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Asymptotic Equivalent Circuits of Interconnects based on Complex Frequency Method

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Abstract— **Nowaday, Spice is widely used for circuit designs and simulations of ICs. Therefore, it is a very important to develop a user friendly simulator with Spice for solving LSI circuits coupled with interconnects, because LSI circuits are usually connected with PCBs(printed circuit boards) which are modeled with the transmission lines in high frequency domain. There have been already published many papers for solving the interconnects terminated by nonlinear subnetworks. However, most of the algorithms are not easy to develop with Spice simulator. In this paper, we propose a new technique of replacing the interconnect by an asymptotic equivalent circuits based on a complex frequency method. We found from many simulation results that we can get the good results even with the low order approximations. Thus, using the equivalent circuits, we can easily obtain the transient responses of LSI circuits coupled with interconnects by Spice simulator.**

1. Introduction

The analysis and design of high speed LSI chips are becoming more and more important, because PCBs connecting LSI chips may cause the signal delays and crosstalks which sometimes happen the faulty switching operations. In the last decade, many papers have been published on the transient analysis of lossy interconnects [1-5]. The recursive convolution methods using moment-matching technique [1-3] can be efficiently applied to the lossy interconnects terminated by nonlinear elements. However, one of problems in the moment-matching method is that the poles obtained by the applications of Maclaurin expansion and Padè approximation may happen to serious errors, when they are located far from the origin. To overcome the problem, Nakhla et al. propose CFH (complex frequency hopping) [4] which can calculate the exact poles, and they introduce a recursive convolution technique for Spice simulation. However, it seems that it is not so easy to develop the user friendly simulator. Another technique based on the inverse Laplace transformation is already programmed in Spice 3 [6], which can be applied to the uniform interconnects. It is known that the simulator sometimes takes much computational time depending on the parameters. In the reference [7], they propose such a technique that the interconnects are replaced by the discrete π-type and/or T-type models, which can be applied to the relatively short interconnect, efficiently.

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In this paper, we propose an asymptotic equivalent circuit technique based on a complex frequency method, which can be applied to the transient response of LSI circuits coupled with interconnects. Firstly, we calculate the exact poles and the residues of admittance matrix obtained from the interconnect. Secondly, the equivalent circuit is synthesized with the essential poles and the residues. Lastly, we get the equivalent circuits coupled with the LSI circuits. Thus, we can calculate the transient responses with Spice. Our simulator can get the good results, especially for the delay, with the relatively low order approximation. In section 2, we show how to calculate the exact poles and residues, and our asymptotic equivalent circuit in section 3. Some interesting illustrative examples are shown in section 4.

2. Calculation of the poles and the residues of interconnect

Now, consider a uniformly coupled N conductor interconnect. The telegraph equation is described by

$$\left.\begin{aligned} \frac{d\mathbf{V}(x,s)}{dx} &= -(\mathbf{R}+s\mathbf{L})\mathbf{I}(x,s) \\ \frac{d\mathbf{I}(x,s)}{dx} &= -(\mathbf{G}+s\mathbf{C})\mathbf{V}(x,s) \end{aligned}\right\} \quad (1)$$

Thus, we have

$$\left.\begin{aligned} \frac{d^2\mathbf{V}(x,s)}{dx^2} &= (\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})\mathbf{V}(x,s) \\ \frac{d^2\mathbf{I}(x,s)}{dx^2} &= (\mathbf{G}+s\mathbf{C})(\mathbf{R}+s\mathbf{L})\mathbf{I}(x,s) \end{aligned}\right\} \quad (2)$$

Let us introduce the transfer matrix $\mathbf{P}_v(s)$ [8], and get the eigenvalues as follows:

$$\mathrm{diag}[\lambda_j(s)^2] = \mathbf{P}_v(s)^{-1}(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})\mathbf{P}_v(s) \quad (3)$$

We further introduce the transfer matrix $\mathbf{P}_c(s)$:

$$\mathbf{P}_c(s) = (\mathbf{R}+s\mathbf{L})^{-1}\mathbf{P}_v(s)\mathbf{\Gamma}(s), \quad \mathbf{\Gamma}(s) = \mathrm{diag}[\lambda_j(s)] \quad (4)$$

