BREAKDOWN OF IN-PHASE SYNCHRONIZATION OF TWO CHAOTIC CIRCUITS COUPLED BY AN INDUCTOR

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ABSTRACT

In this study, in-phase synchronization phenomena observed from chaotic circuits coupled by an inductor are investigated especially paying attention to breakdown of chaos synchronization. It is shown that breakdown of chaos synchronization occurs when chaotic attractor contains an unstable orbit splitting the attractor to two bands. Further, it is also confirmed by calculated results of Lyapunov exponents that hyperchaos appears at the same time.

1. INTRODUCTION

Many nonlinear dynamical systems in various fields have been clarified to exhibit chaotic oscillations and recently applications of chaos to engineering systems are expected such as chaos noise generators, control of chaos, synchronization of chaos, and so on. In those applications, we are specially interested in synchronization of chaos. As far as we know, synchronization of chaos in a simple circuit was first reported by Saito et al. [1]. Since Pecora et al. [2] studied theory of such phenomena, a large number of studies on synchronization of chaos have been reported. However, the study on bifurcation phenomena of chaos synchronization has been started recently [3][4]. There remain a large number of problems related with bifurcation of synchronization of chaos.

In this study, we investigate in-phase synchronization observed from two autonomous three-dimensional chaotic circuits coupled by an inductor. This circuit model is shown in Fig. 1. Each chaotic subcircuit is the circuit proposed by Inaba et al. [5]. This system is a two subcircuits case of a coupled system in Ref [4]. However, in Ref [4], we did not analyze this circuit with exact solutions in detail. In this study, we analyze it in detail especially paying attention to breakdown of in-phase chaos synchronization. For the purpose of the detailed analysis, we consider the case that diodes in the circuit are assumed to operate as ideal switches. In this case the circuit equation in a piecewise linear region is degenerated to a four-dimensional one. Therefore, Poincaré map is derived as three-dimensional map using exact solutions. As a result, Lyapunov exponents can be also calculated by exact solutions. This simplified technique was first proposed by Inaba et al. [6] and was confirmed to be extremely effective in analyzing chaotic phenomena in dissipative electrical circuits. Further, this technique was extended to higher-dimensional circuits [7] or circuits including two diodes by Nishio [8]. However, it has not been applied for the analysis of chaos synchronizations in coupled chaotic circuits. It is shown that breakdown of in-phase chaos synchronization occurs when chaotic attractor contains an unstable orbit splitting the attractor to two bands. Further, it is also confirmed by calculated results of Lyapunov exponents that hyperchaos appears at the same time. We have to notice that two types of synchronization modes coexist in this circuit model; in-phase and anti-phase. We can produce one of the two modes by inputting a certain initial conditions. However, we investigate only the in-phase synchronization mode in this study.

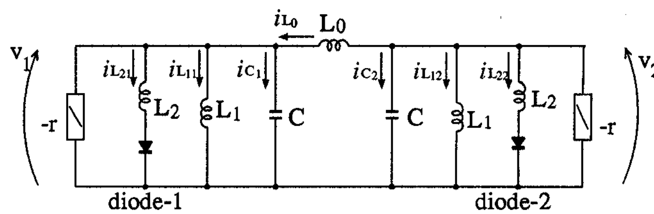

Fig. 1 Circuit model.

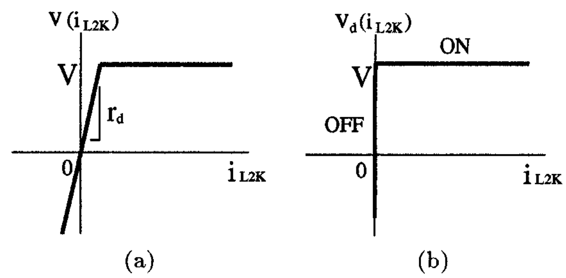

Fig. 2 Diode model.

2. CIRCUIT MODEL

Fig. 1 shows the circuit model used in this article. In our model, two chaotic circuits which are simple chaotic subcircuits proposed by Inaba et al. [5] with the same parameters are coupled by an inductor L_0. This subcircuit consists of only three memory elements, one linear negative resistor and one diode, and is one of the simplest autonomous chaotic circuits.

