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ON SYNCHRONIZATION PHENOMENA IN CHAOTIC SYSTEMS COUPLED BY TRANSMISSION LINE

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ABSTRACT

In this study, synchronization phenomena in chaotic oscillators coupled by a transmission line are investigated. In particular investigation using real circuits is done for the first time. We report the very interesting results.

1. INTRODUCTION

Since synchronization of chaos has been reported by Pecora and Carroll [1], it has received a great deal of attention. A large number of recent papers have been published in this area. However, almost studies on chaos synchronization reported so far, have dealt with synchronization phenomena observed from chaotic oscillators coupled by lumped elements as in [2].

A few studies on systems coupled by elements with time delay have been reported. In [3] two chaotic systems coupled by two delay lines are investigated. It has been shown via numerical experiments that two chaotic circuits synchronized when the time delay exists.

We have also reported synchronization phenomena in a chaotic system coupled by transmission line [4]. It is confirmed by simulations that the subsystems synchronize by adjusting the characteristic impedance of the line when the time delay increases. However the experimental studies on such systems have not been reported so far.

In this study, following our previous study, we investigate two chaotic circuits coupled by a transmission line.In particular the systems are assessed by laboratory experiments as well as computer simulations. These circuits are simulated by using the method of characteristics [5]. In our experiments, delay lines and coaxial cables are used as transmission line. The delay line is a element that transmits input signal waveforms fidelity but delays time.

We show by experiments that v_{C2}-coupled synchronization is achieved, which coincide our previous results in [4]. It is also confirmed by simulations and experiments that v_{C1}-coupled subsystems synchronize only for small delays. A very interesting phenomena, that oscillation of the Chua's circuit stops when coaxial cables are used, is observed from v_{C1}-coupled system.

2. CIRCUIT MODEL

Fig. 1 shows v_{C1}-coupled system that is one of the circuit models used in this paper. v_{C2}-coupled system is omitted because of limitations of space. In these models the Chua's circuit is used as each chaotic subcircuit and two subcircuits are coupled by a transmission line. The Chua's circuit is a extremely simple system but it exhibits the complex dynamics of bifurcation and chaos.

In our simulations, the transmission line is just modeled using the method of characteristics, where the transmission line is replaced by the characteristic model [5] as shown in Fig. 2. After normalizing, we obtain the following circuit equations.

v_{C1}-coupled system:

$$\begin{aligned} \dot{x}_k &= \alpha(y_k - x_k - f(x_k) + \gamma(w_k - x_k)), \\ \dot{y}_k &= x_k - y_k + z_k, \\ \dot{z}_k &= -\beta y_k \end{aligned} \tag{1}$$

v_{C2}-coupled system:

$$\begin{aligned} \dot{x}_k &= \alpha(y_k - x_k - f(x_k)), \\ \dot{y}_k &= x_k - y_k + z_k + \gamma(w_k - y_k), \\ \dot{z}_k &= -\beta y_k \end{aligned} \tag{2}$$

Where

$$f(x) = bx + \frac{1}{2}(a-b)[|x+1| - |x-1|] \tag{3}$$

and

$$\begin{aligned} &\alpha = C_2/C_1, \quad \beta = C_2/LG^2, \quad \gamma = Y_0/G, \\ &\hat{t} = Gt/C_2, \quad \hat{\tau} = G\tau/C_2, \quad \text{“}\cdot\text{”} = d/d\hat{t}, \\ &x_k = v_{C1k}/B_p, \quad y_k = v_{C2k}/B_p, \quad z_k = i_{Lk}/GB_p, \\ &w_k = e_k(\hat{t} - \hat{\tau})/B_p, \quad a = m_0/G, \quad b = m_1/G. \end{aligned}$$

Y_0 and τ are the characteristic admittance and time delay of the transmission line respectively, and e_k is the waveform generator to simulate the reflection in the characteristic model.

