REFLECTION AND TRANSMISSION OF PHASE-INVERSION-WAVES IN OSCILLATORS COUPLED BY TWO KINDS INDUCTORS

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ABSTRACT

In this study, nonlinear wave phenomena related to reflections and transmissions of the phase-inversion-waves around the discontinuity of the system consisting of two parts of coupled van der Pol oscillators are investigated. Behavior of the phase-inversion-waves around a discontinuity in a coupled system consisting of two kinds of arrays of van der Pol oscillators classified into 8 types by computer simulations. Further, the mechanisms of the reflection of the phase-inversion-waves at the discontinuity are explained.

1. INTRODUCTION

Large number of coupled limit-cycle oscillators are useful as models for a wide variety of systems in natural fields, for example, diverse physiological organs including gastrointestinal tracts and axial fiber of nervous systems, convecting fluids, arrays of Josephson junctions and so on. Hence, it is very important to analyze synchronization and the related phenomena observed in coupled oscillators in order to clarify mechanisms of generations or in order to control the generating-conditions of various phenomena in such natural systems. In the field of the electrical engineering, a lot of studies on synchronization phenomena of coupled van der Pol oscillators have been carried out up to now ([1]-[5] and references therein). Recently, we have discovered very interesting wave propagation phenomena of phase states between two adjacent van der Pol oscillators coupled by inductors as a ladder [6]. In the study, we named the continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase as "phase-inversion-wave." Further, we reported the detail on "phase-wave" in [7], which is propagation of the phase difference less than 180 degrees between two adjacent oscillators. The phase-waves exist only in the transient states unlike the phase-inversion-waves.

In this study, we observe behavior of the phase-inversion-waves around a discontinuity in a coupled system consisting of two kinds of arrays of van der Pol oscillators, namely van der Pol oscillators coupled by inductors L_{00} and those coupled by L_{01}. Further, we explain the mechanisms of the reflection of the phase-inversion-waves at the discontinuity.

2. CIRCUIT MODEL

Circuit model is shown in Fig. 1. From the 1st to the N_1th oscillators they are coupled by inductors L_{00}, while from the N_1th to the N_2th oscillators they are coupled by L_{01}. Namely, in this model there is a discontinuity at the N_1th oscillator. At first, the $v-i$ characteristics of nonlinear negative resistors in the circuit are assumed by the following functions.
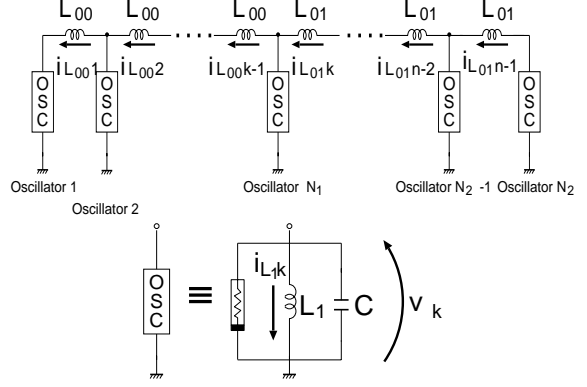

Figure 1: Circuit Model.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are expressed as

[1st oscillator]

$$\begin{aligned}\dot{x}_1 &= y_1 \\ \dot{y}_1 &= -x_1 + \alpha_0(x_2 - x_1) + f(y_1)\end{aligned} \tag{2}$$

[2nd $\sim (N_1 - 1)$th oscillators]

$$\begin{aligned}\dot{x}_k &= y_k \\ \dot{y}_k &= -x_k + \alpha_0(x_{k+1} - 2x_k + x_{k-1}) + f(y_k)\end{aligned} \tag{3}$$

[N_1th oscillator]

$$\begin{aligned}\dot{x}_{N_1} &= y_{N_1} \\ \dot{y}_{N_1} &= -x_{N_1} + \alpha_0(x_{N_1+1} - x_{N_1}) \\ &\quad + \alpha_1(x_{N_1-1} - x_{N_1}) + f(y_{N_1})\end{aligned} \tag{4}$$

[$(N_1 + 1)$th $\sim (N_2 - 1)$th oscillators]

$$\begin{aligned}\dot{x}_k &= y_k \\ \dot{y}_k &= -x_k + \alpha_1(x_{k+1} - 2x_k + x_{k-1}) + f(y_k)\end{aligned} \tag{5}$$

