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COLLISIONS BETWEEN TWO PHASE-INVERSION-WAVES IN AN ARRAY OF OSCILLATORS

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ABSTRACT

In this study, the phenomena related to collisions between two phase-inversion-waves in an array of van der Pol oscillators are investigated. Behavior of the two phase-inversion-waves after they collide with each other is classified by computer simulations. Further, the mechanisms of the complete extinction and reflection of the two waves by using the relationship between phase states and oscillation frequencies are explained.

1. Introduction

A lot of studies on synchronization phenomena of coupled oscillators have been carried out up to now. Endo *et al.* have reported a details of theoretical analysis and circuit experiments about some coupled oscillators as a ladder, a ring and a two-dimensional array [1]-[3]. Recently, wave propagation phenomenon observed from coupled chaotic circuits is also reported [4][5]. However, such studies treat only transient states for a given set of initial conditions and there seems to be very few studies on continuously existing wave propagation phenomenon observed simple coupled oscillators circuits.

On the other hand, we have reported in our past study [6] wave propagation phenomena of phase states in van der Pol oscillators coupled by inductors as a ladder. In that study, we found "phase-inversion-wave" which is continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or anti-phase to in-phase. Further, we explained the mechanisms of the propagation and the reflection at the edge of the ladder by using the relationship between phase states and oscillation frequency

In this study, we pay our attentions on the phenomena related to collisions between two phase-inversion-waves in the same circuit. We can confirm by computer simulations that the behavior of the two phase-inversion-waves after they collide with each other is classified into three types; completely extinction, completely reflection and intermediate complex phenomenon. Further, we can explain the mechanisms of the completely extinction and the completely reflection of the two waves by using the relationship between phase states and oscillation frequency.

2. Circuit Model

Circuit model is shown in Fig. 1. N van der Pol oscillators are coupled by coupling inductors L_0. We carry out computer calculations for the case of $N = 17$. In the computer calculations, we assume the $v-i$ characteristics of nonlinear negative resistors in the circuit by the following functions.

$$i_r(v_k) = -g_0 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0). \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are expressed as

$$\dot{x}_k = y_k \tag{2}$$

$$\dot{y}_k = -x_k + \alpha(x_{k+1} - 2x_k + x_{k-1}) + \varepsilon\left(y_k - \frac{1}{3}y_k^3\right)$$

$$(k = 1, 2, \cdots, N, \quad x_0 = x_1, \quad x_{N+1} = x_N)$$

where

$$t = \sqrt{L_1 C}\tau\ ,\ i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_k\ , v_k = \sqrt{\frac{g_1}{3g_3}}y_k\ ,$$

$$\alpha = \frac{L_1}{L_0}\ ,\ \varepsilon = g_1\sqrt{\frac{L_1}{C}}\ ,\ \frac{d}{d\tau} = \text{“}\cdot\text{”}. \tag{3}$$

It should be noted that α corresponds to the coupling and that ε corresponds to the nonlinearity. Equations (2) are calculated by using the fourth-order Runge-Kutta method.

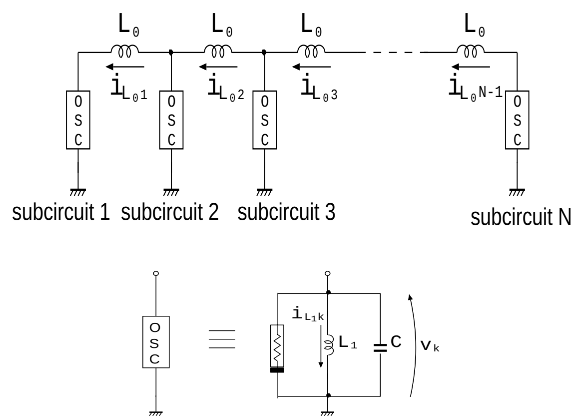


Figure 1: Array of van der Pol oscillators.

Figure 2 shows a typical example of observed phase-inversion-waves. In upper diagrams, vertical axes are sum of voltages of adjacent oscillators and horizontal axes are time. Hence, the diagrams show how phase differences between adjacent oscillators change as time goes. White regions in the diagram correspond to the states that two adjacent oscillators are anti-phase synchronization. While, black regions correspond to the in-phase synchronization. In lower figures, snapshots of attractor of each oscillator and phase states between adjacent oscillators are shown.

In our previous study [6], we have investigated the propagation and the reflection at the edge of the ladder of the wave.

