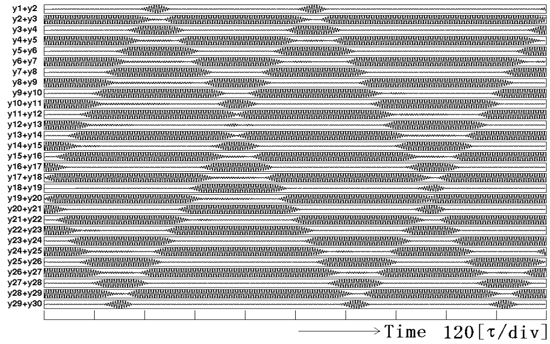
Figure 7 shows phase differences and oscillation frequencies of $OSC_4 \sim OSC_6$. Single wave is propagated in the figure. It should be noted that $\Phi_{k,k+1}$ is phase difference between OSC_k and OSC_{k+1} and that f_k is oscillation frequency of OSC_k. We define them as follows:

$$\Phi_{k,k+1} = \frac{\tau_k(n) - \tau_{k+1}(n)}{\tau_k(n) - \tau_k(n-1)} \times \pi \qquad (4)$$

$$f_k(n) = \frac{1}{2(\tau_k(n) - \tau_k(n-1))} \qquad (5)$$



(a) Single wave ($N = 29$).



(b) Double waves ($N = 30$).

Figure 5: Examples of phase-inversion-waves. $\alpha = 0.10$, $\varepsilon = 0.30$, $\Delta\tau = 0.01$.

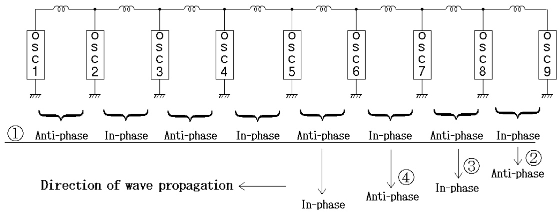


Figure 6: Single wave (outline chart).

where $\tau_k(n)$ is time when the voltage of OSC_k crosses 0[V] at n-th time.

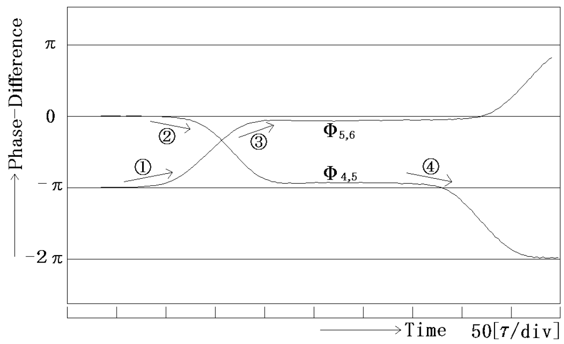
Mechanism of wave propagation can be explained as follows:

1. Let us assume that the circuit synchronizes at in-and-anti-phase and that single wave is going to reach OSC_6 from the direction of OSC_N.

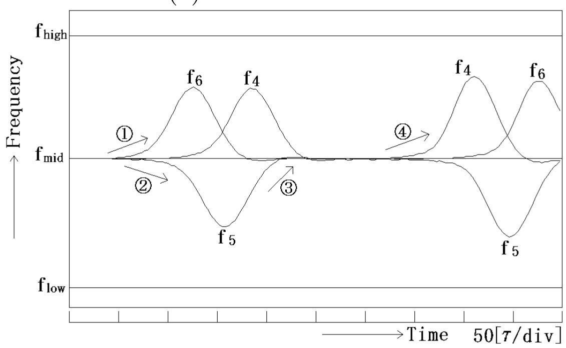
2. As the phase state between OSC_6 and OSC_7 approaches anti-phase from in-phase, f_6 changes from f_{mid} to f_{high}. Hence, $\Phi_{5,6}$ starts to change. (① in Fig. 7.)

3. As the phase state between OSC_5 and OSC_6 approaches in-phase from anti-phase, f_5 starts to change from f_{mid} to f_{low}. (② in Fig. 7.)

4. As f_5 changes from f_{mid} to f_{low}, the phase state between OSC_4 and OSC_5 approaches anti-phase from in-phase. (②in Fig. 7.) Before f_6 reaches f_{high}, it approaches f_{mid} by inversion of the phase states between adjacent oscillators.



(a) Phase difference.