3. NUMERICAL AND EXPERIMENTAL RESULTS

We have already investigated the v_{C2}-coupled system by simulations [4]. We first make experiment to confirm the results. The Chua's circuits are built, based on [6]. The circuit parameters are adjusted so that the circuits exhibit the double scroll attractor, and so that they synchronize when they are directly coupled by a resistor as reported in [2]. The measured values of synchronizing system are as follows:

$$R = 1.435[\text{k}\Omega], \quad C_1 = 9.50[\text{nF}], \quad C_2 = 103.8[\text{nF}],$$
$$L = 19.98[\text{mH}], \quad r_L = 40.25[\Omega],$$
$$m_0 = -0.75[\text{mS}], \quad m_1 = -0.41[\text{mS}], \quad B_p = 1.06[\text{V}]$$

Remaining the circuit parameters, the resistor is replaced by a delay line or coaxial cable. The delay line is a element that transmits input signal waveforms fidelity but delays time only when it is matched with suitable impedance at both ends. Namely how to use the delay lines is the same as the usage of coaxial cables. Note that the delay lines and the coaxial cables are not matched in our experiments. Table 1 shows the characteristics of the delay lines and coaxial cables we used[1],

Table 1: The characteristics of transmission lines.

Transmission Line	Z_0	τ
Delay Line #1 FSL05-050A	50[Ω]	5[nsec]
Delay Line #2 FN-500B	500[Ω]	500[nsec]
Coaxial Cable #1 3D2V 100 m	50[Ω]	500[nsec]
Coaxial Cable #2 3D2V 2 km	50[Ω]	10[μsec]

where Z_0 denotes the characteristic impedance. The experimental results are illustrated in Fig. 3. The subsystems synchronize for all cases. For the case that 2 km long of coaxial cable was used, surprisingly enough, two Chua's circuits synchronized. These results have good agreement with the simulation results [4]. Unfortunately we do not have any transmission lines with larger delay, thus we could not confirm whether what kinds of phenomena will happen for larger delay.

Next we investigate the v_{C1}-coupled system by experiments. With changing the circuit configuration, experimental investigations were done. Fig. 4 shows the results. For the case that the delay lines are used, the subsystems synchronized as shown in Fig. 4(a),(b). However both Chua's circuits stopped oscillating when the coaxial cables are used (see Fig. 4(c),(d)). These let us find the following two issues.

- For the same characteristic impedance, oscillations of the system stop when time delay increases.
- By varying the characteristic impedance, the subsystems synchronize even if time delay increases.

Second item can be also applied to the v_{C2}-coupled system [4].

To make these phenomena more reliable, we further simulate the v_{C1}-coupled system with the use of the method of characteristics [5]. In our numerical experiments, 4th order Runge-Kutta method is used, and we set the stepsize as 0.0001 and the parameters and initial conditions of two subcircuits as follows:

$$\alpha = 10, \quad \beta = 15, \quad a = -1.2, \quad b = -0.75, \quad B_p = 1,$$
$$x_1 = 1.0, \quad y_1 = -0.2, \quad z_1 = 0.3,$$
$$x_2 = 0.7, \quad y_2 = 0.4, \quad z_2 = 0.08.$$

Simulation results are shown in Fig. 5. The phenomena in circuit experiments could be also observed in computer simulations.

4. CONCLUSION

In this study, we have reported very interesting phenomena in a chaotic system coupled by a transmission line. Chaos synchronization is also achieved in such system as well as systems coupled with lumped elements.

Hereafter we intend to analyze the systems qualitatively. The systems may have more exciting phenomena according to the characteristic impedance and time delay.

5. REFERENCES

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[5] F. H. Branin, Jr., "Transient analysis of lossless transmission line," Proc. IEEE, vol. 55, pp. 2012-2013, Nov. 1967.

[6] M. P. Kennedy, "Robust OP amp realization of Chua's circuit," Frequenz, vol. 46, pp. 66-80, 1992.

[1] The delay lines are made by Showa Densen Corporation. The 3D2V cables are made in accordance with JIS (Japanese Industrial Standard). With binding the coaxial cables round drum, all experiments were done.

