On the Properties of Hopfield Neural Networks with Chaotic Noise for Traveling Salesman Problem

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Abstract— In this paper, we investigate general solving abilities of the Hopfield neural network with chaotic noise injected into each neuron for several kinds of the traveling salesman problems. Especially, based on several numerical simulation studies, we inspect which number of periodic intermittency chaotic noise for Hopfield neural network is most effective in the solving performance against the variety of TSP problem types.

I. Introduction

Solving the traveling salesman problem (abbr. TSP)[1], a kind of combinatorial optimization problems, is an important application of Hopfield neural network (abbr. H-NN)[2]. Generally, there are many local minima in TSP[1] and the state in the H-NN is often trapped into these local minima. It remains, therefore, an unsettled questions how to escape easily from them and reach the global minimum. About these points, recently, many researchers suggested that H-NN with chaotic sequences is effective.

Hayakawa et al.[4] mentioned that chaotic noise is more effective than stochastic one, and they also pointed out the intermittency chaos near the periodic-3 window of the logistic map gains the best performance for a kind of TSP. On the other hand, Ueta et al.[6] and Okahisa et al. [7]. concluded in their papers that the effective intermittency chaotic noise is not only 3-window but also periodic-5 and 7 for same kind of TSP.

In this paper, hence, we investigate general solving abilities of the H-NN with chaotic noise injected into each neuron for several kinds of TSPs. Especially, based on several numerical simulation studies, we inspect which number of periodic intermittency chaotic noise for Hopfield neural network is most effective in the solving performance against the variety of TSP problem types, for example, difference of number of cities or reshuffling the placement of cities. We use two performance criterion for this object, one is whether the global minimum is detected or not (the detecting rate of global minimum per one trial) and the other is how many local minima is detected (the average number of detected local minima per one trial).

II. Hopfield Neural Network for TSP

In this paper, we use the original H-NN for comparison with former studies. For solving N-city TSP, $N\times N$ neurons are required and the following energy function is defined to fire (i,j) neuron at the optimal position:

$$\begin{aligned} E &= \frac{A}{2}\sum_{i=1}^{N}\left(\sum_{j=1}^{N}(x_{ij}(t)-1)^2\right) \\ &+ \frac{B}{2}\sum_{j=1}^{N}\left(\sum_{i=1}^{N}(x_{ij}(t)-1)^2\right) \\ &+ \frac{D}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} d_{ij}x_{ik}(t)(x_{i\ k+1}(t)+x_{j\ k-1}(t)) \end{aligned} \quad (1)$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between (j,l)-th neuron and the threshold of the (i,k)-th neuron are described by:

$$\begin{aligned} w_{ikjl} &= -A\{\delta_{ij}(1-\delta_{kl})+\delta_{kl}(1-\delta_{ij})\} \\ &\quad -d_{ij}(\delta_{l\ k+1}+\delta_{l\ k-1}) \\ \theta_{i,k} &= A+B \end{aligned} \quad (2)$$

where A and B are positive constant, and $\delta_{i,j}$ is Kronecker's delta. The states of $N\times N$ neurons are updated due to the following difference equation:

$$x_{ik}(t+1) = f\left(\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ikjl}x_{jl}(t) + \theta_{ik} + \beta z_{ik}(t)\right) \tag{3}$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{2}\left(1 + \tanh\frac{x}{\varepsilon}\right) \tag{4}$$

where $\varepsilon = 0.5$, $z_{ik}(t)$ is an additional noise, and β limits its amplitude of the noise. Figure 1 shows a conceptual neuron model for this H-NN.

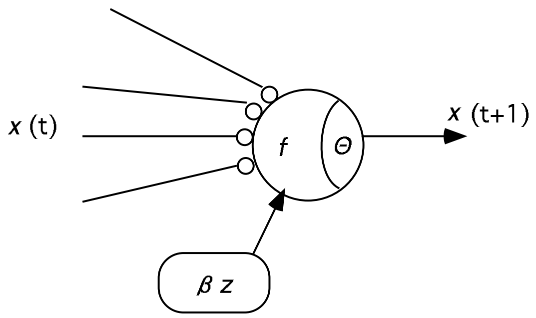

Figure 1: A neuron model

We also use the method of how to distinct each neurons fire or not refered from [4][7]. In this method, the state of a neuron $x_{ik}(t)$ at the time t is described by:

$$\bar{x}_{ik}(t) = \frac{1}{\tau}\sum_{l=t-\tau+1}^{t} x_{ik}(l) \tag{5}$$

$$\hat{x}_{ik}(t) = \begin{cases} 1, & if \quad \bar{x}_{ij}(y) \geq \tilde{x}(t) \\ 0, & otherwize \end{cases} \tag{6}$$

where $\hat{x}_{ik}(t)$ is new state of $x_{ik}(t)$, and where $\bar{x}_{ik}(t)$ is the $(i,k)th$ neuron's average value of output of from $t-\tau$ to t. And where $\tilde{x}(t)$ is N-th value in the $\bar{x}_{ij}(t)$ of $N \times N$ neurons. And we set $\tau = N$ as stated in [5].

