CHAOTIC ITINERANCY PHENOMENA ON COUPLED N-DOUBLE SCROLLS CHAOTIC CIRCUITS

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ABSTRACT

In this study, chaotic itinerancy phenomena on coupled chaotic circuits are investigated. A piecewise linear element included in a chaotic circuit consists of $4n - 1$ segments. In this circuit, we can confirm n-double scrolls or various scrolls attractors. The features of the obtained results are respected for various engineering systems such as chaos communication systems with robustness against various interferences.

1. INTRODUCTION

Many types of coupled chaotic circuits have been researched as one of the most interesting phenomena of the nonlinear circuits in the electrical fields. Many chaotic circuits constructed by electrical elements are also existing. Since they had been investigated in detail up to now, and many paper have reported some interesting chaotic phenomena[1–5]. Chua's circuit is one of them, which is well known chaotic circuit as we can observe a variety of chaotic behavior. There are some papers which treated the case of coupled two Chua's circuits[1,2] or many coupled Chua's circuits[3]. Further, complicated chaotic behavior were found in several coupled systems. In large scale coupled chaotic circuits we can see spatio-temporal chaotic phenomena and chaotic itinerancy phenomena. On the other hand, Suykens et al.[6] proposed a concept which they modify the characteristics of the nonlinear resistor included in the Chua's circuit due to generate n-double scrolls, and they utilized it to the CNNs by one dimensional array of the circuits[7]. By using such method, more complicated chaotic behavior can be generated in simpled chaotic circuits. However, there seems to be no report on n-double scrolls by circuit experiment. Moreover, coupled systems of the n-double scrolls have not been investigated in detail.

Firstly, we consider a modified Chua's circuit which can generate n-double scrolls chaotic attractors. We carry out both numerical simulation and circuit experiment, then we realize 2-double scrolls chaotic attractor on electrical circuits. Secondly, we consider the modified Chua's circuits coupled by one resistor. In this study, chaotic itinerancy phenomena on coupled modified Chua's circuits are investigated in detail. Moreover we calculate autocorrelation function of the modified Chua's circuit. We consider that such complicated chaotic waveforms are expected to be utilized for realization of several chaotic applications such a chaos communication system with robustness against various interferences including multi-user access.

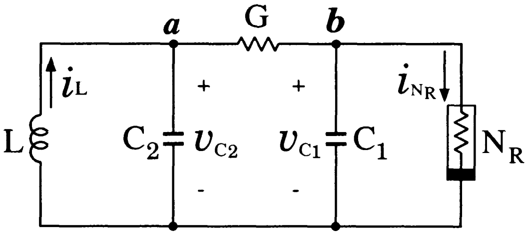

Figure 1: Chua's circuit

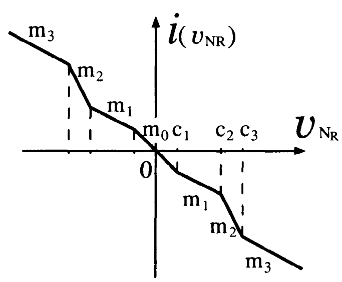

Figure 2: Chua's diode with piecewise linear segments

2. CIRCUIT MODEL & CIRCUIT SIMULATION

At first, we consider the Chua's circuit which can generate n-double scrolls attractor. The concept of this circuit was proposed by Suykens et al[6]. When a piecewise linear resistor included in the circuit is approximated by $4n - 1$ segments, we can obtain n-double scrolls attractor.

The circuit diagram is illustrated in Fig. 1. This circuit consists of only three memory elements, one re-

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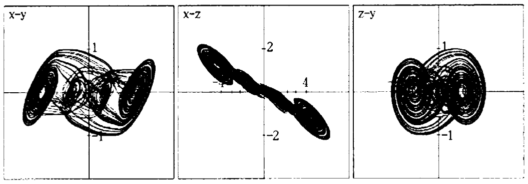

Figure 3: 2-double scrolls chaotic attractors. $\alpha = 10.0$, $\beta = 0.265$. m = [-1.15, -0.71, -1.75, -0.71], c = [1.0, 2.2, 3.0].

