ON SYNCHRONIZATION PHENOMENA IN COUPLED CHAOTIC CIRCUITS NETWORKS

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ABSTRACT

In this study spatial patterns of synchronization states observed from two types of chaotic circuits networks are investigated. For the parameter value at which chaotic subcircuits generate two asymmetric attractors, number of synchronization states increases and more complicated irregular self-switching phenomenon is observed.

1. INTRODUCTION

In our previous study [1], we have proposed two types of coupled oscillators networks and have investigated the generation of various phase patterns. Because the coupled oscillators networks have only local interconnection and generate various synchronization states, it has possibility to be utilized as a parallel information processing system such as cellular neural networks. However, the stability of the phase patterns observed from the Lattice type network is weak and some generating patterns sometimes vanishes by small error of real circuit elements. Further, the Honeycomb type network can generate only two phase states in spite of network size.

On the other hand, we also proposed two types of chaotic circuits networks whose structural element is chaotic autonomous circuit [2][3]. These networks were obtained by replacing oscillators in the above-mentioned coupled oscillators networks by chaotic circuits. By computer simulations, we investigated the generation of interesting self-switching phenomenon of spatial patterns characterized by phase-difference of quasi-synchronization of chaos [4]. However, we considered only the case that each subcircuit generates symmetric chaotic attractor. Namely, we have not investigated what types of synchronization phenomena are observed as control parameter of each subcircuit changes. Further, because only phase difference was available freedom, total number of spatial patterns were limited; especially, Honeycomb type network could produce only two patterns in spite of network size.

In this study we pay our attentions to the case that each chaotic subcircuit in two types of chaotic circuits networks generates chaotic attractors with asymmetric shape. Because of the symmetry of the subcircuit, two asymmetric attractors located symmetrically with respect to the origin coexist in each subcircuit. Hence, each subcircuit can take two different states. This means that the network have two different freedoms; attractors' shape and phase difference, and can produce more complicated irregular self-switching phenomena.

2. CIRCUIT MODEL

The two types of chaotic circuits networks [2][3] are shown in Fig. 1. These networks have local connection structure such that four or three subcircuits around one coupling resistor R are coupled. Though we treat the networks with the minimum size in this article, it is easy to extend to two-dimensional arrays.

Each chaotic subcircuit can produce chaotic attractors as shown in Fig. 2. In our previous study we treated the case that each subcircuit generates symmetric chaotic attractor in Fig. 2(a). In this study we consider the case that each subcircuit generates asymmetric chaotic attractors in Fig. 2(b).

At first, we approximate the $v-i$ characteristics of the diodes by the function $v_d = \sqrt[9]{r_d i_k}$. The circuit equation for the Lattice type network is described by 42nd order ordinary differential equation including 9 nonlinear functions, while for the Honeycomb type network by 30th order differential equation including 6 nonlinear functions. In the followings, The differential equations are calculated by using Runge-Kutta method.

Because of the limitation of the page space, we show only the normalization of variables and parameters.

$$I_k = a\sqrt{\frac{C}{L_1}}x_k, \quad i_k = a\sqrt{\frac{C}{L_1}}y_k, \quad v_k = az_k,$$

$$\alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \quad \gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = R_e\sqrt{\frac{C}{L_1}}, \quad (1)$$

$$\left(\text{where} \quad a = \sqrt[8]{r_d\sqrt{\frac{C}{L_1}}}\right),$$

3. SYNCHRONIZATION PHENOMENA

Figure 3 shows an example of synchronized patterns observed from Lattice type network. We represent the information of the attractors' shape as $\begin{pmatrix} U & L & U \\ U & L & L \\ U & L & L \end{pmatrix}$ where the notations U and L means chaotic attractors in Figs. 2(a) and 2(b), respectively. Simply said, there coexist $2^9 = 512$ different synchronized patterns with respect to the attractors' shape.