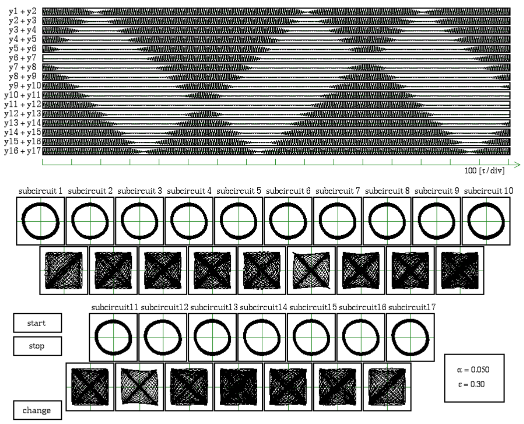


Figure 2: Typical example of phase-inversion-waves.

Remark: Wave propagation phenomenon seems to be observed for any number of coupled oscillators. We could observe similar phenomenon from both of even and odd numbers of oscillators and from both of small (5) and large (100) numbers.

3. Collision between Two Phase-Inversion-Waves

We can see in Fig. 2 that the two phase-inversion-waves completely reflect when they collide with each other. However, by giving different initial conditions we can also observe that they completely extinct after the collision (see Fig. 3).

In order to make clear the condition of the generation of the two completely different phenomena, we investigate the mechanisms of these phenomena by using the relationship between phasestates and oscillation frequency. It has been already known that oscillation frequency of in-phase synchronization of oscillators coupled by inductors is different from that of anti-phase synchronization. Namely, f_{in}, oscillation frequency of in-phase synchronization, is smaller than f_{anti}, oscillation frequency of anti-phase synchronization. Further, the difference between f_{in} and f_{anti} increases as coupling inductance increases [7].

Throughout the paper, we define the phase difference between two adjacent oscillators and the frequency of OSCk as follows:

$$\Phi_{k,k+1}(n) = \frac{\tau_k(n) - \tau_{k+1}(n)}{\tau_k(n) - \tau_k(n-1)} \times \pi$$

$$f_k(n) = \frac{1}{2(\tau_k(n) - \tau_k(n-1))} \qquad (4)$$

where $\tau_k(n)$ is time when the voltage of OSCk crosses 0[V] at n-th time.

Remark: The $f_k(n)$ in Eq. (4) should be called zero-crossing-time instead of frequency. However, since $f_k(n)$ must be compared with f_{in} and f_{anti}, which is oscillation frequency observed from the two coupled oscillators, we use the term frequency.

3.1 Mechanism of complete extinction

1. Let us assume that OSC5 ∼ OSC7 are anti-phase synchronization and that the waves changing from anti-phase into in-phase are going to reach OSC5 and OSC7 at the almost same time from the directions of OSC1 and OSC17 respectively.
2. Reaching the waves causes the changes of oscillation frequencies of OSC5 f_5 and OSC7 f_7 changes from f_{anti} to f_{in}.
3. Accordingly $\Phi_{5,6}$ and $\Phi_{6,7}$ change from π to 0 and 2π respectively.
4. Changes of $\Phi_{5,6}$ and $\Phi_{6,7}$ cause that the change of oscillation frequency of OSC6 f_6 from f_{anti} to f_{in}.
5. When $\Phi_{5,6}$ and $\Phi_{6,7}$ reach 2π and 0 respectively, f_6 become to be equal to f_{in}. The whole array results in stable in-phase synchronization.

Waves changing from in-phase to anti-phase can be explained in a similar manner.

Figure 3 shows computer calculated results corresponding to the completely extinction. We can see a wave extinction occurs as the manner explained above.

3.2 Mechanism of complete reflection

1. Let us assume that OSC7 ∼ OSC10 are in-phase synchronization and that the waves changing from in-phase into anti-phase are going to reach OSC7 and OSC10 at the almost same time from the directions of OSC1 and OSC17, respectively.
2. Waves reach OSC7 and OSC10 almost equal timing. $\Phi_{7,8}$ and $\Phi_{9,10}$ approach π and $-\pi$ by almost equal timing.
3. Because $\Phi_{7,8}$ and $\Phi_{9,10}$ approach π and $-\pi$ respectively, oscillation frequencies of OSC8 f_8 and OSC9 f_9 change from f_{in} toward f_{anti}.
4. However, because f_8 and f_9 change almost simultaneously, in-phase synchronization between OSC8 and OSC9 does not break. Namely, $\Phi_{8,9}$ remains almost 0. Hence, f_8 and f_9 do not reach f_{anti}. Accordingly $\Phi_{7,8}$ and $\Phi_{9,10}$ continue to change until reaching 2π and -2π, respectively.
5. By the effect of the decreases of ($\Phi_{7,8}$ mod 2π) and ($\Phi_{9,10}$ mod 2π), f_8 and f_9 begin to decrease toward f_{in} again.