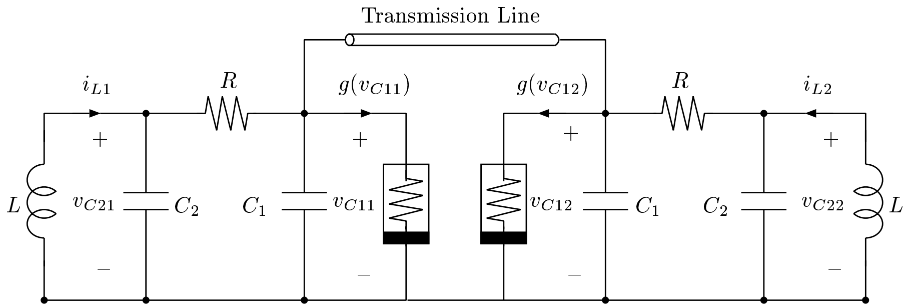


Figure 1: The circuit model. v_{C1}-coupled system.

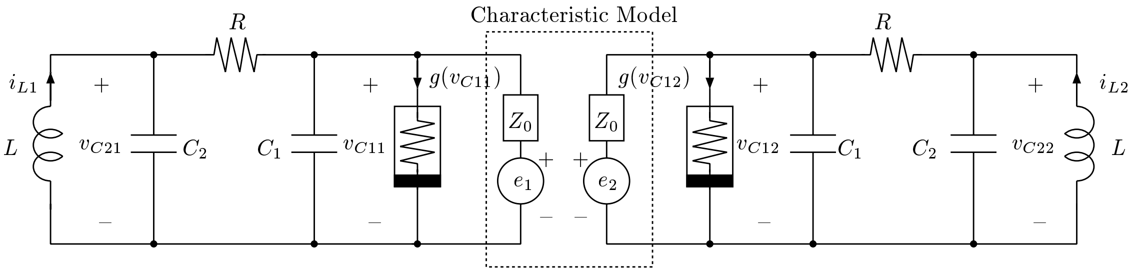
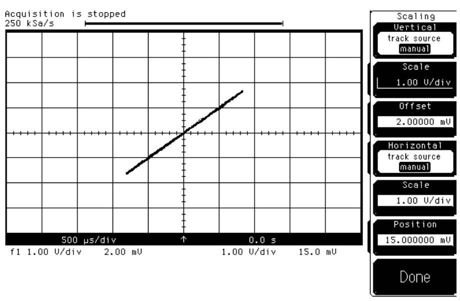
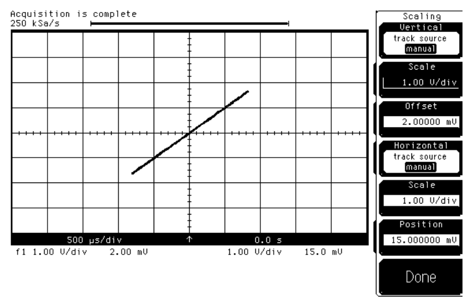


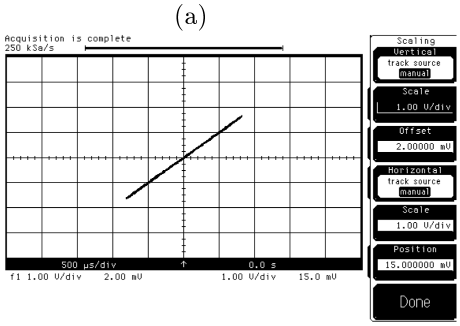
Figure 2: The equivalent circuit of Fig. 1.



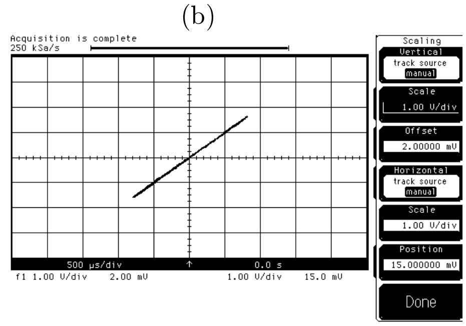
(a)



(b)



(c)



(d)

Figure 3: Synchronization in v_{C2}-coupled system. v_{C11} vs. v_{C12}. (a) Delay Line #1, (b) Delay Line #2, (c) Coaxial Cable #1, (d) Coaxial Cable #2.

