A DESIGN METHOD OF CHAOTIC CIRCUITS USING AN OSCILLATOR AND A RESONATOR

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ABSTRACT

In this study, a simple design method of chaotic circuits is proposed, which utilizes an autonomous oscillator, a resonator and two diodes. By applying three kinds of oscillators and two types of resonators, six circuit models are obtained. It is confirmed that chaotic phenomena are observed in all of six models by both circuit experiments and computer calculations.

1. INTRODUCTION

Recently, engineering applications using chaotic circuits attract many researchers' attentions such as chaos communication systems, chaos controls, chaos cryptosystems, chaos noise generators, and so on. In those systems, individual characteristics of chaotic circuits usually much influence the performance of the systems. Therefore, researchers have to select a chaotic circuit appropriate to their systems. However, the number of known chaotic circuits is not so large and it is not easy to design new chaotic circuits except special skillful people [1]-[5]. Hence, they have to abandon to search the best circuit and compromise with relatively better circuit. Therefore, simple techniques to design new chaotic circuits are necessary[8][10]. We have already proposed a simple design technique which utilizes two oscillators coupled by diodes and have comfirmed we could produce several new chaotic circuits by circuit experiments and computrer calculations[6][7][9][11].

In this study, we propose a new design method to produce simpler chaotic circuits. The method utilizes an autonomous oscillator, a resonator and two diodes. By applying three kinds of oscillators and two types of resonators, six circuit models are obtained. It is confirmed that chaotic phenomena are observed in all of six models by both circuit experiments and computer calculations.

2. SYSTEM MODEL

In our design method, we use a system model shown in Fig. 1, which consists of an oscillator, a resonator and two diodes. In this study, three kinds of second order oscillators: two types of van der Pol oscillators (parallel type and series type) and a Wien bridge oscillator as shown in Fig. 2 and two types of resonators: parallel and series type LC resonators as shown in Fig. 3 are applied to the system. For computer calculations, the coupling diodes are modeled with piecewise linear function as shown in Fig. 4. When we use the parallel type resonator, the voltage controlled type diode model is used. While, the current controlled type diode model is used for the series type resonator. Further, we assume that the nonlinear resistors and input-output characteristics of the operational amplifier in all oscillators are described by linear functions, because we observed attractors only in their linear regions in circuit experiments.

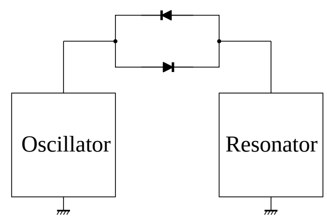


Figure 1: System model.

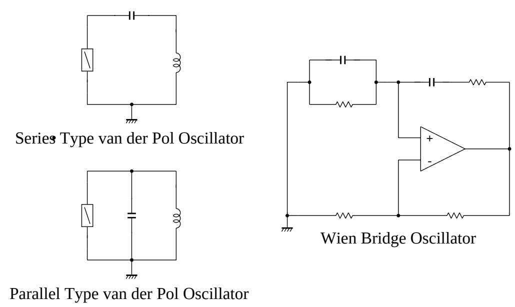


Figure 2: Three kinds of oscillators.

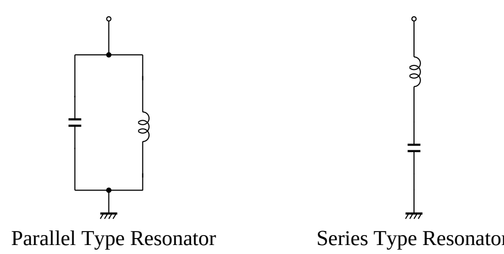


Figure 3: Two types of resonators.

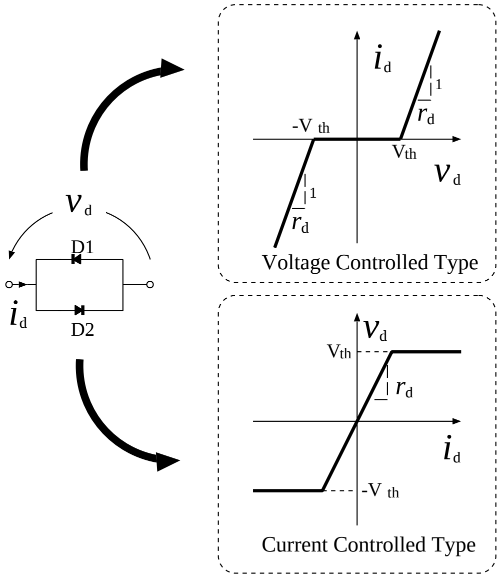


Figure 4: Two types of diode models.

