Spatiotemporal Chaos in Four Chaotic Circuits Coupled by One Resistor

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ABSTRACT

In this study, four simple autonomous chaotic circuits coupled by one resistor are investigated. By carrying out computer calculations and circuit experiments, it is shown that our very simple coupled circuit can exhibit spatiotemporal chaos as well as quasi-synchronizations of chaos in spite of that the number of chaotic subcircuits is only four.

I. Introduction

Many nonlinear dynamical systems in the various fields have been clarified to exhibits chaotic oscillations and recently applications of chaos to engineering systems attract many researchers' attentions, for example, chaos noise generator, control of chaos, synchronization of chaos, and so on. Among the studies on such applications, synchronization of chaotic systems or signals is extremely interesting [1]~[3], because the chaotic solution is unstable and small error of initial values must be expanded as time goes. As far as we know, such phenomena have been firstly reported to be genareted in simple real circuits by a group of Saito [1]. Since Pecora *et al.* have investigated such phenomena theoretically [2], many papers have been published until now. Further, secure communication systems using chaos synchronizations [4][5] and coupled chaoptic circuits generating various types of quasi-synchronizations [6]~[8] are also proposed.

On the other hand, a network of chaotic one-dimensional maps have been investigated earnestly by Kaneko [9]~[12]. He has discovered varios kinds of phenomena called as spatiotemporal chaos such as diffusion and Brownian motion of defect, clustering, spatiotemporal intermittency and so on. Recently, Chua and his colleagues published their papers on spatiotemporal chaos observed in a chain of coupled Chua's circuits [13][14]. However, they treated only the parameter sets for which each Chua's circuits generate simple one-periodic attractor. The study of such systems are very important not only as models for nonlinear systems with many degrees of freedom, but also for the clarification of biological information processing and for engineering applications.

In this study, we investigate four simple autonomous chaotic circuits coupled by one resistor. By carrying out computer calculations and circuit experiments, we found that our very simple coupled circuit can exhibit spatiotemporal chaos as well as quasi-synchronizations of chaos in spite of that the number of chaotic subcircuits is only four. We consider that our model would be good model to clarify mechanism and characteristics of spatiotemporal chaos because it is the simplest real physical system exhibiting spatiotemporal chaos. Further, we would like to emphasize that all phenomena introduced in this paper have been observed from real physical circuit model made up easily in the laboratory.

II. Circuit Model

Circuit model is shown in Fig. 1. In our system four same chaotic circuits are coupled by one resistor. Each chaotic subcircuit is a symmetric version of the circuit model proposed by Inaba *et al.* [15]. It consists of three memory

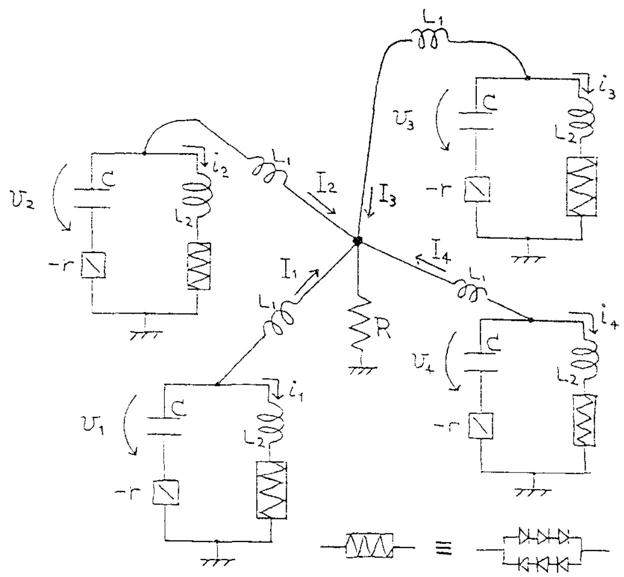

Fig. 1 Circuit model.