Although Fig. 3 cannot show the information on the phase difference, the two pairs of opposite phases quasi-synchronizations [2][4] generates around each coupling resistor. The information of the phase difference is represented by the notation in [2] and the number of different spatial

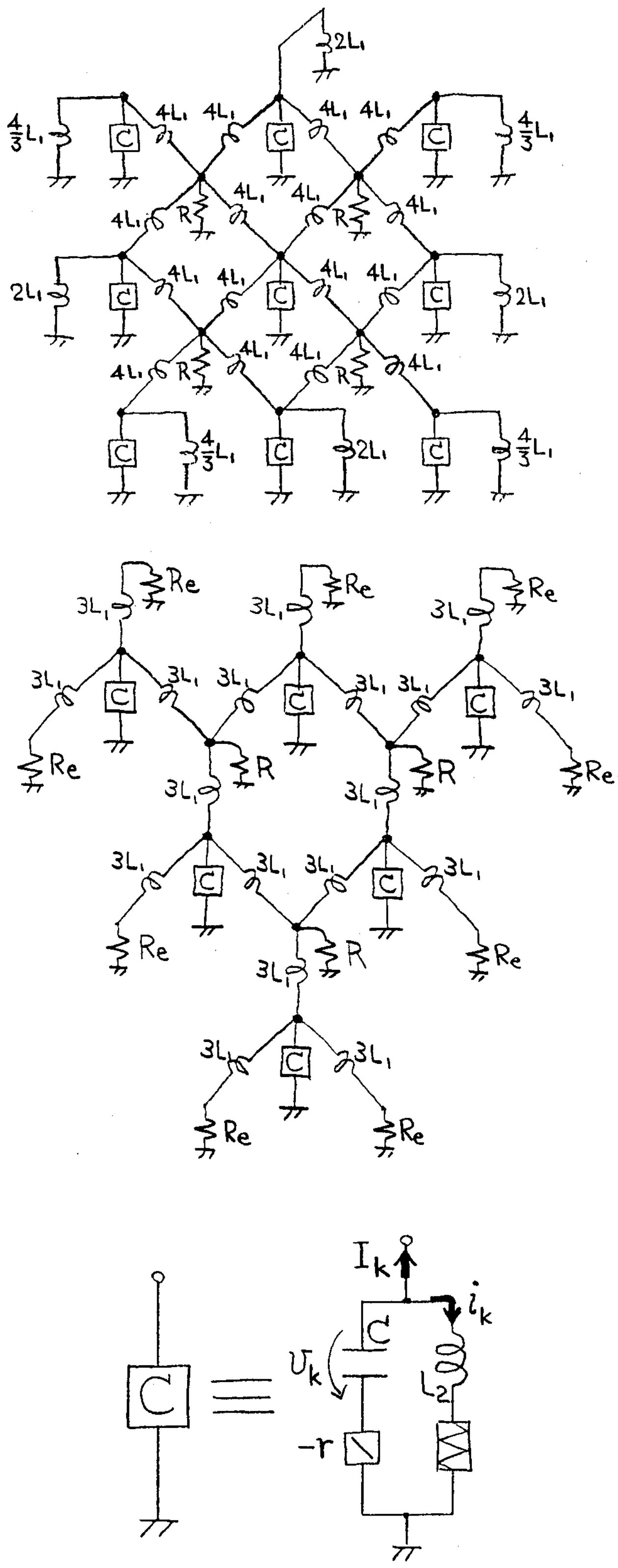
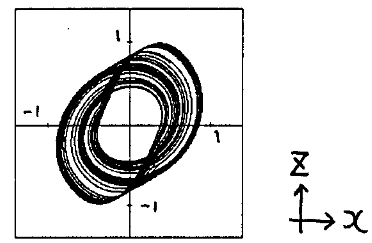
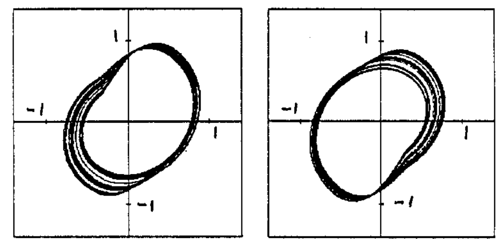


Fig. 1 Coupled oscillators networks.



(a)



(b)

Fig. 2 Chaotic attractors observed feom each subcircuit. (a) Symmetric attractor ($\alpha = 24.0$ and $\beta = 0.300$). (b) Coexistence of two asymmetric attractor ($\alpha = 24.0$ and $\beta = 0.283$).