Obviously, we can obtain six circuit models using the oscillators in Fig. 2 and the resonators in Fig. 3. Circuit equations governing these six circuit models are described by piecewise linear four-dimensional differential equations. For two of the six circuit models, we show the circuit equations in the followings.

2.1. Model 1

The Circuit Model 1, which is shown in Fig. 5, consists of a parallel type van der Pol oscillator and a series type LC resonator.

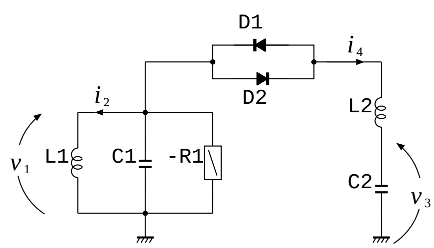


Figure 5: Circuit Model 1.

Circuit equations of the Circuit Model 1 are described as follows:

$$\left\{\begin{array}{lcl} C_1\dfrac{dv_l}{dt} & = & \dfrac{1}{R_1}v_1 - i_2 - i_4, \\ L_1\dfrac{di_2}{dt} & = & v_1, \\ C_2\dfrac{dv_3}{dt} & = & i_4, \\ L_2\dfrac{di_4}{dt} & = & v_1 - v_3 - v_d, \end{array}\right. \tag{1}$$

where

$$v_d = \frac{1}{2}r_d\left\{\left|i_4 + \frac{V_{th}}{r_d}\right| + \left|-i_4 + \frac{V_{th}}{r_d}\right|\right\} \tag{2}$$

By substituting the variables and parameters,

$$\begin{gathered} x_1 = \frac{v_1}{V_{th}}, \quad x_2 = \sqrt{\frac{L_1}{C_1}}\frac{i_2}{V_{th}}, \quad x_3 = \frac{v_3}{V_{th}}, \quad x_4 = \sqrt{\frac{L_1}{C_1}}\frac{i_4}{V_{th}}, \\ x_d = \frac{v_d}{V_{th}}, \quad \text{“}\cdot\text{”} = \frac{d}{dt}, \quad \tau = \frac{1}{\sqrt{L_1C_1}}t, \\ \alpha = \frac{1}{R_1}\sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{C_1}{C_2}, \quad \gamma = \frac{L_1}{L_2}, \quad \delta = r_d\sqrt{\frac{C_1}{L_1}}, \end{gathered} \tag{3}$$

(1) is normalized as

$$\left\{\begin{array}{lcl} \dot{x}_1 & = & \alpha x_1 - x_2 - x_4, \\ \dot{x}_2 & = & x_1, \\ \dot{x}_3 & = & \beta x_3, \\ \dot{x}_4 & = & \gamma(x_1 - x_2 - x_d), \end{array}\right. \tag{4}$$

where

$$x_d = \frac{1}{2}\{|\delta x_4 + 1| + |-\delta x_4 + 1|\}. \tag{5}$$

2.2. Model 2

The Circuit Model 2, which is shown in Fig. 6, consists of a series type van der Pol oscillator and a parallel type LC resonator.

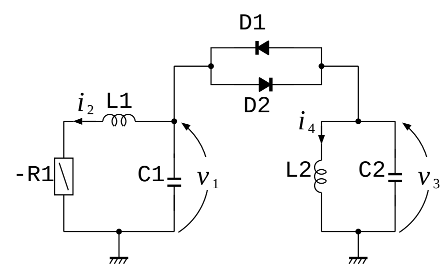


Figure 6: Circuit Model 2.

Circuit equations of the Circuit Model 2 are describe as follows:

$$\left\{\begin{array}{lcl} C_1\dfrac{dv_l}{dt} & = & -i_2 - i_d, \\ L_1\dfrac{di_2}{dt} & = & v_1 + R_1 i_2, \\ C_2\dfrac{dv_3}{dt} & = & -i_4 + i_d, \\ L_2\dfrac{di_4}{dt} & = & v_3, \end{array}\right. \tag{6}$$

where

$$\begin{aligned} i_d = \frac{1}{r_d}\Big\{ & v_1 - v_3 - \frac{1}{2}\left(|v_1 - v_3 - V_{th}|\right. \\ & \left. - |v_1 - v_3 + V_{th}|\right)\Big\}. \end{aligned} \tag{7}$$