III. H-NN with Chaotic Noise

For solving TSP by H-NN with noise, we generate a time series by the logistic map:

$$z_{ik}(t+1) = az_{ik}(t)(1 - z_{ik}(t)) \tag{7}$$

where a is a bifurcating parameter of the logistic map.

Hayakawa[4] shows the period-3 intermittent chaos of the logistic map is more effective than other chaotic noise. Figure 2 shows a time series of period-3 intermittent chaos of the logistic map in the case of $a = 3.8250, \lambda = 0.4114$, where λ is Lyapunov index.

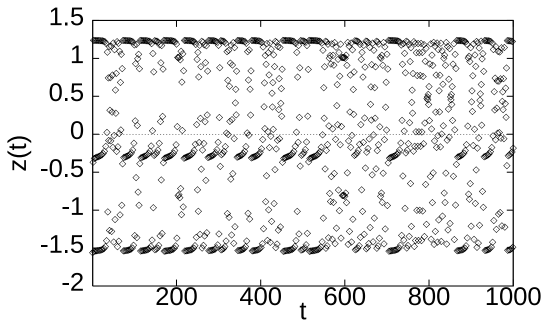

Figure 2: period-3 intermittency chaos

IV. Simulations

We also simulate H-NN with chaotic time series noise for several kinds of TSPs. Figure 3 shows some type of TSPs. The initial values of all neurons and noises are selected in randam, and the values of A, B, D (parameters of energy function) and β (amplitude of thenoise) are set as stated in [4][5]. Table 1 shows these value.

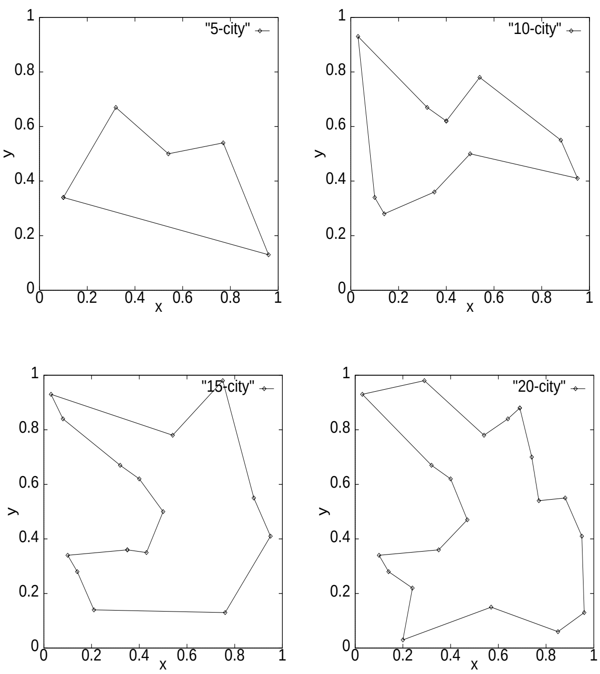

Figure 3: 5-city, 10-city, 15-city and 20-city TSPs

Table 1: The value of parameters

City number	The value of parameters
5	A=1.0, B=1.0, D= 1.0, $\beta = 0.45$
10	A=1.1, B=1.1, D= 4.0, $\beta = 0.47$
15	A=1.1, B=1.1, D=16.9, $\beta = 0.50$
20	A=1.0, B=1.0, D=25.0, $\beta = 0.45$

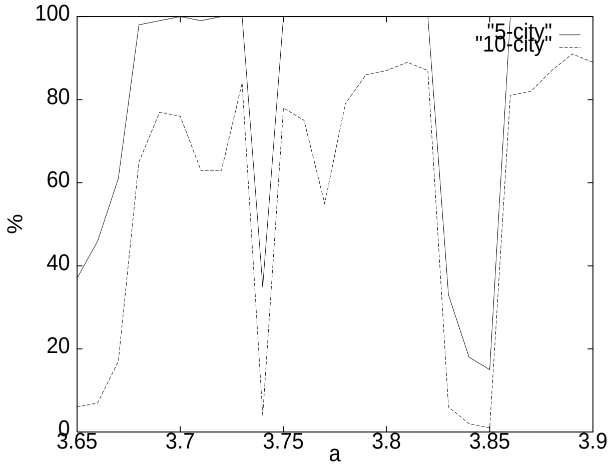

Figure 4: The detecting rate of global minimum (5, 10-city)

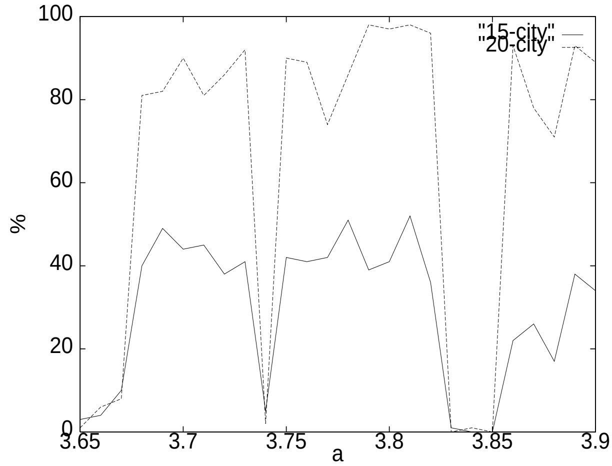

Figure 5: The detecting rate of global minimum (15, 20-city)