At first we approximate the characteristics of two diodes by the following two-segment piecewise linear functions as Fig. 2(a).

$$v_d(i_{L2k}) = \frac{r_d}{2}\left\{i_{L2k} + \frac{V}{r_d} - \left|i_{L2k} - \frac{V}{r_d}\right|\right\} \qquad (1)$$

where $k = 1, 2$. The circuit equations are explained by six-dimensional piecewise linear differential equations as follows.

$$\left\{\begin{array}{rcl} L_{1k}\dfrac{di_{L1k}}{dt} & = & v_k \\ L_{2k}\dfrac{di_{L2k}}{dt} & = & v_k - v_d(i_{L2k}) \\ C\dfrac{dv_k}{dt} & = & (-1)^k\dfrac{L_1}{L_0}(i_{L11} - i_{12}) \\ & & -(i_{L1k} - i_{L2k}) + \dfrac{v_k}{r} \end{array}\right. \quad (2)$$

(k = 1,2.)

By changing the variables and parameters,

$$t = \sqrt{L_1 C}\ \tau, \quad \text{“ . ”} = \frac{d}{d\tau}, \quad i_{L1k} = V\sqrt{\frac{C}{L_1}}x_k,$$
$$i_{L2k} = V\sqrt{\frac{C}{L_1}}y_k, \quad v_k = V z_k. \quad (3)$$
$$\alpha = \frac{L_1}{L_0},\ \beta = \frac{L_1}{L_2},\ \gamma = \frac{1}{r}\sqrt{\frac{L_1}{C}},\ \delta = r_d\sqrt{\frac{C}{L_1}}$$

(2) is normalized as

$$\left\{\begin{array}{rcl} \dot{x}_k & = & z_k \\ \dot{y}_k & = & \beta(z_k - f(y_k)) \\ \dot{z}_k & = & (-1)^k\alpha(x_1 - x_2) - (x_k + y_k) + \gamma z_k \end{array}\right. \quad (4)$$

$$f(y_k) = \frac{1}{2}\left\{\delta y_k + 1 - |\delta y_k - 1|\right\} \quad (5)$$

where $k = 1, 2.$ and $f(y_k)$ is a piecewise linear function of y_k corresponding to the characteristic of the diode. When we use this diode model, we can calculate attractors by using exact solution of Eq. (4). However, because it is hard to give analytical representation of the Poincaré map and its Jacobian, Lyapunov exponents cannot be calculated analytically.

In order to calculate Lyapunov exponents by using analytical form, we consider the case that the diodes in the circuit are assumed to operate as ideal switches as shown in Fig. 2(b). When this simplified technique is used, the circuit equation is degenerated by OFF state of the diode. Namely, when a diode is in the OFF state, the current i_{L2k} is constrained to zero. Hence, when both diodes are in the OFF state, the circuit equation is degenerated to four-dimensional equation. It enables us to derive three-dimensional Poincaré map analytically.

3. SYNCHRONIZATION PHENOMENA

Fig. 3 shows a typical example of chaotic attractors obtained from computer calculation. The figures show, from left, attractors on the $z_1 - z_2$ plane, $z_1 - x_1$ plane and $z_2 - x_2$ plane. For computer calculations, the parameter values of α and β are fixed and γ is varied as a control parameter.

Fig. 3(a)$\sim$(f) shows that attractors are synchronized completely. For these parameter values, the attractors are