By substituting the variables and parameters,

$$x_1 = \frac{v_1}{V_{th}}, \quad x_2 = \sqrt{\frac{L_1}{C_1}}\frac{i_2}{V_{th}}, \quad x_3 = \frac{v_3}{V_{th}}, \quad x_4 = \sqrt{\frac{L_1}{C_1}}\frac{i_4}{V_{th}},$$
$$x_d = \frac{v_d}{V_{th}}, \quad \text{“}\,\cdot\,\text{”} = \frac{d}{dt}, \quad \tau = \frac{1}{\sqrt{L_1 C_1}}t,$$
$$\alpha = R_1\sqrt{\frac{C_1}{L_1}}, \quad \beta = \frac{C_1}{C_2}, \quad \gamma = \frac{L_1}{L_2}, \quad \delta = \frac{1}{r_d}\sqrt{\frac{L_1}{C_1}}, \tag{8}$$

(6) is normalized as

$$\begin{cases} \dot{x}_1 &= \alpha x_1 - x_2 - \varepsilon y_d, \\ \dot{x}_2 &= x_1, \\ \dot{x}_3 &= \gamma(\beta x_3 - x_4 + \varepsilon y_d), \\ \dot{x}_4 &= \delta x_3, \end{cases} \tag{9}$$

where

$$y_d = x_1 - x_3 - \frac{1}{2}(|x_1 - x_3 - 1| - |x_1 - x_3 + 1|). \tag{10}$$

3. ANALYSIS

3.1. Circuit Model 1

Figure 7 shows circuit experimental results (a), computer calculated results (b) and it's Poincaré maps (c) of the Circuit Model 1. We can observe the Hopf bifurcation from periodic orbit (1) to quasi-periodic orbit (2). Further, chaotic attractors are observed after torus-breakdown (3).

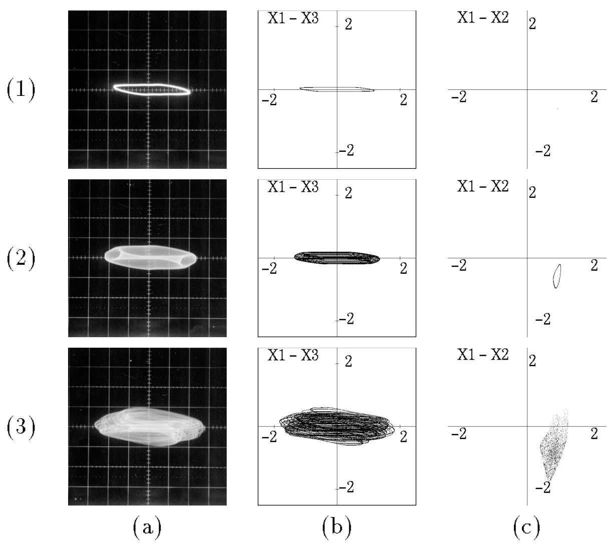


Figure 7: Attractors of Circuit Model 1. (a) circuit experimental results, (b) computer calculated results and (c) it's Poincaré maps.
Circuit experiment: Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. C_1 = 33.0 [nF], C_2 = 100.0 [nF], L_1 = 50.0 [mH] and L_2 = 20.0 [mH], (1) R_1 = 2.43 [kΩ], (2) R_1 = 2.23 [kΩ] and (3) R_1 = 1.80 [kΩ].
Computer calculation: $\beta = 0.33$, $\gamma = 2.50$, $\delta = 30.0$, (1) $\alpha = 0.20$, (2) $\alpha = 0.25$ and (3) $\alpha = 0.50$.

3.2. Circuit Model 2

Figure 8 shows circuit experimental results (a), computer calculated results (b) and it's Poincaré maps (c) of the Circuit Model 2. We can also observe the Hopf bifurcation from periodic orbit (1) to quasi-periodic orbit (2). Further, chaotic attractors are observed after torus-breakdown (3).

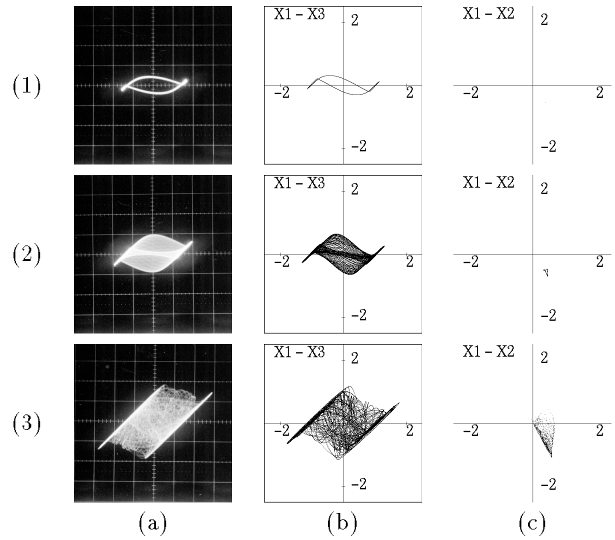


Figure 8: Attractors of Circuit Model 2. (a) circuit experimental results, (b) computer calculated results and (c) it's Poincaré maps.
Circuit experiment: Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. C_1 = 33.0 [nF], C_2 = 33.0 [nF], L_1 = 50.0 [mH], L_2 = 20.0 [mH], (1) R_1 = 0.472 [kΩ], (2) R_1 = 0.464 [kΩ] and (3) R_1 = 0.757 [kΩ].
Computer calculation: $\beta = 0.33$, $\gamma = 2.50$, $\delta = 30.0$, (1) $\alpha = 0.20$, (2) $\alpha = 0.26$ and (3) $\alpha = 0.50$.