Then, the admittance matrix is described as follows:

$$\begin{bmatrix} \mathbf{I}(0,s) \\ \mathbf{I}(d,s) \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11}(d,s) & \mathbf{Y}_{12}(d,s) \\ \mathbf{Y}_{21}(d,s) & \mathbf{Y}_{22}(d,s) \end{bmatrix} \begin{bmatrix} \mathbf{V}(0,s) \\ \mathbf{V}(d,s) \end{bmatrix} \quad (5)$$

where

$$\begin{aligned} \mathbf{Y}_{11}(d,s) = \mathbf{Y}_{22}(d,s) &= \mathbf{P}_c(s)\mathrm{diag}[\coth\lambda_j(s)d]\mathbf{P}_v(s)^{-1} \\ \mathbf{Y}_{12}(d,s) = \mathbf{Y}_{21}(d,s) &= -\mathbf{P}_c(s)\mathrm{diag}[\sinh^{-1}\lambda_j(s)d]\mathbf{P}_v(s)^{-1} \end{aligned} \quad (6)$$

Observe from (6) that poles of the admittance matrix are obtained only by solving $\sinh\lambda_j(s) = 0$. In this case, we have the following theorem for calculations of the poles.

Theorem 1: The locations of poles satisfying relations (6) are found by solving the following equation:

$$\det\left|(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})+\left(\frac{n\pi}{d}\right)^2\mathbf{I}\right|=0,\quad n=0,1,2,\ldots \tag{7}$$

Proof: From (6), the poles satisfy the following relation:

$$\det|\mathbf{P}_c(s)\mathrm{diag}[\sinh^{-1}\lambda_j(s)d]\mathbf{P}_v(s)^{-1}|=0 \tag{8}$$

Note that since the transfer matrices $\mathbf{P}_v(s)$ and $\mathbf{P}_c(s)$ are nonsingular, we have

$$\sinh\lambda_j(s)d=0, j=1,2,\ldots,N \tag{9}$$

The poles are found by solving (9). The relation can be rewritten as follows:

$$\lambda_j(s)d=jn\pi,\quad j=1,2,\ldots,N\quad n=0,1,2,\ldots \tag{10}$$

Note that $\lambda_j^2(s)$ from (10) is equal to the eigenvalue satisfying (3). Hence, shifting the eigenvalue of (3) $-(n\pi/d)^2$, the characteristic equation is written as follows:

$$\det\left|(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})-\left(\lambda-\left(\frac{n\pi}{d}\right)^2\right)\mathbf{I}\right|=0 \tag{11}$$

Therefore, if we can find the solutions satisfying $\lambda=0$ given by (11), they are equal to the eigenvalues satisfying (10). Hence, from the relation between the roots and coefficients in the characteristic equation, we have

$$\prod_{j=1}^{N}\lambda_j=\det\left|(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})+\left(\frac{n\pi}{d}\right)^2\mathbf{I}\right|=0 \tag{12}$$

Thus, if we can find s satisfying (12) for each n, they are poles of the admittance matrix given by (6).

Q.E.D.

Observe that (12) is an algebraic equation of s, so that they are numerically calculated for each n.

Now, let us evaluate the residues of admittance matrices (6).

Theorem 2: The residues of $\mathbf{Y}_{12}(d,s)$ and $\mathbf{Y}_{21}(d,s)$ in (6) for the pole s_j is given by

$$\mathbf{k}_{12,j}=-\mathbf{P}_c(s)\mathrm{diag}\left[\frac{1}{\cosh(\lambda_j(s)d)\frac{\partial\lambda_j(s)}{\partial s}d}\right]\mathbf{P}_v(s)^{-1}\Bigg|_{s=s_j} \tag{13}$$

where $\frac{\partial\lambda_j(s)}{\partial s}$ is obtained as follows:

$$\begin{bmatrix}\dfrac{\partial\mathbf{U}_j}{\partial s}\\ \dfrac{\partial\lambda_j(s)}{\partial s}\end{bmatrix}=\begin{bmatrix}\mathbf{Z}(s)\mathbf{Y}(s)-\lambda_j(s)^2\mathbf{I} & -2\lambda_j(s)\mathbf{U}_j\\ \mathbf{U}_j^T & 0\end{bmatrix}^{-1}$$

$$\times\begin{bmatrix}-\dfrac{\partial}{\partial s}\{\mathbf{Z}(s)\mathbf{Y}(s)\}\mathbf{U}_j\\ 0\end{bmatrix} \tag{14}$$

where

$$\mathbf{Z}(s)=\mathbf{R}+s\mathbf{L},\quad\mathbf{Y}(s)=\mathbf{G}+s\mathbf{C} \tag{15}$$

and $\mathbf{U}_j$ is the eigenvector for $\lambda_j(s_j)$.

Proof: We have

$$(\mathbf{Z}(s)\mathbf{Y}(s)-\lambda_j(s)^2\mathbf{I})\mathbf{U}_j=\mathbf{0} \tag{16}$$

Differentiating (16) by s, we have

$$\left(\frac{\partial}{\partial s}\{\mathbf{Z}(s)\mathbf{Y}(s)\}-2\lambda_j(s)\frac{\partial\lambda_j(s)}{\partial s}\mathbf{I}\right)\mathbf{U}_j$$

$$+(\mathbf{Z}(s)\mathbf{Y}(s)-\lambda_j(s)^2\mathbf{I})\frac{\partial\mathbf{U}_j}{\partial s}=\mathbf{0} \tag{17}$$

Since $\mathbf{Z}(s)\mathbf{Y}(s)$ is the symmetric matrix, the eigenvector is described by a normalized orthogonal vector. Thus, we have

$$\mathbf{U}_j^T\mathbf{U}_j=1\ \Rightarrow\ \mathbf{U}_j^T\frac{\partial\mathbf{U}_j}{\partial s}=0 \tag{18}$$

Combining (17) with (18), we have

$$\begin{bmatrix}\mathbf{Z}(s)\mathbf{Y}(s)-\lambda_j(s)^2\mathbf{I} & -2\lambda_j(s)\mathbf{U}_j\\ \mathbf{U}_j^T & 0\end{bmatrix}\begin{bmatrix}\dfrac{\partial\mathbf{U}_j}{\partial s}\\ \dfrac{\partial\lambda_j(s)}{\partial s}\end{bmatrix}$$

$$=-\begin{bmatrix}\dfrac{\partial}{\partial s}(\mathbf{Z}(s)\mathbf{Y}(s))\mathbf{U}_j\\ \mathbf{0}\end{bmatrix} \tag{19}$$

Thus, we have $\frac{\partial\lambda_j(s)}{\partial s}$ for s_j and $\lambda_j(s)$, and the residue is calculated by (13).

Q.E.D.

Corollary: The residue of $\mathbf{Y}_{11}(d,s)$ is given by

$$\mathbf{k}_{11,j}=\mathbf{P}_c(s)\mathrm{diag}\left[\frac{1}{\frac{\partial\lambda_j(s)}{\partial s}d}\right]\mathbf{P}_v(s)\Bigg|_{s=s_j} \tag{20}$$

Then, using these poles and the residues, the element of admittance matrix are described by the partial fractions as follows:

$$Y_{ij}(s)=\frac{k_{1,ij}}{s-p_1}+\frac{k_{2,ij}}{s-p_2}+\frac{k_{3,ij}}{s-p_3}+\cdots \tag{21}$$

Consider the special case of a single interconnect. The admittance equation is given by

$$\begin{bmatrix}I(0,s)\\ I(d,s)\end{bmatrix}=\frac{1}{Z_0(s)}\begin{bmatrix}\coth\lambda(s)d & -\sinh^{-1}\lambda(s)d\\ -\sinh^{-1}\lambda(s)d & \coth\lambda(s)d\end{bmatrix}$$

$$\times\begin{bmatrix}V(0,s)\\ V(d,s)\end{bmatrix} \tag{22}$$

where

$$Z_0(s)=\sqrt{(R+sL)/(G+sC)},\quad\lambda(s)=\sqrt{(R+sL)(G+sC)}$$

Thus, the poles are solution of $\sinh\lambda(s)d=0$. Namely, we have

$$s_0=-R/L \tag{23.1}$$

and[1]

$$\sqrt{(R+sL)(G+sC)}=jn\pi/d,\quad n=0,1,2,\ldots \tag{23.2}$$

Thus, we have

$$s_n=\frac{-(RC+LG)}{2CL}\pm j\frac{\sqrt{-4CL(RG+(\frac{n\pi}{d})^2)+(RC+LG)^2}}{2CL} \tag{24}$$