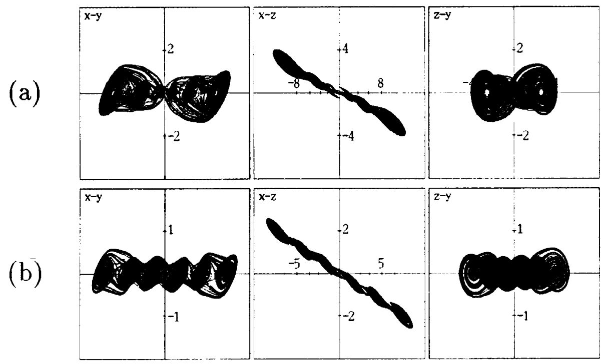

Figure 4: 3-double scrolls chaotic attractors. $\alpha = 10.0$, $\beta = 0.265$. (a) m = [-1.15, -0.73, -1.35, -0.73, -1.30, -0.73], c = [1.0, 2.2, 3.5, 5.5, 8.0], (b) m = [-1.15, -0.78, -1.35, -0.78, -1.23, -0.78], c = [1.0, 2.5, 3.5, 5.0, 6.5].

sistor and one nonlinear element. The nonlinear element is defined as piecewise linear as shown in Fig. 2 and it is shaped symmetrically for the origin point. If we choose adequate parameters, we can confirm n-double scrolls chaotic attractors.

The circuit dynamics of general Chua's circuit are as follows:

$$\begin{aligned} \dot{x} &= \alpha\beta(y - x - f(x)) \\ \dot{y} &= \beta(x - y) + z \\ \dot{z} &= -y \end{aligned} \qquad (1)$$

where characteristics of the nonlinear resistor and the circuit parameters are

$$f(x) = \delta_{2n-1}x + \frac{1}{2}\sum_{i=1}^{2n-1}(\delta_{i-1} - \delta_i)(|x + c_i| - |x - c_i|) \qquad (2)$$

$$\begin{aligned} &t = \sqrt{LC_2}\tau, \quad \text{``}\,.\,\text{''} = \frac{d}{d\tau}, \\ &v_{C1} = Vx, \quad v_{C2} = Vy, \quad i_L = V\sqrt{\frac{C_2}{L}}z, \\ &\alpha = \frac{C_2}{C_1}, \quad \beta = G\sqrt{\frac{L}{C_2}}, \quad \delta_i = \frac{m_i}{G}. \end{aligned} \qquad (3)$$

where m_i and c_i are values of the slope of segments and breakpoints, respectively. V is any value, hereafter V is fixed at 1.0[V]. We show some numerical calculation results obtained by using the Runge-Kutta method with time step size $\Delta\tau = 0.001$. Fig. 3 shows 2-double scrolls attractors. We can see such chaotic attractors when adequate parameters are given. Similarly, we can obtain 3-double scrolls attractors as shown in Fig. 4. Thus we can obtain n-double scrolls attractors easily.

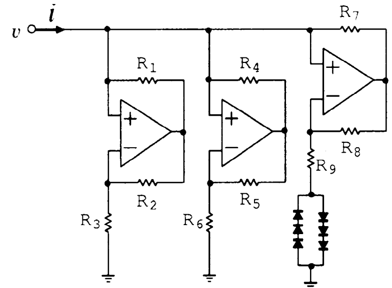

Figure 5: Realization of piecewise linear resistor by using O.P.Amps. OP Amp type is TL082CP, $R_1 = R_2$ = 21.7[kΩ], R_3 = 1.69[kΩ], $R_4 = R_5$ = 237[Ω], R_6 = 1.56[kΩ], $R_7 = R_8$ = 5.06[kΩ], R_9 = 1.46[kΩ], threshold value of the diodes is about 3.0[V].