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elements, one linear negative resistor and one nonlinear resistor, which is realized by connecting some diodes, and is one of the simplest chaotic circuits. Fig. 2 shows a typical example of chaotic attractors obtained from the uncoupled chaotic subcircuit. In the following circuit experiments, the values of the inductors and the capacitor in each chaotic subcircuit are fixed and those values are measured as $L_1 = 204.15mH \pm 0.073\%$, $L_2 = 9.933mH \pm 0.030\%$ and $C = 0.03425\mu F \pm 0.29\%$.

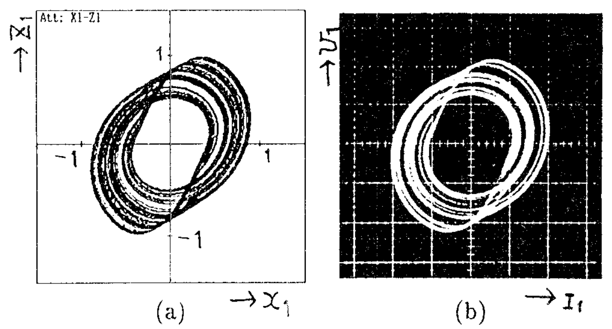

Fig. 2 Typical example of chaotic attractors observed from the chaotic subcircuit. (a) Computer calculation for $\alpha = 24.0$ and $\beta = 0.295$. (b) Circuit experiment fot $L_1 = 204.15mH$, $L_2 = 9.933mH$, $C = 0.03425\mu F$ and $r = 630\Omega$. Horizontal: 0.4mA/div. Vertical: 1V/div.

At first, we approximate the $i - v$ characteristics of the nonlinear resistor consisting of diodes by the following function.

$$v_d(i_k) = \sqrt[9]{r_d i_k}. \tag{1}$$

By changing the variables and parameters,

$$\begin{aligned} & t = \sqrt{L_1 C}\tau, \qquad \text{" . "} = \frac{d}{d\tau}, \\ & I_k = a\sqrt{\frac{C}{L_1}}x_k, \quad i_k = a\sqrt{\frac{C}{L_1}}y_k, \quad v_k = a z_k, \\ & \alpha = \frac{L_1}{L_2}, \qquad \beta = r\sqrt{\frac{C}{L_1}}, \qquad \gamma = R\sqrt{\frac{C}{L_1}}, \\ & \left(\text{where} \quad a = \sqrt[8]{r_d\sqrt{\frac{C}{L_1}}}\ \right), \end{aligned} \tag{2}$$

(2) is normalized as

$$\begin{aligned} \dot{x_k} &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_k \\ \dot{y_k} &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z_k} &= x_k + y_k \end{aligned} \tag{3}$$

$$(k=1,\ 2,\ 3,\ 4)$$

where

$$f(y_k) = \sqrt[9]{y_k}. \tag{4}$$

For computer calculations, in order to consider the difference of real circuit elements, (4) is rewritten as follows.

$$\begin{aligned} \dot{x_k} &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_k \\ \dot{y_k} &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z_k} &= \{1 + (k-1)\Delta\omega\}(x_k + y_k) \end{aligned} \tag{5}$$

$$(k=1,\ 2,\ 3,\ 4).$$

In the following computer calculations, we fix the parameter α as 24.0 and (5) is calculated by using the Runge-Kutta method with step size $\Delta t = 0.01$.

III. In and opposite-phases quasi-synchronization

Before we treat spatiotemporal chaos, we introduce two types of quasi-synchronizations of chaos in this section and next one.

Fig. 3 shows an example of in and opposite-phases quasi-synchronizations. In this case, each subcircuit exhibits chaos as Fig. 2. But, two of four subcircuits are almost synchronized at the in-phase and the rest is almost synchronized to the two subcircuits with π phase difference. Namely, phase difference with respect to the subcircuit 1 is described as $\{\pi,\ 0,\ \pi\}$ for the example in Fig. 3. Though we omit other phase states in Fig. 3, there coexist more two different phase states, namely $\{0,\ \pi,\ \pi\}$ and $\{\pi,\ \pi,\ 0\}$.