[N_2th oscillator]

$$\begin{aligned}\dot{x}_{N_2} &= y_{N_2} \\ \dot{y}_{N_2} &= -x_{N_2} + \alpha_1(x_{N_2-1} - x_{N_2}) + f(y_{N_2})\end{aligned} \tag{6}$$

where

$$f(y_k) = \varepsilon\left(y_k - \frac{1}{3}y_k^3\right) \tag{7}$$

and

$$t = \sqrt{L_1 C}\,\tau \;,\; i_{L_1 k} = \sqrt{\frac{C g_1}{3 L_1 g_3}} x_k \;,\; v_k = \sqrt{\frac{g_1}{3 g_3}} y_k \;,$$

$$\alpha_0 = \frac{L_1}{L_{00}} \;,\; \alpha_1 = \frac{L_1}{L_{01}} \;,\; \varepsilon = g_1\sqrt{\frac{L_1}{C}} \;,\; \frac{d}{d\tau} = \text{`` $\cdot$ ''}.\tag{8}$$

It should be noted that α corresponds to the coupling and that ε corresponds to the nonlinearity. Equations (2)-(8) are calculated for the cases of $(N_1, N_2) = (9, 17)$ and $(21, 30)$ by using the fourth-order Runge-Kutta method.

3. REFLECTION AND TRANSMISSION

In this section we observe reflection and transmission of the phase-inversion-waves for the case of $(N_1, N_2) = (9, 17)$. We fix the parameters $\varepsilon = 0.20$ and $\alpha_0 = 0.050$ and α_1 is varied as a control parameter. In the computer calculations, we produced phase-inversion-waves as follows. Almost same initial conditions are given for all oscillators to produce complete in-phase synchronization, which is one stable steady states in the system. Next, invert the voltage and the current of the first oscillator.

Fig. 2 shows the observed result. In diagrams, vertical axes are sum of voltages of adjacent oscillators and horizontal axes are time. The observed phenomena are classified into eight types as follows.

(a) ∼around 0.001: The phase-inversion-waves are reflected at the discontinuity almost completely.

(b) around 0.002∼0.018: The phase-inversion-waves are reflected and transmitted. But, the phase-inversion-waves change to the phase-waves after the transmission.

(c) around 0.019∼0.025: The phase-inversion-waves are reflected and transmitted and do not change to the phase-waves after the transmission.

(d) around 0.026∼0.043: The phase-inversion-waves are reflected and transmitted. But, the phase-inversion-waves change to the phase-waves after the reflection.

(e) around 0.044∼0.056: The phase-inversion-waves are transmitted almost completely. The phase-inversion-waves transmitted completely when α_1 is equal to 0.050.

(f) around 0.057∼0.130: The phase-inversion-waves are transmitted. The phase-waves are generated after the phase-inversion-waves are transmitted.

(g) around 0.131∼0.173: The phase-inversion-waves are reflected and transmitted. But, the phase-inversion-waves change to the phase-waves after the reflection and transmission.

(h) around 0.174 ∼: The phase-inversion-waves are reflected and transmitted. But, the phase-inversion-waves change to the phase-waves after the transmission.

It should be noted that we could observe different transmission phenomena for the cases of $L_{01} \gg L_{00}$ and $L_{01} \ll L_{00}$. Namely, for $L_{01} \gg L_{00}$ ($\alpha_1 \ll 0.05$) phase-inversion-wave is not transmitted beyond the discontinuity. While, for $L_{01} \ll L_{00}$ ($\alpha_1 \gg 0.05$) phase-inversion-wave is transmitted as phase-wave beyond the discontinuity. This interesting difference will be explained in the Sec. 5.

Figure 3 summarizes the above bifurcation.

(a) $\alpha_1 = 0.0001$.

(b) $\alpha_1 = 0.018$.

(c) $\alpha_1 = 0.021$.

(d) $\alpha_1 = 0.027$.

(e) $\alpha_1 = 0.046$.

(f) $\alpha_1 = 0.100$.

