CONTROL OF PHASE STATES IN RING OSCILLATORS SHARING INDUCTORS

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ABSTRACT

In this study we show control of phase states in ring oscillators sharing inductors by infusing the voltage of the first oscillator into the other one. We confirm the suppression of the phase states of the original system and the generation of new phase states which we have not seen before infusing the voltage.

1. INTRODUCTION

Since coupled oscillator systems can describe various phenomena in natural fields, there have been many investigations on such systems and various interesting phenomena have been reported by several researchers. For example, Mori *et al* and Kimura *et al* have investigated synchronization phenomena observed in two oscillators coupled by a capacitor [1] and a resistor [2] respectively. Endo *et al* and Nishio *et al* have investigated synchronization phenomena observed in many oscillators coupled by inductors [3]-[5] and resistors [6]-[8]. We have confirmed the oscillation death in many oscillators coupled by resistors [9][10]. However, there are not many investigations on oscillators sharing elements. Scott has investigated synchronization phenomena in 1-dimensional (1D) and 2-dimensional (2D) arrays of almost-linear and almost lossless networks [11][12]. He has only treated these system as distributed circuits. We investigated synchronization phenomena in oscillators sharing inductors [13].

On the other hand recently many researchers pay their attentions to the possibility of application to parallel information processing architecture using coupled oscillators or coupled chaotic circuits. As the first step for this purpose, we need to control phase states in coupled oscillators by any methods. Namely, we have to restrict generating phase states, make some oscillators stop, or produce new previous unseen phase states.

In this study we investigate control of phase states in ring oscillators sharing inductors with infusing voltage of the first oscillator into the other one. In this system the voltage of the first oscillator is infused into the other one through the buffer so as to control phase states. We can expect the generation of interesting phase states by infusing voltage because all oscillators which are synchronized at in-phase are unstable in the original system. We confirm the suppression of the phase states in the original system without voltage infusion and the generation of new previously unseen phase states after infusing the voltage. These phenomena are confirmed by both numerical calculations and circuit experiments. The present study contributes to the development of coupled oscillators networks which are expected to be one of the future parallel information processing architectures.

2. CIRCUIT MODEL

Fig. 1 shows our method. By using the switch, it is possible to infuse the voltage of the first oscillator into the *i*-th oscillator. The synchronization phenomena of the original system without infusing has reported in [13].

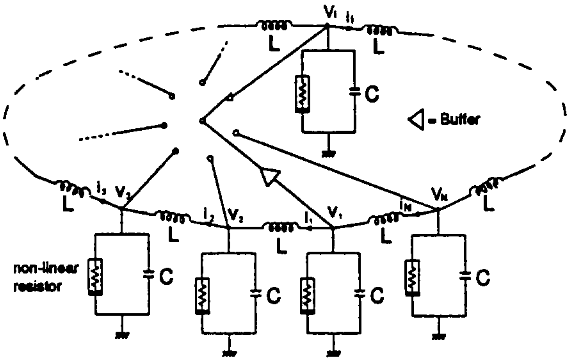


Fig. 1 Circuit model.

We assume that v-i characteristics of the nonlinear resistor is represented by third order polynomial equation

$$i_r(v) = -g_1 v + g_3 v^3 \quad (\, g_1, g_3 > 0 \,). \tag{1}$$

Changing the variables

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \quad i_k = \sqrt{\frac{Cg_1}{3Lg_3}}y_k \tag{2}$$

and defining

$$\varepsilon = g_1\sqrt{\frac{L}{C}} \tag{3}$$

then the normalized equations are represented as follows

$$\dot{x}_k = -y_k + y_{k-1} + \varepsilon\left(x_k - \frac{1}{3}x_k^3\right) \tag{4}$$

$$\dot{y}_k = x_k - x_{k+1} \tag{5}$$

$$(\, k = 1, 2, 3, \ldots, N \text{ except } k = i \,).$$

The equations of *i*-th infused oscillator are

$$\dot{x}_i = -y_i + y_{i-1} + \varepsilon\left(x_1 - \frac{1}{3}x_1^3\right) \tag{6}$$

$$\dot{y}_{i-1} = x_{i-1} - x_1 \tag{7}$$

$$\dot{y}_i = x_1 - x_{i+1} \tag{8}$$

where ε is the largeness of nonlinearity. For numerical calculations, to consider the difference between the natural frequencies of the real oscillators, the symmetry of the normalized equation (3) is broken by using additional parameters $\Delta_k = 0.001k$ as follows

$$\dot{x}_k = \left\{-y_k + y_{k-1} + \varepsilon\left(x_k - \frac{1}{3}x_k^3\right)\right\}(1+\Delta_k^2) \quad (9)$$
$$(k = 1, 2, 3, ..., N).$$