3.3. Other circuit models

We carried out circuit experiments and computer calculations for the other four circuit models. Figures 9, 10, 11 and 12 show the four circuit models (a) and examples of chaotic attractors obtained from the corresponding circuit models. We confirmed that all of the circuit generated chaotic attractors for relatively wide range of circuit parameters.

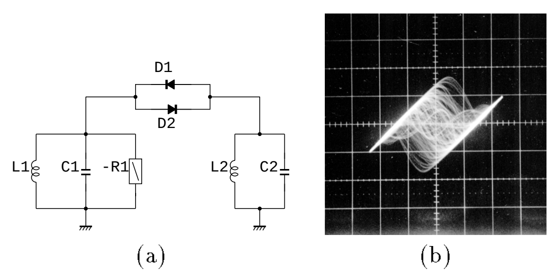
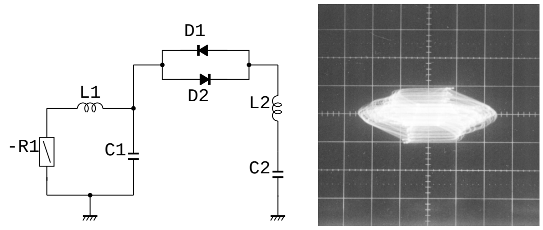
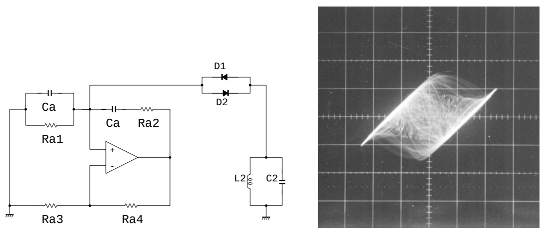


Figure 9: (a) Circuit Model 3 and (b) chaotic attractor (circuit experimental result). Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. C_1 = 68.0 [nF], C_2 = 33.0 [nF], L_1 = 100.0 [mH], L_2 = 20.0 [mH] and R_1 = 1.34 [kΩ].



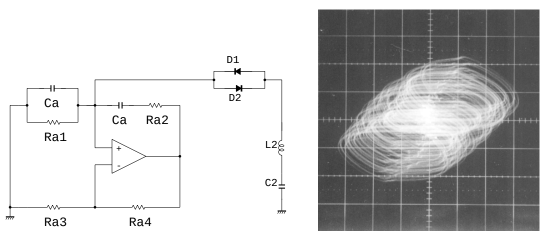
(a) (b)

Figure 10: (a) Circuit Model 4 and (b) chaotic attractor (circuit experimental result). Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. C_1 = 33.0 [nF], C_2 = 100.0 [nF], L_1 = 50.0 [mH], L_2 = 20.0 [mH] and R_1 = 0.68 [kΩ].



(a) (b)

Figure 11: (a) Circuit Model 5 and (b) chaotic attractor (circuit experimental result). Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. R_{a1} = 2.10 [kΩ], R_{a2} = 1.00 [kΩ], R_{a3} = 1.00 [kΩ], R_{a4} = 2.00 [kΩ], C_a = 33.0 [nF], C_2 = 33.0 [nF] and L_2 = 10.0 [mH].



(a) (b)

Figure 12: (a) Circuit Model 6 and (b) chaotic attractor (circuit experimental result). Horizontal axis is v_1 [0.5v/div]. Vertical axis is v_3 [0.5v/div]. R_{a1} = 0.680 [kΩ], R_{a2} = 1.00 [kΩ], R_{a3} = 1.00 [kΩ], R_{a4} = 2.00 [kΩ], C_a = 33.0 [nF], C_2 = 68.0 [nF] and L_2 = 20.0 [mH].

4. CONCLUSIONS

In this study, we have proposed a new design method of chaotic circuits utilizing an autonomous oscillator, a resonator and two diodes. We investigated six kinds of circuit models and confirmed that chaotic phenomena were generated in all of the six circuit models by both circuit experiments and computer calculations. We consider that further investigation on designing methods of chaotic circuits including this study could contributes for increase of possible engineering applications of chaotic systems.

5. REFERENCES

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