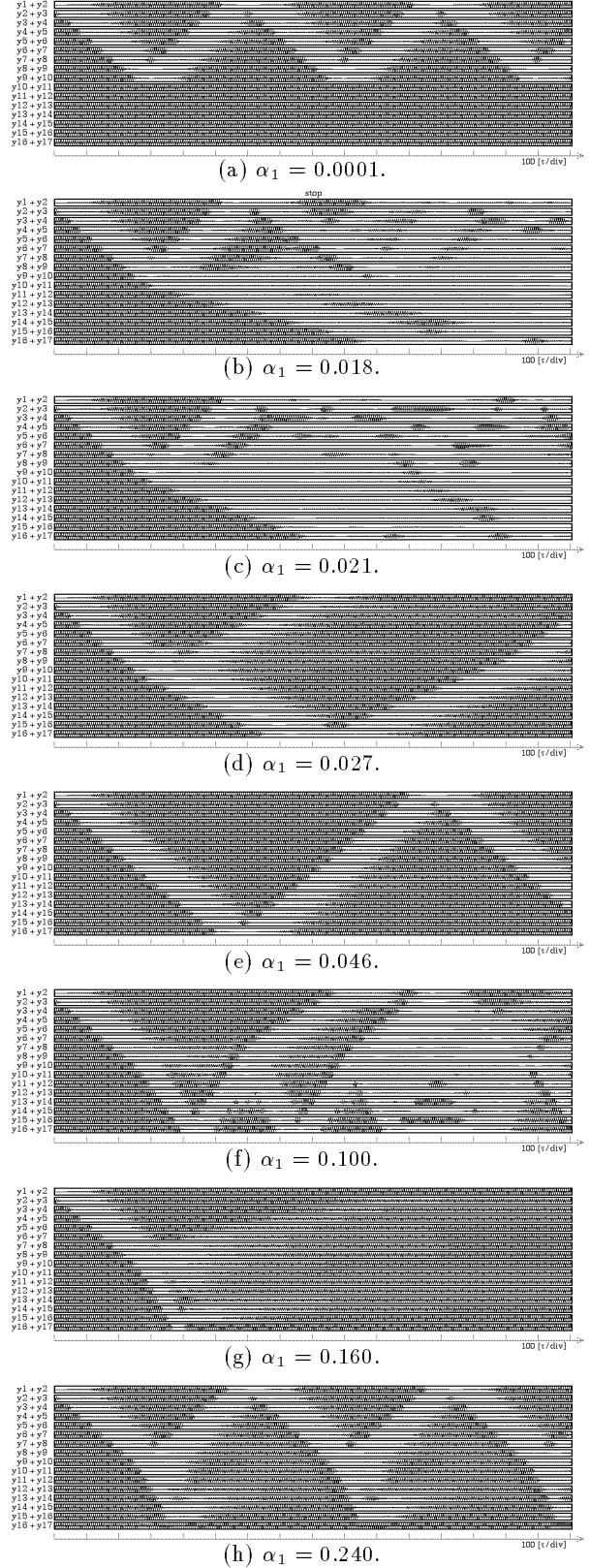(g) $\alpha_1 = 0.160$.

(h) $\alpha_1 = 0.240$.

Figure 2: Observation of the reflection and the transmission of the phase-inversion-waves. $N_1 = 9$, $N_2 = 17$, $\alpha_0 = 0.050$ and $\varepsilon = 0.20$.
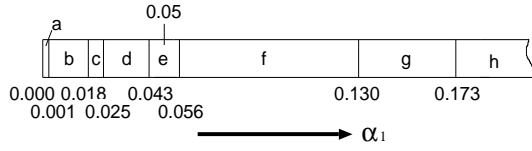
Figure 3: Bifurcation diagram. $N_1 = 9$, $N_2 = 17$, $\alpha_0 = 0.050$ and $\varepsilon = 0.20$.

4. COMPARISON WITH WAVES IN TRANSMISSION LINE

The reflection and transmission observed in our system seem to be similar to wave phenomena observed in two transmission lines with different characteristic impedances joined in tandem. However, we found two major different phenomena in our system as follows:

1. After the reflection or the transmission, the phase-inversion-waves often changes their characteristics, namely we can observe that the phase-inversion-waves change to phase-waves with phase difference less than 180 degrees for wide range of control parameter value.

2. The reflection or the transmission may occurs two times when the phase-inversion-waves arrives at the discontinuity. The reason of this phenomena is could be explained as follows. The oscillator at the discontinuity needs longer time to returns to in-phase synchronization frequency than time the other oscillators need, after the phase-inversion-waves arrived at the oscillator. This produces new phase difference between the oscillator at the discontinuity and its adjacent oscillators and it results the generation of the second phase-wave.