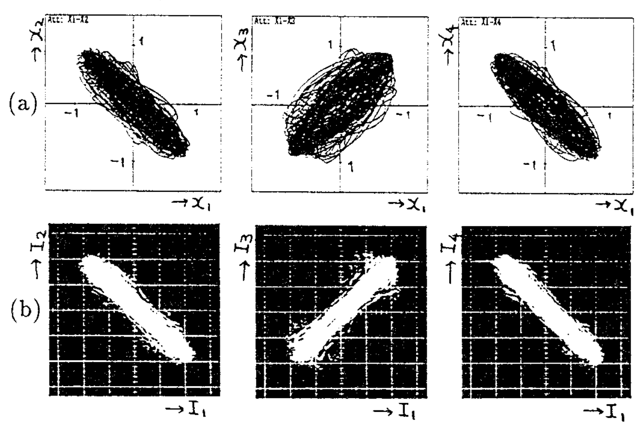

Fig. 3 In and opposite-phases quasi-synchronization. (a) Computer calculation for $\beta = 0.29$, $\gamma = 0.40$ and $\Delta\omega = 0.0$. (b) Circuit experiment fot $r = 570\Omega$ and $R = 2.4k\Omega$. Horizontal and Vertical: 0.4mA/div.

IV. Two pairs of opposite-phases quasi-synchronization

Fig. 4 shows an example of two pairs of opposite-phases quasi-synchronizations. In this case, subcircuits 1 and 2 are almost synchronized at the opposite-phase. Also subcircuits 3 and 4 are almost synchronized at the opposite-phase. However, a pair of subcircuits 1−2 and the other pair of 3−4 are independent. We had considered that there coexist more two different phase states, namely {1−3, 2−4} and {1−4, 2−3}. However actually the combination of the decoupling into two pairs is decided by the slight difference of real circuit elements and other combination states cannot be observed. Namely, this quasi-synchronization is based on the asymmetry of the coupling and cannot be generated in the case of completely symmetric coupling as $\Delta\omega = 0.0$. We consider that this phenomenon is deeply related with the clustering [12]. However we do not call this phenomenon as spatiotemporal chaos because the spatial pattern is always the same.

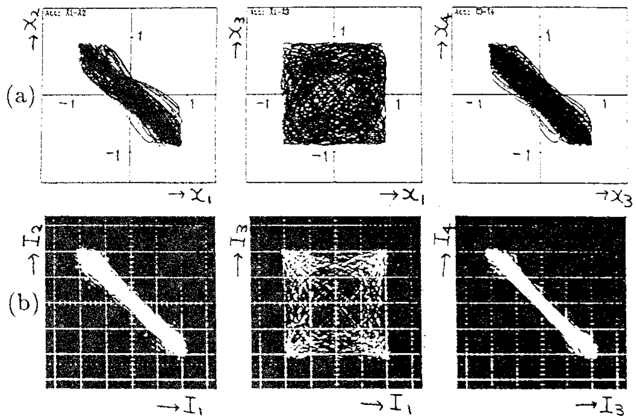

Fig. 4 Two-pairs of opposite-phases quasi-synchronization. (a) Computer calculation for $\beta = 0.295$, $\gamma = 0.340$ and $\Delta\omega = 0.01$. (b) Circuit experiment fot $r = 580\Omega$ and $R = 850\Omega$. Horizontal and Vertical: 0.4mA/div.

V. Spatiotemporal chaos

For large region in parameter space, we observed complex chaotic motion. Namely, we observed that three phase states of in and opposite-phases quasi-synchronizations are switched automatically and randomly as shown in Fig. 5. The order of the appearance of three phase states is truly unpredictable. Further switching period is also chaotic, namely a state may be switched to the next state instantly and a state may be switched after about a few second. We also observed similar self-switching phenomenon of three phase states of two pairs of opposite-phases quasi-synchronizations as shown in Fig. 6. We can call these phenomena as saptio-temporal chaos because spatial pattern corresponding to three synchronization states changes chaotically as time goes and it is caused by the local chaotic motion of subcircuits.