Waves changing from anti-phase to in-phase can be explained in a similar manner.

Figure 4 shows computer calculated results corresponding to the completely reflection. We can see wave reflections occur as the manner explained above.

According to the above-explained mechanisms of the completely extinction and the completely reflection, we can conclude that the two phase-inversion-waves colliding each other will;

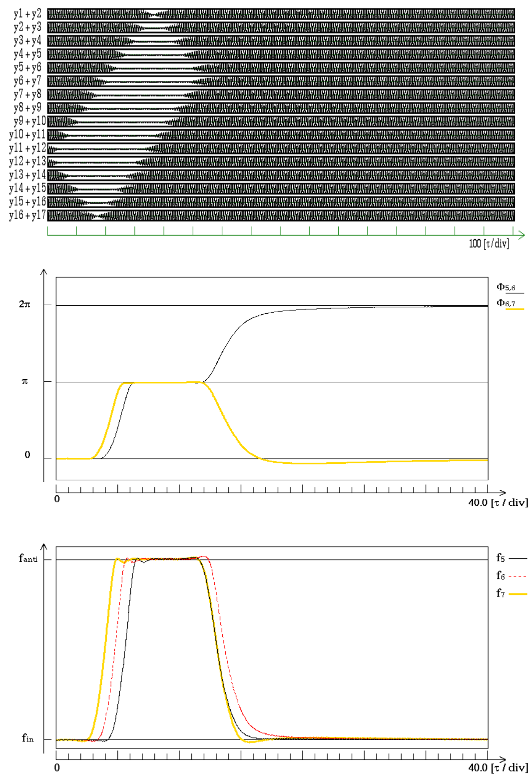


Figure 3: Completely extinction.

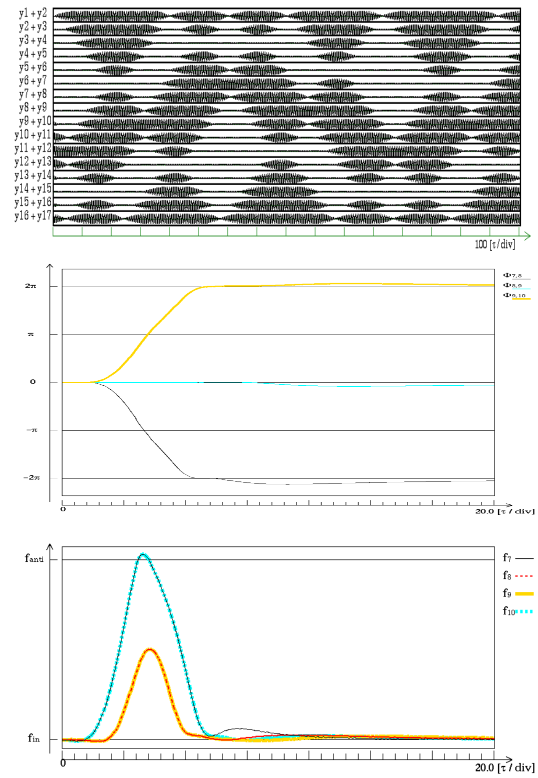


Figure 4: Completely reflection.

1. **Completely reflect** when the waves reach OSCk and OSC$k+1$ at the almost equal timing.

2. **Completely extinct** when the waves reach OSC$k-1$ and OSC$k+1$ at the almost equal timing.

3.3 Between complete extinction and complete reflection

From the above conclusion, our next question is what kind of phenomena will be generated when the two phase-inversion-waves collide each other with time difference.

In order to investigate the intermediate phenomena, we generate two phase-inversion-waves from both edges of the array with time delay for one side. T_{in} is the period of oscillation frequency of the in-phase synchronization ($T_{in} = 1/f_{in}$).

Obtained computer simulated results from various values of the time delay are shown in Fig. 5.

1. Figure 5(a): After the collision, two phase-inversion-waves completely extinct.

2. Figure 5(b): After the collision, small level of phase difference remains. The phase difference propagate to one direction. It is attenuated as time goes and extinct soon. This phenomenon are observed for the time delay smaller than 0.9 $T_{in}[\tau]$.

3. Figure 5(c): After the collision, a relatively large level of phase difference remains. The phase difference grows to π and propagates as a phase-inversion-wave.