Now, let us compare the poles obtained by the *moment matching method* with *our exact method.*

[1] The root $s=-G/C$ is excluded, because $\lim_{s\to-G/C}Z_0(s)\sinh\lambda(s)d\neq0$.

Table 1: Comparison between our method and the moment matching method [2][2]. The length of line is $d = 5[mm]$, and the circuit parameters are

$$R = 0.5[\Omega/mm], \qquad L = 10[nH/mm]$$
$$C = 4[pF/mm] \qquad G = 0.5[mS/mm]$$

No.	Exact poles with our method	Poles from the moment matching method
p_0	-0.500E-01	-0.500E-01
p_1	-0.875E-01+ j0.314E+01	-0.875E-01+j0.314E+01
p_2	-0.875E-01+ j0.628E+01	-0.875E-01+j0.628E+01
p_3	-0.875E-01+ j0.942E+01	-0.875E-01+j0.942E+01
p_4	-0.875E-01+ j0.125E+02	-0.867E-01+j0.121E+02
p_5	-0.875E-01+ j0.157E+02	-0.196E+01+j0.128E+02
p_6	-0.875E-01+ j0.188E+02	0.178E+01+j0.120E+02
p_7	-0.875E-01+ j0.219E+02	-0.470E+01+j0.133E+02

Observe that the moment matching method has the serious error after no.5 pole. It seems that the error happens from the Padè approximation, and the Maclaurin expansion which can be applied only around the origin.

3. Asymptotic equivalent circuit

Note that, for the lossy interconnects, the poles far from the imaginary axis in s-plane don't give large effect in the transient analysis, and the residues usually become the smaller as the poles leave from the origin [2]. Therefore, the lossy interconnect can be well-modeled with relatively few terms of (21). In this section, we consider the modeling of the admittance matrix described by the partial fractions in the form of (21). For simplicity, we consider an uncoupled uniform interconnect.
Put the poles as follows:

$$p_0 = -u_0, \quad p_i = -u_i \pm jv_i, \quad i = 1, 2, \ldots \tag{25}$$

where

$$u_0 = R/L, \quad u_i = \frac{LG + RC}{2LC}$$
$$v_i = \frac{\sqrt{4LC(RG + (\frac{n\pi}{d})^2) - (LG + RC)^2}}{2LC}$$

Then, the corresponding residues are given by

$$k_{0,11} = \frac{2}{Ld}, \quad k_{i,11} = \left.\frac{1}{Z_0(s)D\{\lambda(s)\}d}\right|_{s=p_i} \quad i = 1, 2, \ldots \tag{26.1}$$

$$k_{0,12} = -\frac{2}{Ld}, \; k_{i,12} = \left.\frac{(-1)^{i-1}}{Z_0(s)D\{\lambda(s)\}d}\right|_{s=p_i} \quad i = 1, 2, \ldots \tag{26.2}$$

where $k_{0,22} = k_{0,11},\ k_{i,22} = k_{i,11},\ k_{0,21} = k_{0,12},\ k_{i,21} = k_{i,12}$ and

$$D\{\lambda(s)\} = \frac{\partial\lambda(s)}{\partial s} = \frac{L}{2}\sqrt{\frac{G + sC}{R + sL}} + \frac{C}{2}\sqrt{\frac{R + sL}{G + sC}} \tag{27}$$