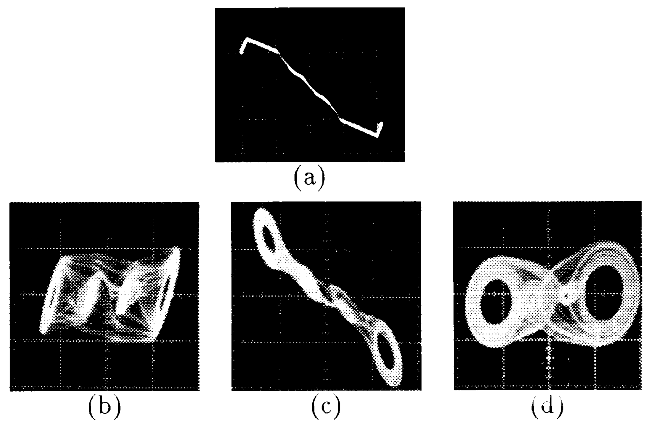

Figure 6: 2-double scrolls chaotic attractor obtained from the circuit experiment. C_1= 4.45[nF], C_2 = 49.3[nF], L = 5.25[mH], $1/G$ = 1.06[kΩ]. (a) characteristic of Chua's diode, v_{NR}[5V/div] vs. i_{NR}[10mA/div], (b) v_{C1}[5V/div] vs. v_{C2}[1V/div], (c) v_{C1}[5V/div] vs. i_L[5mA/div], (d) i_L[5mA/div] vs. v_{C2}[1V/div].

In this study, we carried out circuit experiment on electrical circuits. We realized the nonlinear resistor by using OP. Amps, some linear resistor and diodes as shown in Fig 5. Some experimental results are shown in Fig. 6. We can realize 2-double scrolls chaotic attractors. In circuit experiments, it is difficult normally to realize a nonlinear resistor which has a lot of segments by using only electrical elements. However, if we will be able to realize that, we will obtain more complicated chaotic phenomena on real electrical circuits.

3. CIRCUIT SIMULATION FOR COUPLED CHUA'S CIRCUITS

In this section, we focus on complicated chaotic phenomena on coupled modified Chua's circuits. Here we consider coupled modified Chua's circuits connected by one resistor as shown in Fig. 7. Our aim is to investigate chaotic itinerancy phenomena on this simple coupled chaotic circuits. This coupled modified Chua's circuits is very simple because the system is governed by six dimensional piecewise linear equations.

We assume that two modified Chua's circuits are coupled by one resistor G_x, and we defined a new following parameter of coupling:

$$\gamma = G_x \sqrt{\frac{L}{C_2}} \qquad (G_x = 1/R_x). \tag{4}$$

For computer calculations, in order to consider the difference of real circuit elements, the circuit equation of coupled modified Chua's circuits is written by using an additional parameter $\Delta\omega$ as follows:

$$\begin{aligned} \dot{x_k} &= \alpha\beta(y_k - x_k - f(x_k)) \\ \dot{y_k} &= \beta(x_k - y_k) + z_k + (-1)^k \gamma(y_1 - y_2) \\ \dot{z_k} &= -\{1 + (k-1)\Delta\omega\} y_k \end{aligned} \tag{5}$$

where, k = 1 and 2. Hereafter, Eq. (5) is calculated by using the Runge-Kutta method with time step size $\Delta\tau$ = 0.002 and $\Delta\omega$ is 0.01.

We show some numerical simulation results. Figs. 8 and 9 show chaotic phenomena of the 2-double scrolls case and 3-double scrolls case, respectively. From the left of the attractors, attractor of one modified Chua's circuit, attractor of the other modified Chua's circuit and synchronization state, respectively. When we focus on the synchronization state of both attractors, we can confirm that the state of two variables of x_1 and x_2 are wandering on the $x_1 - x_2$ plane. Namely, chaotic itinerancy phenomena were confirmed in this system. We can also confirm several interesting chaotic phenomena such as chaos synchronization, bursting of chaos synchronization and chaotic itinerancy phenomena. In order to clarify stability of synchronization between both circuits, we investigate the relation between existence time and place. Fig. 10 shows a probability of existence on the $x_1 - x_2$ plane for 3-double scrolls case. Vertical axis means time τ where synchronization state exist in the corresponding area. These figures indicate that frequency of chaotic itinerancy can be controlled by tunning coupling parameter value.