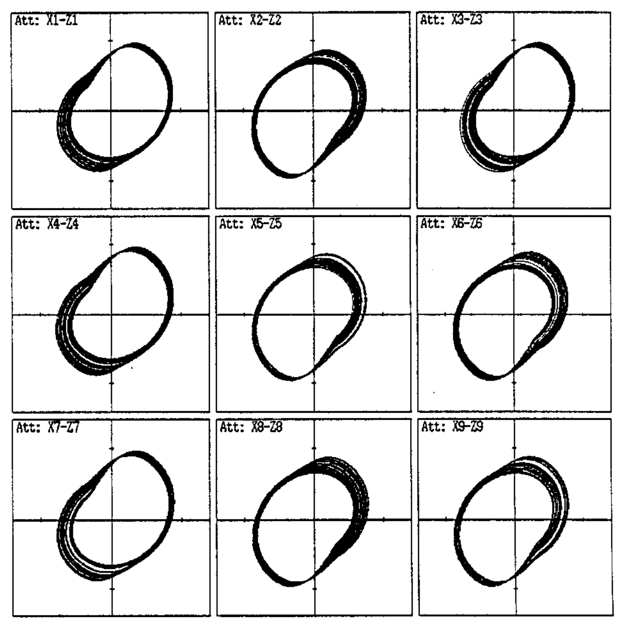


Fig. 3 An example of synchronized patterns observed from Lattice type network ($\alpha = 24.0$, $\beta = 0.280$ and $\gamma = 0.01$).

patterns is seven as follows.

$$\begin{pmatrix} A & \overline{A} & A \\ B & \overline{B} & B \\ C & \overline{C} & C \end{pmatrix}, \begin{pmatrix} A & B & C \\ \overline{A} & \overline{B} & \overline{C} \\ A & B & C \end{pmatrix}, \begin{pmatrix} A & B & A \\ \overline{A} & \overline{B} & \overline{A} \\ B & A & B \end{pmatrix},$$

$$\begin{pmatrix} A & \overline{A} & B \\ B & \overline{B} & A \\ A & \overline{A} & B \end{pmatrix}, \begin{pmatrix} A & B & \overline{B} \\ \overline{B} & \overline{A} & A \\ A & B & \overline{B} \end{pmatrix}, \begin{pmatrix} A & B & A \\ \overline{B} & \overline{A} & \overline{B} \\ B & A & B \end{pmatrix},$$

$$\begin{pmatrix} A & B & A \\ \overline{B} & \overline{A} & \overline{B} \\ A & B & A \end{pmatrix}, \tag{2}$$

where different characters represent independent phase and $\overline{A}$ means the opposite phase of A.

Because attractors' shape and phase difference seem to be independent, it is considered that there coexist $512 \times 7 = 3584$ spatial patterns.

On the other hand, Fig. 4 shows an example of synchronized patterns observed from Honeycomb type network. The information of the attractors' shape is represented as $\begin{pmatrix} U & & L & & L \\ & U & & U & \\ & & U & & \end{pmatrix}$. Simply said, there coexist $2^6 = 64$ different synchronized patterns with respect to the attractors' shape.

The Honeycomb type network produce only the following two different phase states in spite of the network size.

$$\begin{pmatrix} 0 & & \frac{2\pi}{3} & & \frac{4\pi}{3} \\ & \frac{4\pi}{3} & & 0 & \\ & & \frac{2\pi}{3} & & \end{pmatrix}, \begin{pmatrix} 0 & & \frac{4\pi}{3} & & \frac{2\pi}{3} \\ & \frac{2\pi}{3} & & 0 & \\ & & \frac{4\pi}{3} & & \end{pmatrix}, \tag{3}$$

where 0, $2\pi/3$ or $4\pi/3$ means the phase difference with respect to the subcircuit located at the upper left corner.

Hence, it is considered that there coexist $64 \times 2 = 128$ spatial patterns.