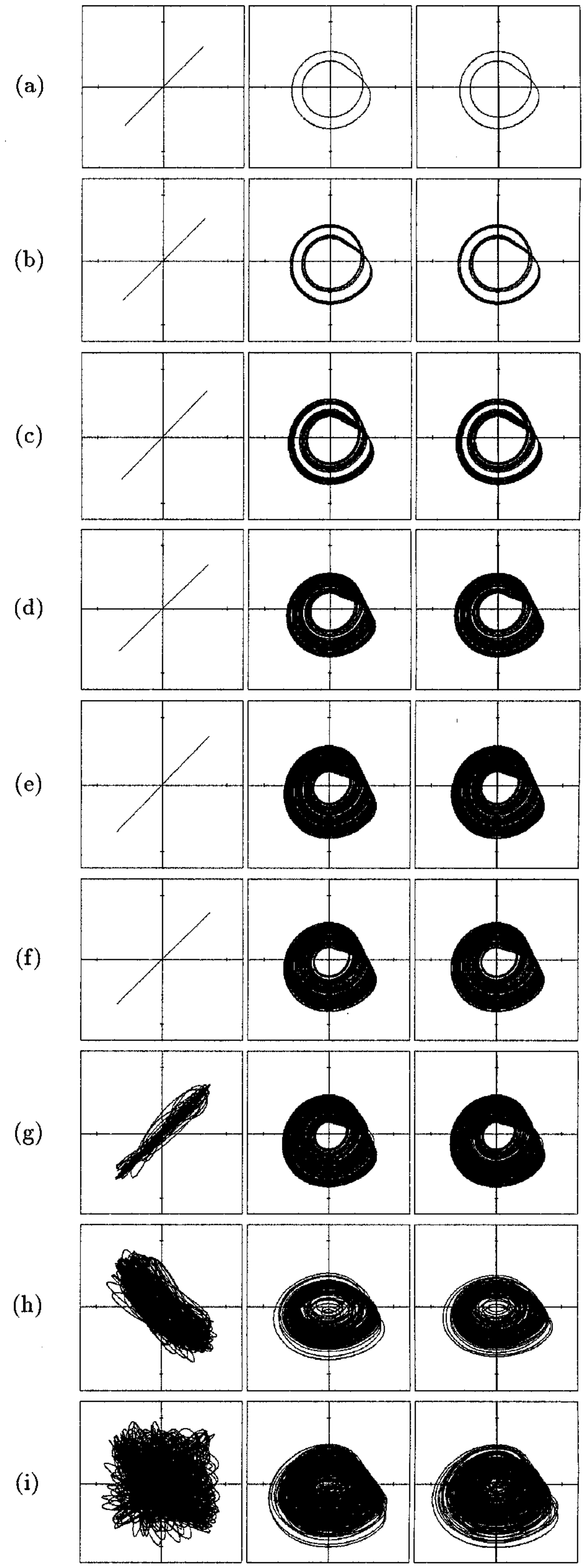

Fig. 3 Attractors obtained from computer calculation. $\alpha = 0.52$, $\beta = 6.0$. γ : (a)0.100, (b)0.125, (c)0.150, (d)0.175, (e)0.200, (f)0.205, (g)0.215, (h)0.235, (i)0.280. From left, attractors of z_1 vs. z_2, z_1 vs. x_1, z_2 vs. x_2.

split to two bands by an unstable orbit. As a parameter value increases, two bands of the chaotic attractor merges to one band and the attractor contains the unstable orbit. At the same time, chaos synchronization becomes incomplete as shown in Fig. 3(g). The reason is explained as follows. In the case that chaotic attractor contains the unstable orbit, synchronized two chaotic solutions can approach the unstable orbit. If two solutions go through different side with respect to the unstable orbit, they go away to the opposite direction each other. This mechanism makes stability of synchronization weak. Though there exist infinitely many unstable orbits in chaotic attractors, another unstable orbits have little influence on the stability of synchronization. As a parameter γ increases, in-phase synchronization becomes unstable and only anti-phase synchronization is observed as Fig. 3(h). Finally, both of synchronization becomes unstable and self-switching phenomenon of in-phase and anti-phase is observed as shown in Fig. 3(i).

We can also observe similar results from circuit experiments as shown in Fig. 4.

4. POINCARÉ MAP

We define the Poincaré section in the four-dimensional region where both diodes are in the OFF state. In that case the Poincaré map T can be derived as a three-dimensional map by using exact solutions. The projections of the Poincaré map onto the $x_1 - x_2$ plane by numerical calculation are shown in Fig. 5. We can see that synchronization of chaos breaks down in Fig. 5(c).