5. MECHANISM OF REFLECTION

In this section, the reflection mechanism of the phase-inversion-waves at the discontinuity is explained according to the computer calculated results for the case of $(N_1, N_2) = (21, 30)$.

It has been already known that oscillation frequency of in-phase synchronization of oscillators coupled by inductors is different from that of anti-phase synchronization. Namely, f_{in}, oscillation frequency of in-phase synchronization, is smaller than f_{anti}, oscillation frequency of anti-phase synchronization. Further, the difference between f_{in} and f_{anti} increases as coupling inductance increases. We use this characteristic to explain the mechanism of the reflection.

5.1 When $L_{01} \gg L_{00}$ (i.e. $\alpha_1 \ll \alpha_0$).

1. Let us assume that OSC20∼OSC30 are in-phase synchronization and that a phase-inversion-wave from in-phase to anti-phase reaches at OSC20 from the direction of OSC1.

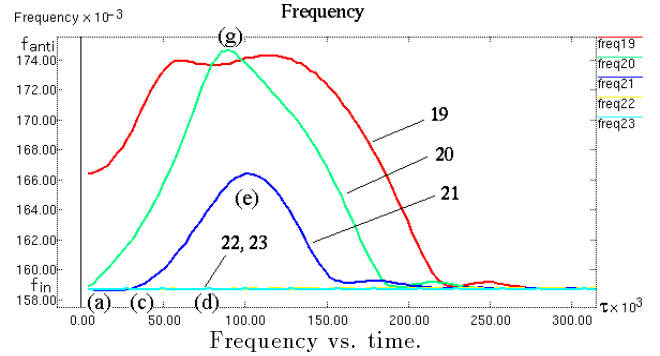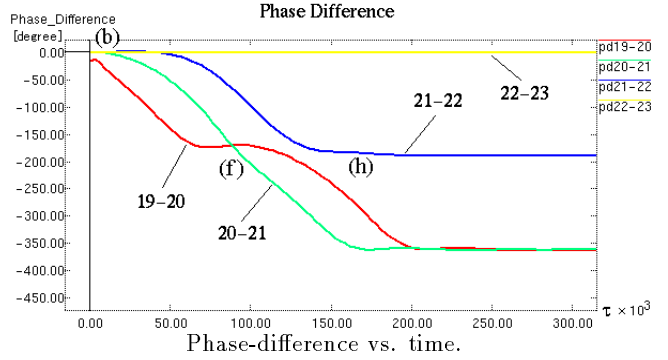

Phase-difference vs. time.

Frequency vs. time.

Figure 4: Reflection at the discontinuity for the case of $\alpha_1 \ll \alpha_0$. $N_1 = 21$, $N_2 = 30$, $\alpha_0 = 0.0500$, $\alpha_1 = 0.0001$ and $\varepsilon = 0.20$.

2. Oscillation frequency of OSC20 f_{20} begins to change from f_{in} toward f_{anti}, because in-phase synchronization between OSC19 and OSC20 breaks by the phase-inversion-wave (see (a) in Fig.4).

3. As oscillation frequency of OSC20 f_{20} changes from f_{in} to f_{anti}, phase difference between OSC20 and OSC21 $\Phi_{20,21}$ approaches $-\pi$ (see (b) in Fig.4).

4. As $\Phi_{20,21}$ approaches $-\pi$, oscillation frequency of OSC21 f_{21} begins to change from f_{in} to f_{anti} (see (c) in Fig.4).

5. Change of f_{21} does not propagate to OSC22 immediately, because L_{01} is much larger than L_{00} (see (d) in Fig.4).

6. Hence, f_{21} cannot reach f_{anti} and begins returning to f_{in} (see (e) in Fig.4).

7. As f_{21} returns to f_{in}, $\Phi_{20,21}$ continues to increase until reaching -2π (see (f) in Fig.4).

8. By the effect of the decrease of $\Phi_{20,21}$, f_{20} begins to decreases again from f_{anti} toward f_{in} (see (g) in Fig.4).

9. $\Phi_{21,22}$ becomes π, because in spite of constant value which is f_{22}, f_{21} changes from f_{in} and returns to f_{in} again (see (h) in Fig.4).