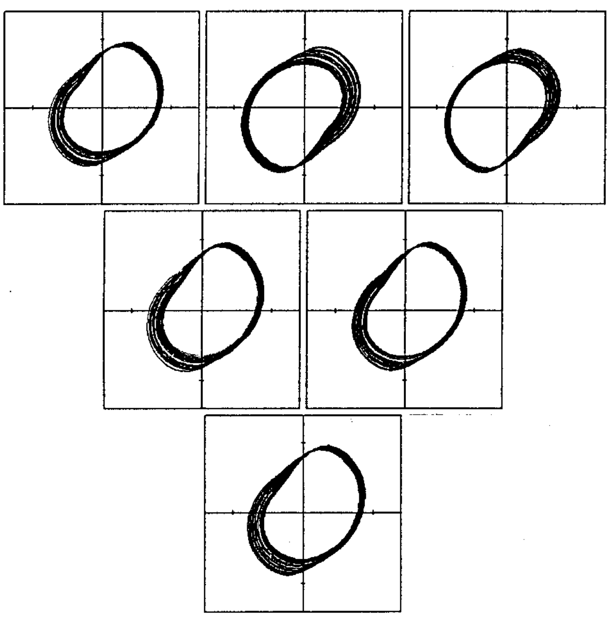


Fig. 4 An example of synchronized patterns observed from Honeycomb type network ($\alpha = 24.0$, $\beta = 0.280$, $\gamma = 0.05$ and $\delta = 0.01$).

4. SELF-SWITCHING OF SPATIAL PATTERNS

In order to investigate self-switching phenomenon of spatial patterns observed from the Lattice type network, we define the Poincaré section as $z_1 = 0$ where $dz_1/dt > 0$ and plot the values of x_i (i=1, 2, $\cdots$, 9) on $x_i - n$ (n denotes the number of iteration of the Poincaré map) plane when the solution hits the Poincaré section.

Figure 5 shows the time series of the Poincaré map obtained from computer simulations where the position of subcircuits corresponding to x_1–x_9 is as follows.

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix} \tag{4}$$

From the figure we can see the self-switching phenomena of various spatial patterns. For example, a spatial pattern with phase states of $\begin{pmatrix} A & B & \overline{B} \\ \overline{B} & \overline{A} & A \\ A & B & \overline{B} \end{pmatrix}$ and attractors' shape of $\begin{pmatrix} L & L & H \\ H & H & H \\ H & L & H \end{pmatrix}$ appear in the shaded area. Also spatial patterns with phase states of $\begin{pmatrix} A & B & C \\ \overline{A} & \overline{B} & \overline{C} \\ A & B & C \end{pmatrix}$ appears in the left side of the figure. Further, switching of attractors' shape can be observed at many points.

Figure 6 shows the result obtained for different parameter sets. Switching speed of phase states becomes slower and switching speed of attractors' shape becomes faster. Namely, it is possible to change the switching speed of phase states and attractors' shape by varying parameters of coupling and negative resistance.

For Honeycomb type network, we investigated time series of Poincaré map. However, we cannot observe self-switching between two different phase states. The phase difference seems to be decided by given initial conditions. Hence, we can observe self-switching of 64 spatial patterns with different attractors' shape.

5. CONCLUSIONS

In this study we have investigated spatial patterns of synchronization states observed from two types of chaotic circuits networks. For the parameter value at which chaotic subcircuits generate two asymmetric attractors, number of synchronization states increases and more complicated irregular self-switching phenomenon are observed.

For the purpose of engineering applications of this network, it will be first step to control generating spatial patterns. Namely, the situation that only some of generating patterns remains stable and others are unstable is desirable. We have tried to add one connections between two subcircuits, it was partly achieved. The detail will be shown in the presentation.