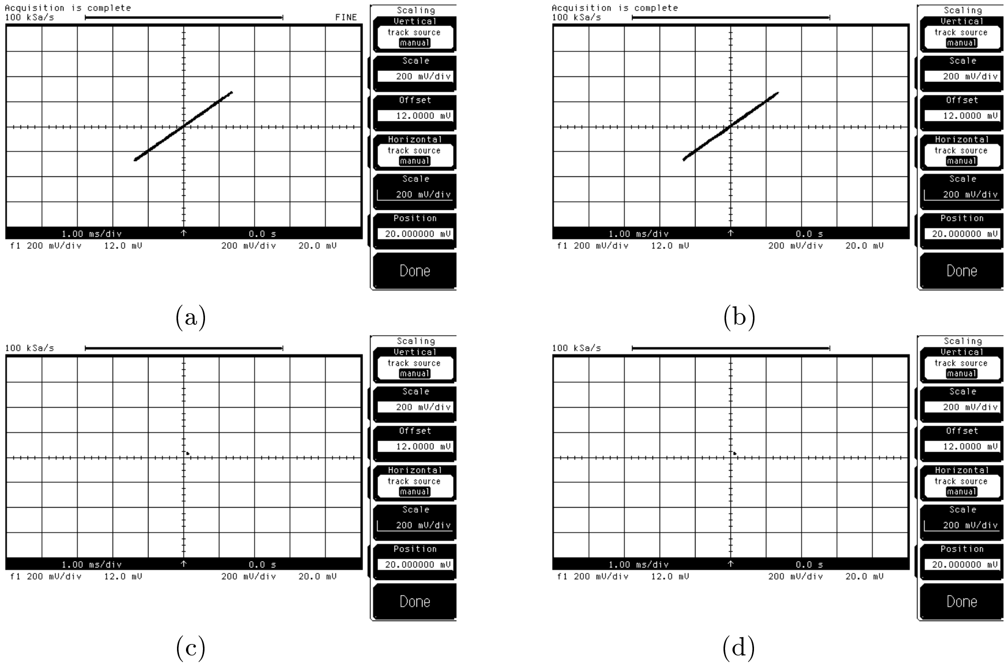


Figure 4: Experimentally observed phenomena in v_{C1}-coupled system. v_{C21} vs. v_{C22}. (a) Delay Line #1, (b) Delay Line #2, (c) Coaxial Cable #1, (d) Coaxial Cable #2.

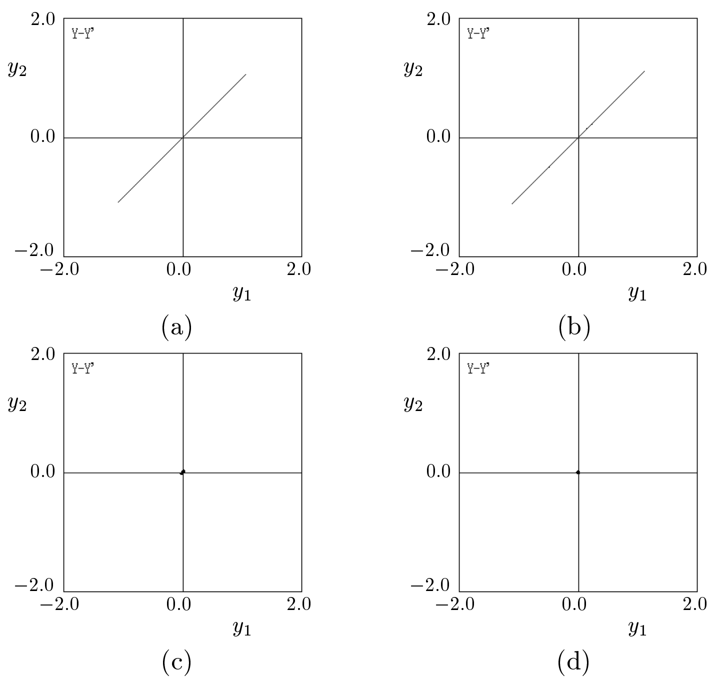


Figure 5: Numerically observed phenomena in v_{C1}-coupled system. (a) $\gamma = 10,\ \hat{\tau} = 0.001$, (b) $\gamma = 1,\ \hat{\tau} = 0.01$, (c) $\gamma = 10,\ \hat{\tau} = 0.01$, (d) $\gamma = 10,\ \hat{\tau} = 0.1$.