In order to investigate the stability of chaos synchronization in detail, we calculate the Lyapunov exponents of the three-dimensional Poincaré map. Note that because the Poincaré map can be represented by analytical form, we can derive its Jacobian DT analytically. The first, second and third Lyapunov exponents ($\lambda_1 > \lambda_2 > \lambda_3$) can be calculated by using DT as follows.

$$\lambda_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \ln | DT_j e_1^j |$$

$$\lambda_1 + \lambda_2 = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \ln | DT_j e_1^j \wedge DT_j e_2^j | \qquad (6)$$

$$\lambda_1 + \lambda_2 + \lambda_3 \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \ln | DT_j e_1^j \wedge DT_j e_2^j \wedge DT_j e_3^j |$$

where $\wedge$ is an exterior product.

Calculated results are shown in Fig. 6. We can classify characteristic of chaos as follows. The attractors in Fig. 3(b)∼(i) are classified to chaos, because Lyapunov exponents are as $\lambda_1 > 0$. While the attractor in Fig. 3(g) is hyperchaos with 2 positive Lyapunov exponents, because Lyapunov exponents are as $\lambda_1 > \lambda_2 > 0 > \lambda_3$. We can see that the hyperchaos appears for the value of $\gamma \geq 0.210$. It just corresponds to breakdown of in-phase synchronization of chaotic attractors. Namely, we can confirm that breakdown of chaos synchronization means the generation of hyperchaos. In this circuit, we can make sure that hyperchaos with three positive Lyapunov exponents exists $\gamma \geq 0.230$. However, such attractors cannot be observed at in-phase mode.

5. CONCLUSIONS

In this study, we investigated breakdown of in-phase synchronization of chaotic circuits coupled by an inductor. It was shown that breakdown of chaos synchronization occurs when chaotic attractor contains an unstable orbit splitting the attractor to two bands. Further it was confirmed that hyperchaos appeared at the same time.

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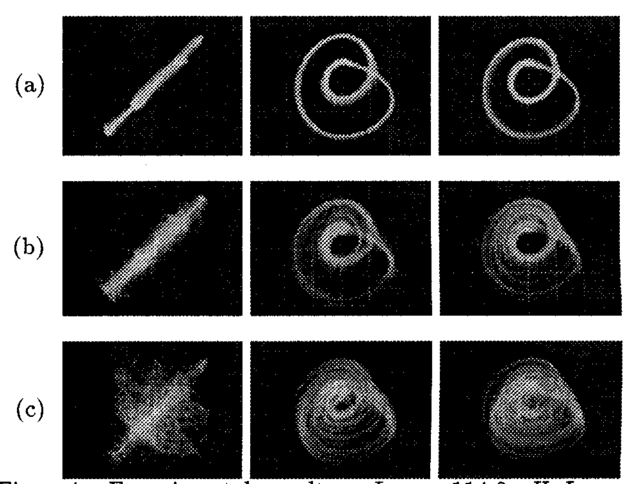

Fig. 4 Experimental results. $L_0 = 114.8mH, L_1 = 56.8mH, L_2 = 10.0mH, C = 68.2nF. (a) r = 2.11k\Omega, (b) r = 1.89k\Omega, (c) r = 1.61k\Omega$.

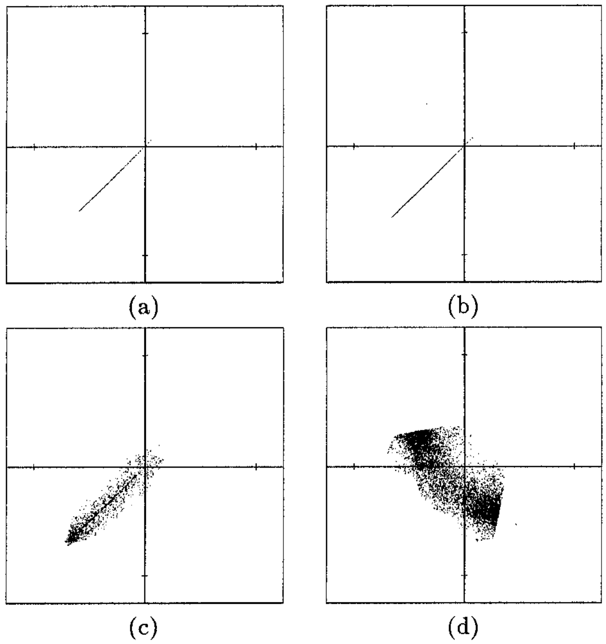

Fig. 5 Poincaré map. x_1 vs. x_2. γ : (a)0.175, (b)0.205, (c)0.215, (d)0.235.