(b) Oscillation frequency.

Figure 7: Wave propagation (computer calculated results).

5. Before f_5 reaches f_{low}, it approaches f_{mid} by inversion of the phase states between adjacent oscillators. f_5 is equal to f_{mid} when the phase states invert almost. (③ in Fig. 7.)

After the wave reflects at the edge of the array, when the wave propagated to the direction of OSC_N reaches OSC_4, the phase states and oscillation frequencies change in a similar manner. (④in Fig. 7.) Although the above explanation is for the case of wave propagation, wave reflection can be explained in a similar manner.

4. Conclusions

In this study, we observed phase-inversion-waves in the ladder of coupled oscillators synchronizing at in-and-anti-phase, in which in-phase and anti-phase are existing alternately. We carried out circuit experiments and computer calculations.

In the previous study, wave propagation, reflection at the edge of array, reflection by collision of two waves and extinction by collision of two waves could be observed. However, extinction could not be observed in this study. Further, single phase-inversion-wave could be observed for the first time.

In phase-inversion-waves in the state of in-and-anti-phase synchronization, oscillation frequencies change around single stable frequency f_{mid}. By using this feature, we could explain the mechanism of wave propagation by using the relationship between phase states and oscillation frequencies.

References

[1] D.A. Linkens "Analytical Solution of Large Numbers of Mutually Coupled Nearly Sinusoidal Oscillators" *IEEE Trans. Circuits Syst.*, vol. cas-21, no. 2, pp. 294-300, Mar. 1974.

[2] T. Endo and S. Mori "Mode Analysis of a Multimode Ladder Oscillator" *IEEE Trans. Circuits Syst.*, vol. 23, no. 2, pp. 100-113, Feb. 1976.

[3] T. Endo and S. Mori "Mode Analysis of Two-Dimensional Low-Pass Multimode Oscillator" *IEEE Trans. Circuits Syst.*, vol. 23, no. 9, pp. 517-530, Sep. 1976.

[4] T. Endo and S. Mori "Mode Analysis of a Ring of a Large Number of Mutually Coupled van der Pol Oscillators" *IEEE Trans. Circuits Syst.*, vol. 25, no. 1, pp. 7-18, Jan. 1978.

[5] K. Radparvar and B.Z. Kaplan "Experimental and Analytical Investigation of Synchronization Dynamics of Two Coupled Multivibrators" *IEEE Trans. Circuits Syst.*, vol. 32, no. 3, pp. 267-273, Mar. 1985.

[6] L. Fabiny and K. Weisenfeld "Clustering Behavior of Oscillator Arrays" *Physical Review A*, vol. 43, no.6, pp. 2640-2648, Mar. 1991.

[7] Y. Setou, Y. Nishio and A. Ushida "Synchronization Phenomena in Many Oscillators Coupled by Resistors as a Ring" *Proc. of IEEE APCCAS'94*, pp. 570-575, Dec. 1994.

[8] S. Moro, Y. Nishio and S. Mori "Synchronization Phenomena in Oscillators Coupled by One Resistor" *IEICE Trans. Fundamentals*, vol. E78-A, no. 2, pp. 244-253, Feb. 1995.

[9] S. Moro, Y. Nishio and S. Mori "On Coupled Oscillators Networks for Cellular Neural Networks" *IEICE Trans. Fundamentals*, vol. E80-A, no. 1, pp. 214-222, Jan. 1997.

[10] H. Daido "Strange Waves in Coupled-Oscillator Arrays: Mapping Approach" *Physical Review Letters*, vol. 78, no.9, pp. 1683-1686, Mar. 1997.

[11] Y. Setou, Y. Inami, Y. Nishio and A. Ushida "On a Ladder of Oscillators Sharing Inductors" *Proc. of NOLTA'97*, vol. 1, pp. 565-568, Nov. 1997.

[12] M. Yamauchi, M. Wada, Y. Nishio and A. Ushida "Wave Propagation Phenomena of Phase States in Oscillators Coupled by Inductors as a Ladder," *IEICE Trans. Fundamentals*, vol. E-82A, no.11, pp. 2592–2598, Nov. 1999.

[13] M. Okuda, M. Yamauchi, Y. Nishio and A.Ushida "Analysis of Wave-Motion in Coupled Oscillators by Simplified Model" *Proc. of NDES2000*, pp. 105-108, May. 2000.