As L_{01} approaches L_{00}, it becomes easier that change of f_{21} propagates to OSC22. Hence, we could observe that the phase-inversion-wave is transmitted beyond the discontinuity as shown in Figs. 3(b)(c)(d). While, the reflection caused by the fact that the change of f_{21} is hard to propagate becomes smaller.

5.2 When $L_{01} << L_{00}$ (i.e. $\alpha_1 >> \alpha_0$).
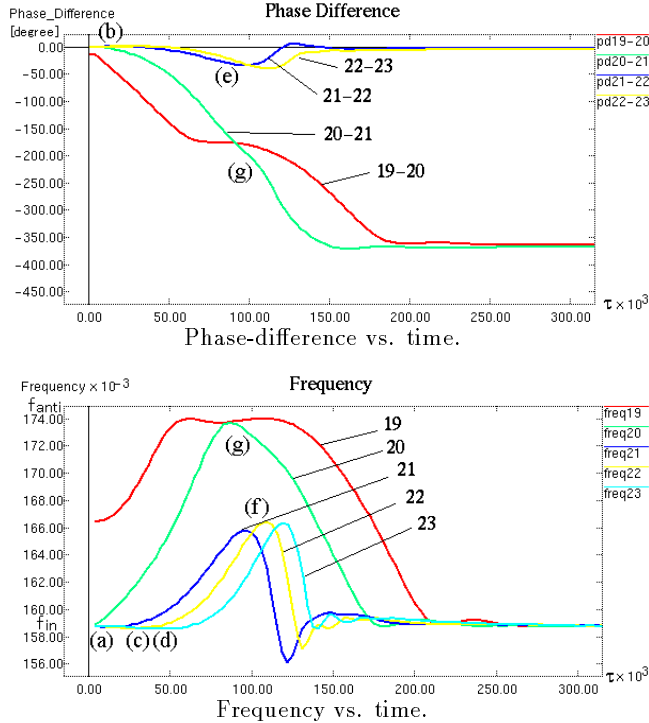

Phase-difference vs. time.

Frequency vs. time.

Figure 5: Reflection at the discontinuity for the case of $\alpha_1 >> \alpha_0$. $N_1 = 21$, $N_2 = 30$, $\alpha_0 = 0.00500$, $\alpha_1 = 0.20000$ and $\varepsilon = 0.20$.

1. Let us assume that OSC20~OSC30 are in-phase synchronization and that a wave from in-phase to anti-phase reaches at OSC20 from the direction of OSC1.

2. Oscillation frequency of OSC20 f_{20} begins to change from f_{in} toward f_{anti}, because in-phase synchronization between OSC19 and OSC20 breaks by the phase-inversion-wave (see (a) in Fig.5).

3. As oscillation frequency of OSC20 f_{20} changes from f_{in} to f_{anti} $\Phi_{20,21}$ approaches $-\pi$ (see (b) in Fig.5).

4. As $\Phi_{20,21}$ approaches $-\pi$, oscillation frequency of OSC21 f_{21} changes from f_{in} to f_{anti} (see (c) in Fig.5).

5. Change of f_{21} propagates to OSC22 immediately, because L_{01} is much smaller than L_{00} (see (d) in Fig.4).

6. Hence, in-phase synchronization between OSC21 and OSC22 is hard to break. (see (e) in Fig.5). And f_{21} and f_{22} return to f_{in} together (see (f) in Fig.5).

7. As f_{21} returns to f_{in}, $\Phi_{20,21}$ continues to increase until reaching -2π (see (g) in Fig.5).

Hence, when L_{01} is much smaller than L_{00}, the transmission waves always exist.

As L_{01} approaches L_{00}, it becomes more difficult that phase states of OSC21 and OSC22 hold in-phase synchronization. Hence, the reflection of the phase-inversion-waves is disappear as shown in Figs. 3(f)(g).

6. CONCLUSIONS

In this study, we observed behavior of the phase-inversion-waves around the discontinuity of the system consisting of two parts of coupled van der Pol oscillators. By computer calculations observed phenomena were classified into 8 types. Further, we could explain the mechanism of the reflection of the phase-inversion-wave at the discontinuity and made clear that different mechanisms reflect the waves fro the cases of $L_{01} > L_{00}$ and $L_{01} < L_{00}$.

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