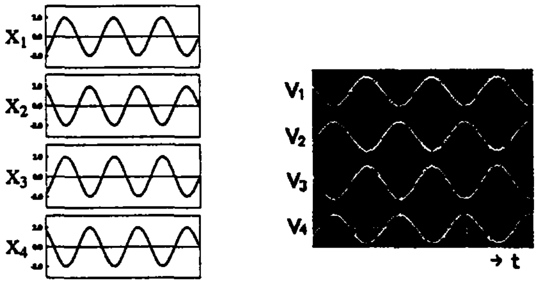
For the following numerical calculations, the largeness of nonlinearity is $\varepsilon = 0.05$. For circuit experiments, we use the parameters of C = 33[nF] and L = 10[mH].

3. NUMERICAL CALCULATIONS AND CIRCUIT EXPERIMENTS

3.1. In the case of N = 4

In our previous work we have confirmed the generation of two types of phase states [13]. One of them is shown in Fig. 2 where phase difference between two adjacent oscillators is π. The other is shown in Fig. 3. For this case phase difference between diagonal oscillators is π, but two pairs are not synchronized. The generation of these two phase states are decided by the initial condition.

When we infuse the voltage of the first oscillator into the third one, the phase state of Fig. 3 is suppressed and only the phase state of Fig. 2 generates. Namely it shows that we can restrict the phase states by infusing the voltage.



(a) (b)

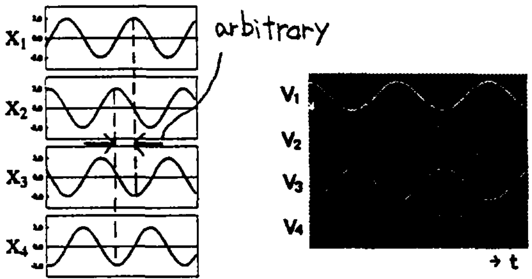
Fig. 2 Phase state observed for both cases without infusing and with infusing into the third oscillator ($N = 4$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

On the other hand we can get the new phase state shown in Fig. 4 by infusing the voltage of the first oscillator into the second one. This phase state have never seen before we had infused the voltage. In the phase state the first and the second oscillators are synchronized at in-phase and the third and the fourth oscillators are synchronized at opposite phase to the first one.

3.2. In the case of N = 8

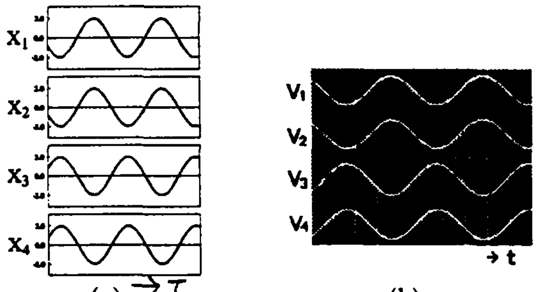
Similarly to the N = 4 we have confirmed the generation of phase states extending Figs. 2 and 3 [13]. When we infuse the voltage of the first oscillator into the third one, we can only observe the phase state in Fig. 5 by suppression to the other phase states. On the other hand we confirm the generation of the new phase state by infusing the voltage of the first oscillator into the second one shown in Fig 6.

We can not observe the new phase state by infusing the voltage of the first oscillator into the third, the fourth or the fifth one. In that case the system can not be synchronized.



(a) (b)

Fig. 3 Phase state observed only for the case without infusing ($N = 4$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).