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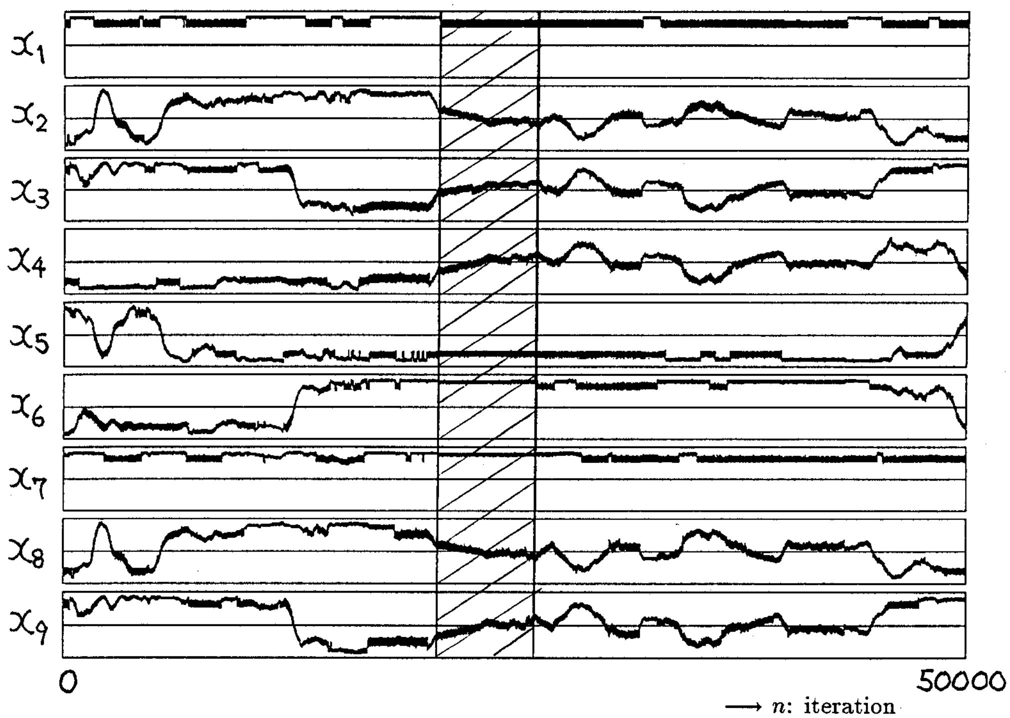

Fig. 5 Time series of Poincaré map: Self-switching of various spatial patterns observed from Lattice type network ($\alpha = 24.0$, $\beta = 0.280$ and $\gamma = 0.30$).

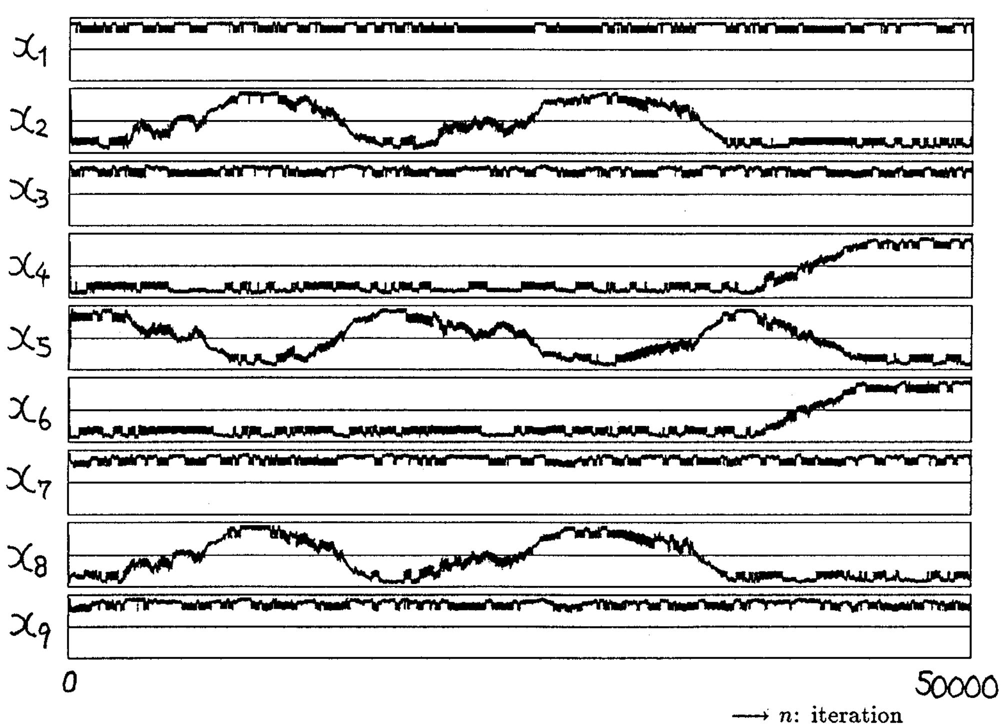

Fig. 6 Time series of Poincaré map: Self-switching of various spatial patterns observed from Lattice type network ($\alpha = 24.0$, $\beta = 0.2805$ and $\gamma = 0.10$).