We also investigate autocorrelation function of this modified n-double scrolls Chua's circuit. In Fig. 11, autocorrelation results obtained from computer calculation are shown. Fig. 11(a) and (b) are 3-double scrolls case and double scrolls case, respectively. Horizontal corresponds to time which time distance is set to 0.5[τ], and vertical corresponds to the value of autocorrelation function. We can confirm the value of autocorrelation function in the case of 3-double scrolls converge faster than the double scrolls case. It means it has robustness for various interferences in the 3-double scrolls case. Therefore, we consider that it can make a communication system with robustness for various interferences by using n-double scrolls features.

4. CONCLUSIONS

In this study, we investigated chaotic itinerancy phenomena on coupled chaotic circuits which have piecewise linear elements. We can confirm n-double scrolls chaotic attractors on computer simulation or circuit experiments. Chaotic itinerancy phenomena on coupled chaotic circuits have been investigated. The autocorrelation functions of both 3-double scrolls and double scrolls were calculated. We consider that such complex chaotic waveforms with chaotic itinerancy are expected to be utilized for realization of chaos communication systems with robustness against various interferences including multi-user access because of its quick decay of correlation functions.

Acknowledgment

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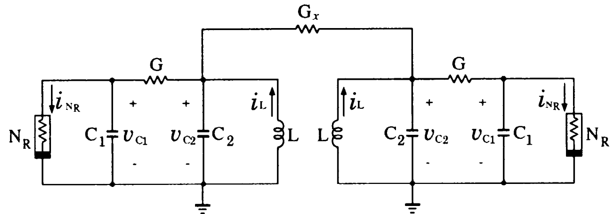

Figure 7: Modified Chua's circuits coupled by one resistor

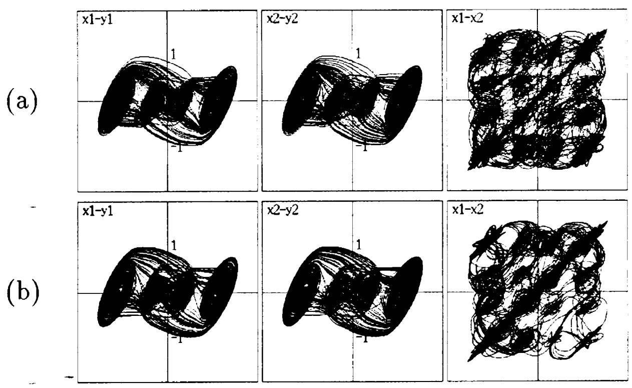

Figure 8: 2-double scrolls case. Chaotic itinerancy phenomenon on coupled modified Chua's circuits which N_R has 7 segments. $\alpha = 10.0$, $\beta = 0.265$, (a) $\gamma = 0.150$ ($R_x \simeq 2.52$[kΩ]), (b) $\gamma = 0.300$ ($R_x \simeq 1.26$[kΩ]),