(a) (b)

Fig. 4 New phase state observed for the case with infusing into the second oscillator ($N = 4$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

3.3. In the case of N = 5 or N = 7

We have confirmed the generation of two types of 5 phase states [13]. One is the phase shift of $2\pi/5$ between adjacent oscillator as in Fig.7 and the other is the phase shift of $4\pi/5$. These phase states disappear by infusing the voltage of the first oscillator into the other one.

We confirm the generation of the new phase pattern by infusing the voltage of the first oscillator into the second one shown in Fig. 8. For the phase state in Fig. 8 the first, the second and the third oscillators are synchronized at in-phase and the fourth and the fifth oscillators are synchronized at opposite phase to the first one. The interest thing is that the phase state in Fig. 8 cannot be observed when we infuse the voltage of the first oscillator into the third one, nevertheless the first and the third oscillators are synchronized at in-phase.

In the case of $N = 7$, three types of 7 phase states exist where the phase shift is $2\pi/7$, $4\pi/7$ or $6\pi/7$ [13]. Similarly to the case of $N = 5$, we confirm the suppression to these phase states and the generation of the new phase states shown in Fig. 9 by infusing the voltage of the first oscillator into the fourth oscillator.

From the above two examples when we infuse the voltage of the first oscillator into the other one in the case that N is odd, we can predict the suppression to the phase states and the generation of the phase states similar to Fig. 9

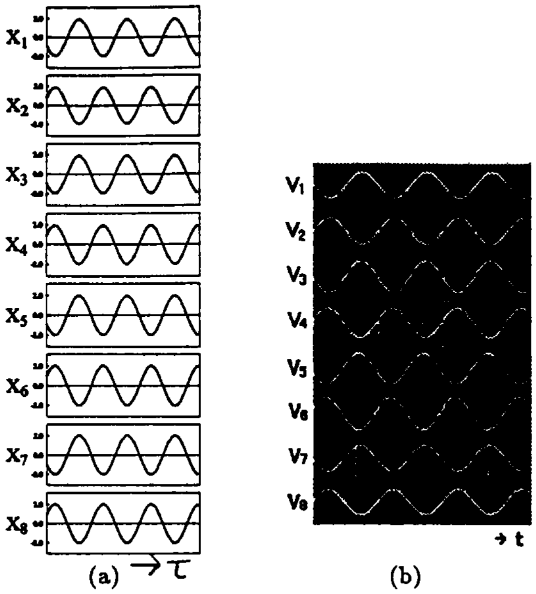


Fig. 5 Phase state observed for both cases without infusing and with infusing into the third oscillator ($N = 8$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

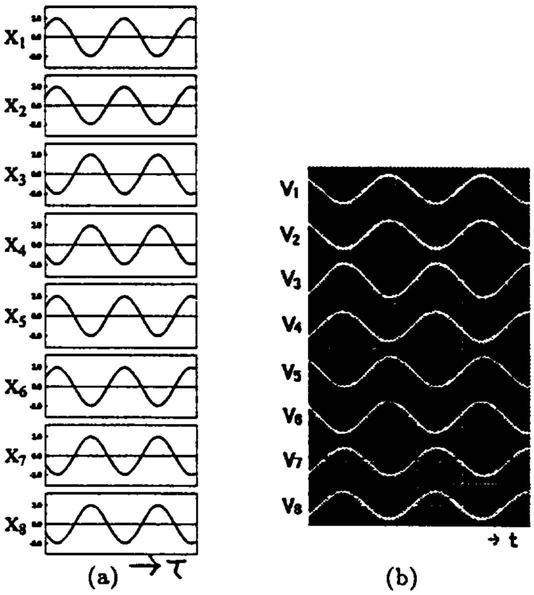


Fig. 6 New phase state observed for the case with infusing into the second oscillator ($N = 8$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

4. OSCILLATION DEATH

We discovered more interest synchronization phenomenon for the case of $N = 6$ by infusing the voltage, namely oscillations of some oscillators stop by infusing the voltage.

The first one shown in Fig. 10(a) is observed by infusing the voltage of the first oscillator into the second one. The first, the second, the fourth and the fifth oscillator are oscillating. On the other hand the third and the sixth oscillators stop to oscillate. The second shown in Fig. 10(b) is observed by infusing the voltage of the first oscillator into the third oscillator. The second, the fourth and the sixth oscillators are oscillating. On the other hand the first, the third and the sixth oscillators stop to oscillate.