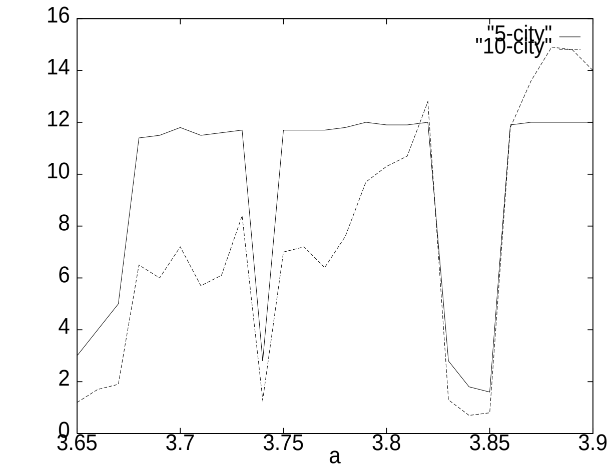

Figure 6: The average number of detected local minima (5, 10-city)

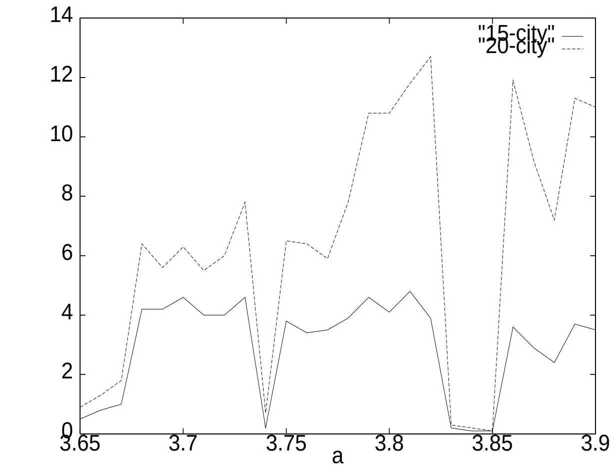

Figure 7: The average number of detected local minima (15, 20-city)

Hayakawa[4] use how long the state of NN stays in the global minimum per one trial as performance criterion. But, generally, the golobal minimum of TSP is not known. We use, therefore, two performance criterion, one is whether the global minimum is detected or not (the detecting rate of global minimum per one trial) and the other is how many local minima is detected (the average number of detected local minima per one trial). And one trial in simulations is defined as follows:

- Fixed values of parameters (A, B, D, β) as shown in table 1.
- Select the initial values of all neurons in randam.
- Repeat 1000 times of transfer of every neurons' input-output signal.

A. Simulation results

Figures 4 and 5 show the detecting rate of global minimum per one trial, and figures 6 and 7 show the average number of detected local minima per one trial. From these results, we can confirm that not only 3-window ($a = 3.8250$) but also other chaotic noises are effective except if the noise is stable periodic sequence. And these results roughly go for other kinds of TSPs that is not shown in figure 3, too. Especially, we can obtain very stable results with the intermittent chaos.

B. The parformance of the intermittent chaos

Table 2 shows the detecting rate of global minimum and the average number of detected local minima of periodic-3,5,7 intermittent chaos. From this table,

Table 2: The performance of the intermittent chaos

City number	Type of intermittent chaos	Lyapunov index	The detecting rate of global minimum(%)	The average number of detected local minima
5	period-7($a = 3.7001$)	0.3576	100	11.6
	period-5($a = 3.7380$)	0.2951	100	11.7
	period-3($a = 3.8250$)	0.4114	100	11.8
10	period-7($a = 3.7001$)	0.3576	100	7.5
	period-5($a = 3.7380$)	0.2951	100	8.8
	period-3($a = 3.8250$)	0.4114	100	13.1
15	period-7($a = 3.7001$)	0.3576	49	5.2
	period-5($a = 3.7380$)	0.2951	43	5.9
	period-3($a = 3.8250$)	0.4114	41	4.8
20	period-7($a = 3.7001$)	0.3576	100	9.2
	period-5($a = 3.7380$)	0.2951	98	9.9
	period-3($a = 3.8250$)	0.4114	96	13.1

these noises almost mark 100% in the detecting rate of global minimum, and get more number of detected local minima than other chaotic noise. So we can concluded that periodic-7 intermittent chaos is the most effective in the detecting rate of global minimum, and periodic-3 intermittent chaos is the most effective in the average number of detected local minima.

V. Conclusion

We investigate the general solving abilities of the H-NN with chaotic noise for several kinds of TSPs. And we confirm that the periodic-7 intermittency chaotic noise is most effective in the solving performance criterion for searching the global minimum and the periodic-3 intermittency chaotic noise is most effective in the solving performance criterion for the detecting rate of local minima. Further more, we also confirm that these periodic chaotic noises have robustness property against the variety of TSP problem types and that the periodic intermittency chaotic noise is one of the most suitable chaotic noise for H-NN as solving method for TSP.

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