Since the poles and residues are composed of the complex conjugate ones, the admittance matrices can be described as follows:

$$Y_{11}(d, s) = Y_{22}(d, s)$$

$$\simeq \frac{k_0}{s + p_0} + \sum_{i=1}^{2M} \frac{2\Re\{k_i\}s + 2\Re\{k_i p_i\}}{(s + u_i)^2 + v_i^2} \tag{28.1}$$

$$Y_{12}(d, s) = Y_{21}(d, s)$$

$$\simeq -\frac{k_0}{s + p_0} + \sum_{i=1}^{2M} \frac{2 \times (-1)^{i-1}(\Re\{k_i\}s + \Re\{k_i p_i\})}{(s + u_i)^2 + v_i^2} \tag{28.2}$$

For simplicity, we set here $k_0 = k_{0,11} = -k_{0,12}$, $k_i = k_{i,11} = |k_{i,12}|$, and $2M + 1$ means the order of approximation. Set each term of (28) as follows:

$$Y_0(s) = \frac{b_0}{s + a_0}, \quad Y_i(s) = \frac{b_{i1}s + b_{i0}}{s^2 + a_{i1}s + a_{i0}}, \quad i = 1, 2, \ldots, 2M \tag{29}$$

Then, the circuits given by (29) are realized by Fig.1.

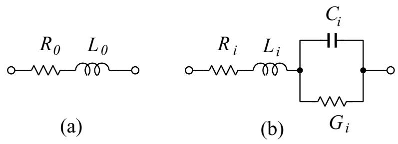


Figure 1: Equivalent circuits of eq.(29.1) and (29.2).

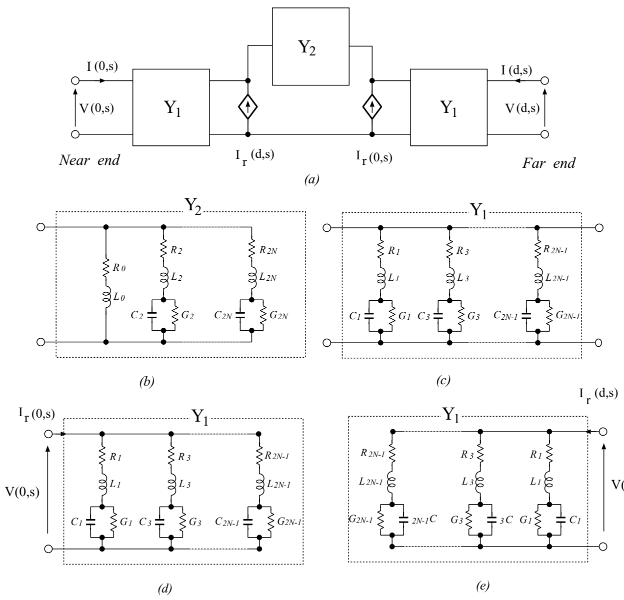


Figure 2: Equivalent circuits of an interconnect.

where the parameters are given by

$$L_0 = \frac{1}{b_0}, \quad R_0 = \frac{a_0}{b_0} \tag{30.1}$$

$$\left.\begin{aligned} L_i &= \frac{1}{b_{i1}}, \quad R_i = \frac{a_{i1}b_{i1} - b_{i0}}{b_{i1}^2} \\ C_i &= \frac{b_{i1}^3}{a_{0i}b_{i1}^2 + (b_{i0} - a_{i1}b_{i1})b_{i0}} \\ G_i &= \frac{b_{i1}^2 b_{i0}}{a_{0i}b_{i1}^2 + (b_{i0} - a_{i1}b_{i1})b_{i0}} \end{aligned}\right\} \tag{30.2}$$

Observe that the admittance of $(Y_{11}(s), Y_{22}(s))$ and $(Y_{12}(s), Y_{21}(s))$ are composed of the same terms except of their signs. Hence, put the admittance composed of the

[2]We used a double precision computer for the moment matching.

same sign to $Y_1(s)$, and that of the opposite sign to $Y_2(s)$ as follows:

$$Y_1(s) = \sum_{i=1}^{N} \frac{2\Re\{k_{2i-1}\}s + 2\Re\{k_{2i-1}p_{2i-1}\}}{(s+u_{2i-1})^2 + v_{2i-1}^2} \quad (31.1)$$