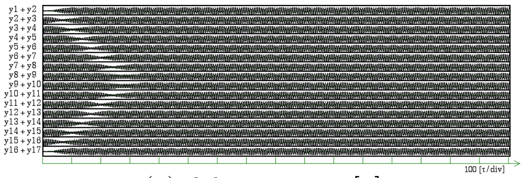
4. Figures 5(d)∼(f): Complex wave propagation phenomenon is observed. Various waves of different level of phase differences are produced after the collision and they influence each other.

5. Figure 5(g)After the collision, two phase-inversion-waves completely reflect. This phenomenon are observed for the time delay larger than $1.7T_{in}[\tau]$.

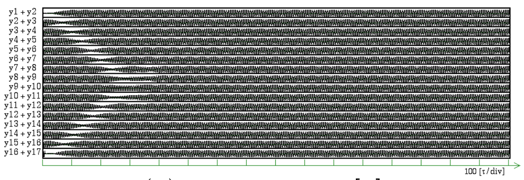
Figure 6 shows the relationship between time delay and observed phenomena. Because the time that the phase-inversion-wave propagate from one oscillator to the next is about $5T_{in}[\tau]$, the structure of this figure will repeat as time delay increases. Further, we can say that the reflection can be generated more easily than extinction.

4. Conclusions

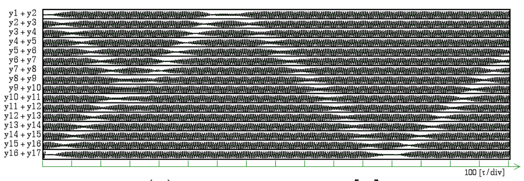
In this study we investigated the phenomena related to collisions between two phase-inversion-waves in an array of van der Pol oscillators. We could confirm by computer simulations that the behavior of the two phase-inversion-waves after they collide with each other was classified into three types; completely extinction, completely reflection and intermediate complex



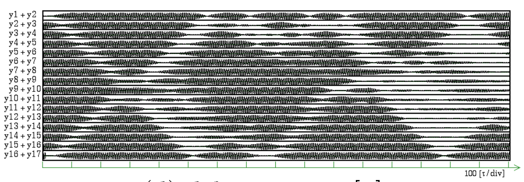
(a) delay=$0.00T_{in}[\tau]$.



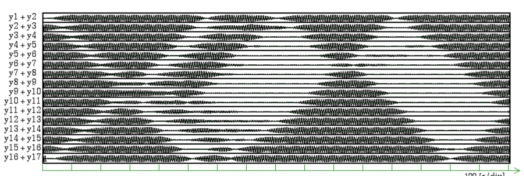
(b) delay=$1.50T_{in}[\tau]$.



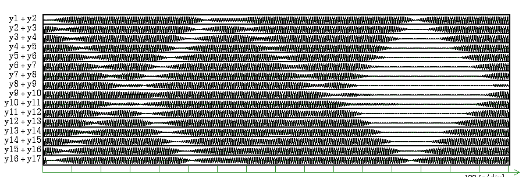
(c) dela y=$100T_{in}[\tau]$.



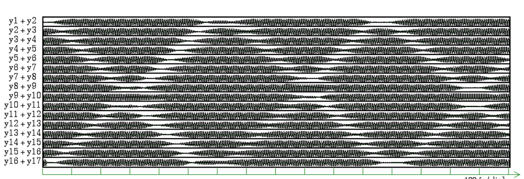
(d) delay=$1.25T_{in}[\tau]$.



(e) dela y=$150T_{in}[\tau]$.



(f) delay=$1.65T_{in}[\tau]$.



(g) delay=$1.70T_{in}[\tau]$.

Figure 5: Change from complete extinction to complete reflection.

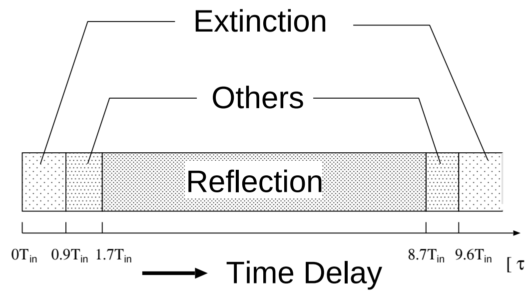


Figure 6: Domain of extinction, reflection and other phenomena.

phenomenon. Further, we could explain the mechanisms of the completely extinction and the completely reflection of the two waves by using the relationship between phase states and oscillation frequency

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