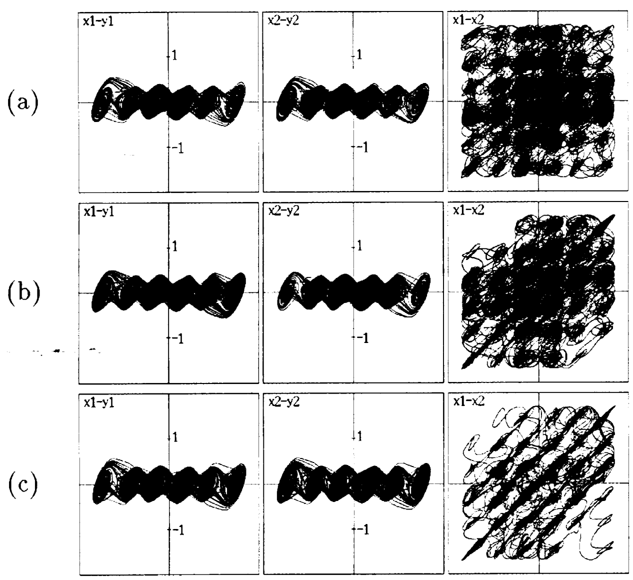

Figure 9: 3-double scrolls case. Chaotic itinerancy phenomena on coupled modified Chua's circuits which N_R has 11 segments. $\alpha = 10.0$, $\beta = 0.265$, (a) $\gamma = 0.100$ ($R_x \simeq 3.78$[kΩ]), (b) $\gamma = 0.150$ ($R_x \simeq 2.52$[kΩ]), (c) $\gamma = 0.300$ ($R_x \simeq 1.26$[kΩ]).

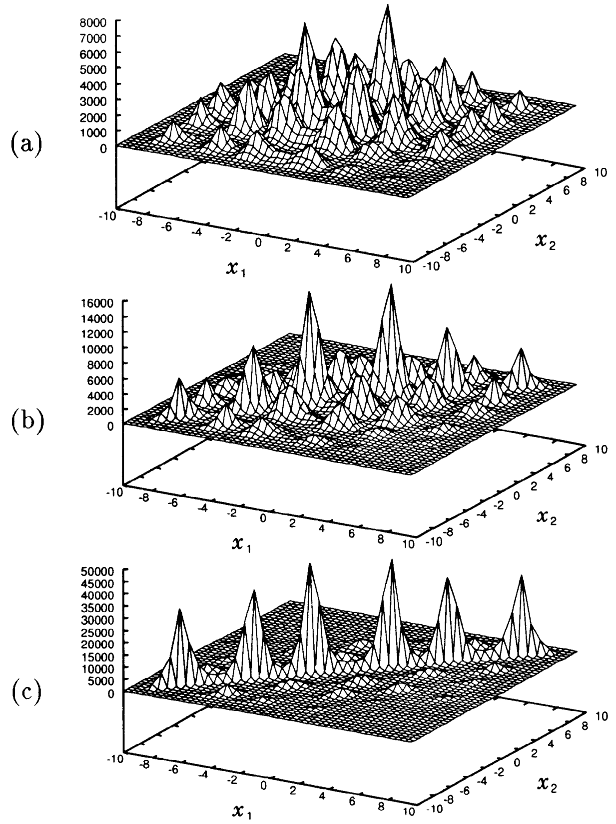

Figure 10: Probability of existence on the $x_1 - x_2$ plane for 3-double scrolls case. Total time [τ]: 10^6. (a) $\gamma = 0.10$, (b) $\gamma = 0.15$, (c) $\gamma = 0.30$.

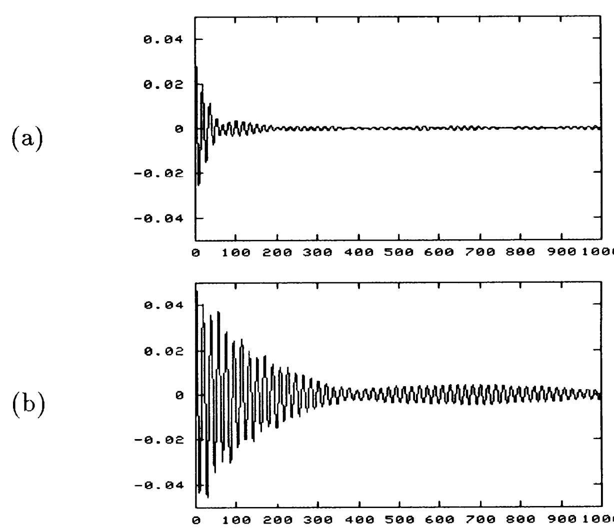

Figure 11: Autocorrelation function for (a) 3-double scrolls, (b) double scrolls. Horizontal: time[$\times 0.5\tau$].