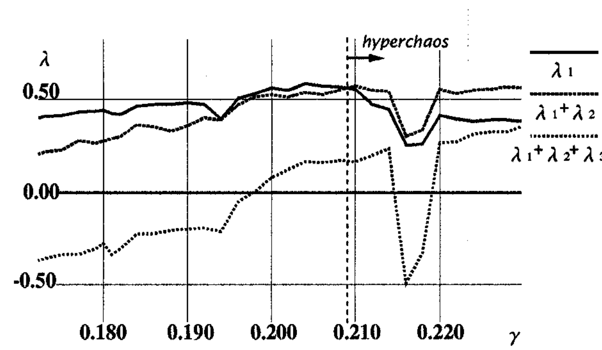

Fig. 7 Expanded boxed portion Fig. 6 Lyapunov exponents.

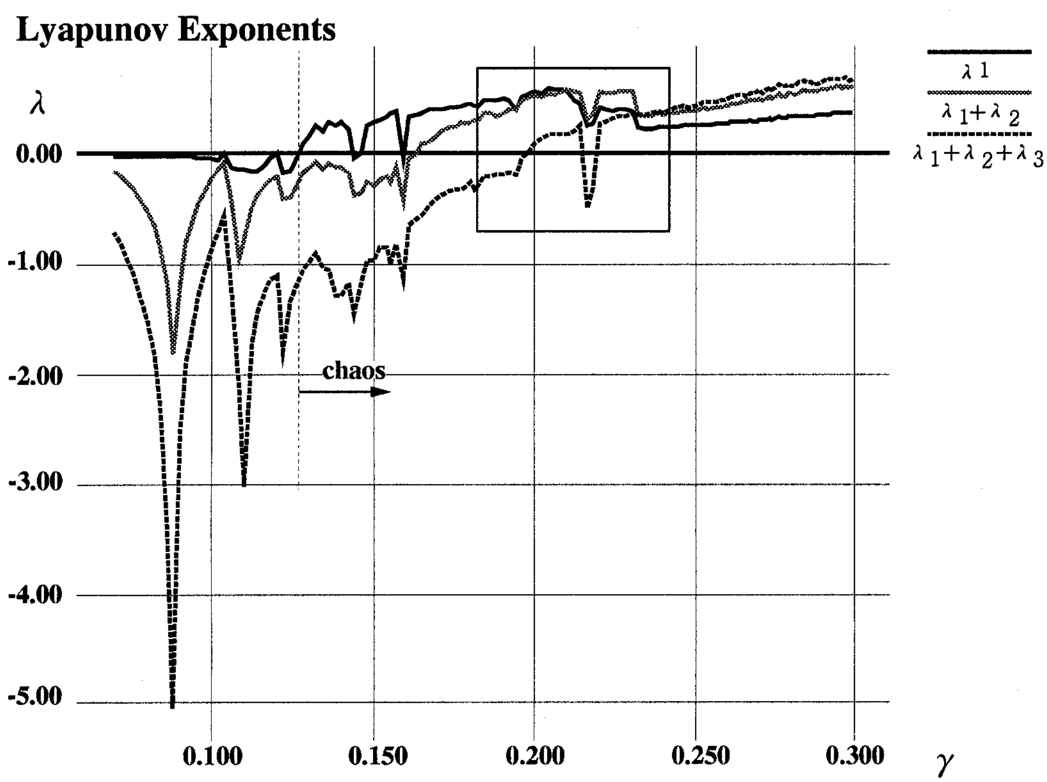

Fig. 6 Lyapunov exponents obtained from computer calculation.