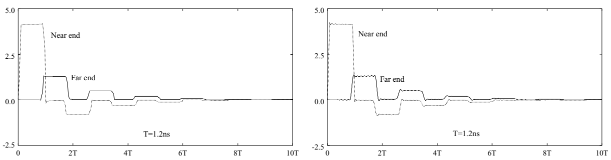
$$Y_2(s) = \frac{k_0}{s+p_0} + \sum_{i=1}^{N} \frac{2\Re\{k_{2i}\}s + 2\Re\{k_{2i}p_{2i}\}}{(s+u_{2i})^2 + v_{2i}^2} \quad (31.2)$$

Then, we can realize the interconnect by the equivalent circuit as shown in Fig.2. Note that the current controlled current sources $I_r(d,s)$ in the figure shows the reflection at the far end, and $I_r(0,s)$ the reflection at the near end.

4. Illustrative examples

4.1 Interconnect terminated by linear resistors

To investigate the accuracy of our asymptotic method, consider an interconnect terminated by $10[\Omega]$ at near and far ends, respectively.



(a) Solution by the numerical inverse Laplace transformation.

(b) Solution by our asymptotic method with our 15th order.

Figure 3: Comparison our asymptotic method to the numerical inverse Laplace transformation.

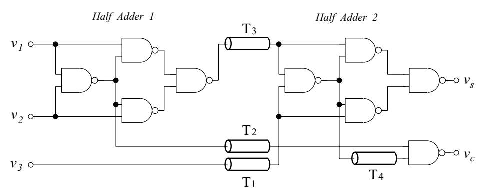
$$L = 1[nH/mm], \quad R = 0.5[\Omega/mm], \quad C = 4[pF/mm],$$

$$G = 0.5[mS/mm], \quad d = 5[mm]$$

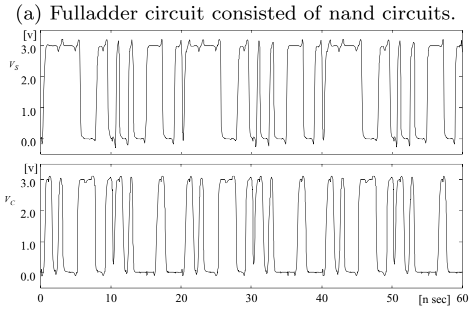
From these results, we found that our method can get the good results with 15th order of the approximation. Furthermore, the time delay is exactly estimated even with our low order model.

4.2 A LSI circuit coupled with interconnects

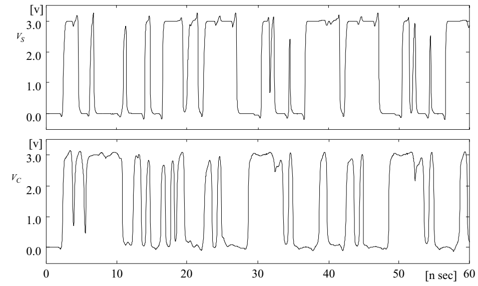
Consider a fulladder circuit coupled with interconnects.



(a) Fulladder circuit consisted of nand circuits.



(b) Transient response of the fulladder without interconnects.



(c) Transient response of the fulladder coupled with interconnects with 37th order.

Figure 4: $R = 500[\Omega/mm]$, $L = 10[\mu/mm]$, $C = 0.01[pF/mm]$, $G = 0.5[\mu S/mm]$, $d = 5[mm]$

In this example, we replaced the interconnects by the asymptotic equivalent circuits, and get the results with transient analysis of Spice. We found from these results that it has large time delay. Our simulator is easily applied to any kind of circuits.

5. Conclusions and remarks

In this paper, we have proposed an asymptotic equivalent circuit for interconnets. Thus, replacing the interconnect by the circuit, we can easily calculate the transient response with Spice. At first step, we calculate the exact poles and the residues of admittance matrix, and describe it in the form of partial fractions. Secondly, we realize the admittance matrix by the equivalent circuit. We found that we can get good result even with our low order approximation.
As the future problem, we need to extend our algorithm to nonuniform interconnects.

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