However this phenomenon is observed only from the circuit experiments. We consider this is because the nonlinear resistor is approximated by third order polynomial equation in numerical calculations.

5. CONCLUSIONS

In this study we investigated control of phase states in ring oscillators sharing inductors with infusing voltage of the first oscillator into the other one. In this system the voltage of the first oscillator is infused into the other one through the buffer so as to control phase states. We confirmed the suppressing the phase states of the original system and the generation of the new phase states which we had not seen before infusing. These phenomena were confirmed by numerical calculations and circuit experiments. For the future problem are the detailed investigation of the oscillation death in the case of $N = 6$ and theoretical proof of the observed phenomena.

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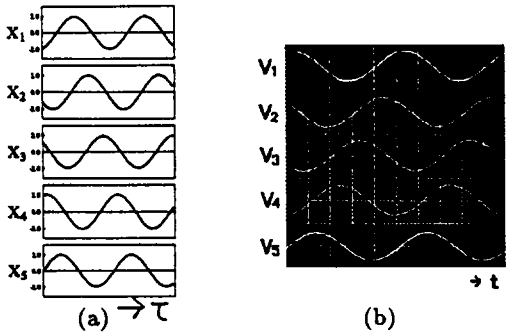

Fig. 7 Phase state observed only for the case without infusing ($N = 5$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

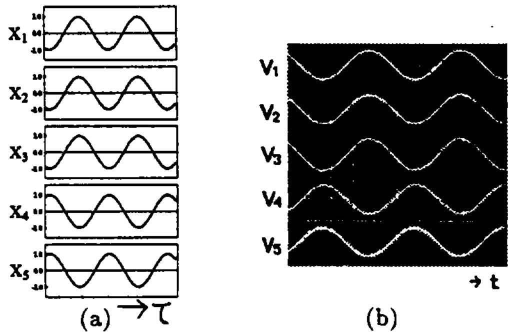

Fig. 8 New phase state observed for case with infusing into the second oscillator ($N = 5$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

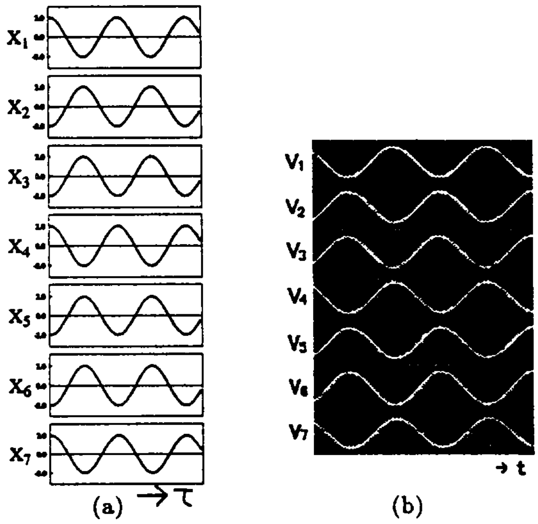

Fig. 9 New phase state observed for the case with infusing into the second oscillator ($N = 7$).
(a) Numerical calculation.
(b) Experimental result
(Vertical 1 V/div. Horizontal 20 μs/div.).

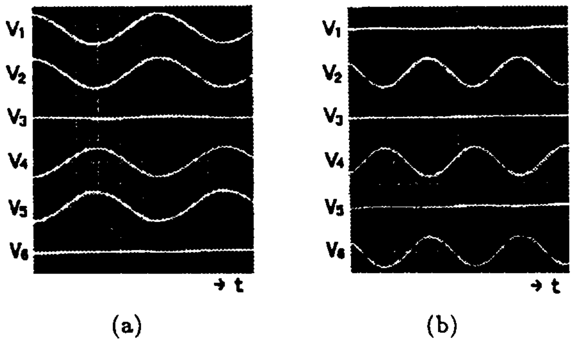

Fig. 10 The oscillation death ($N = 6$).
(a) Infusing the voltage of the first oscillator into the second one.
(b) Infusing the voltage of the first oscillator into the third one.
Vertical 1 V/div. Horizontal 20 μs/div.