In order to investigate such phenomena, we define the Poincaré section as $z_1 = 0$ where $dz_1/dt > 0$ and plot the values of x_i (i=1, 2, 3, 4) on $x_i - n$ (n denotes the number of iteration of the Poincaré map) plane when the solution hits the Poincaré section. Fig. 7 shows time series of attractors corresponding to the self-switching of in and opposite-phases quasi-synchronizations. For example, phase state of {π, π, 0} appears in the shaded area.

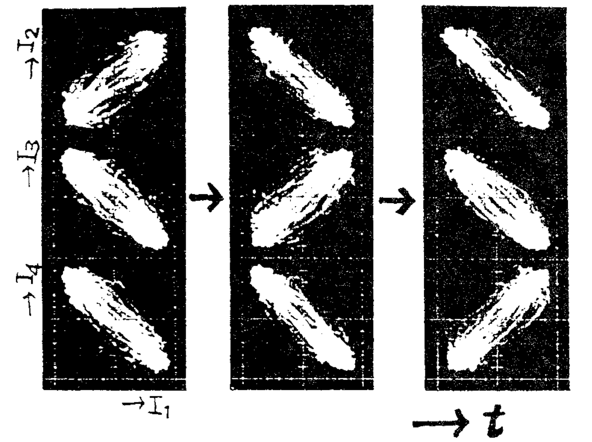

Fig. 5 Spatio-temporal chaos: Self-switching of in and opposite-phases quasi-synchronizations. $r = 630\Omega$ and $R = 1.3k\Omega$. Horizontal and Vertical: 1.0mA/div.

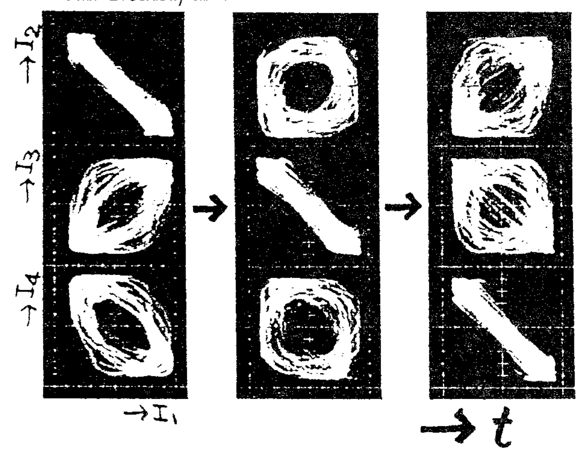

Fig. 6 Spatio-temporal chaos: Self-switching of two-pairs of opposite-phases quasi-synchronizations. $r = 630\Omega$ and $R = 96\Omega$. Horizontal and Vertical: 1.0mA/div.

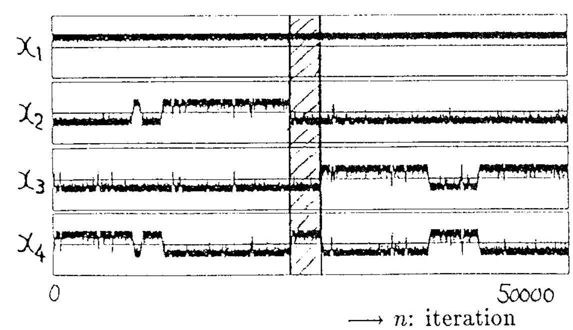

Fig. 7 Spatio-temporal chaos: Self-switching of in and opposite-phases quasi-synchronizations. $\beta = 0.295$, $\gamma = 0.10$ and $\Delta\omega = 0.0$.

In the figure we can confirm that three phase states appear chaotically. Fig. 8 shows time series of attractors corresponding to the self-switching of two-pairs of opposite-phases quasi-synchronizations. For example, phase state of $\{1-4,\ 2-3\}$ appears in the shaded area. In the figure we can confirm that three phase states appear chaotically.

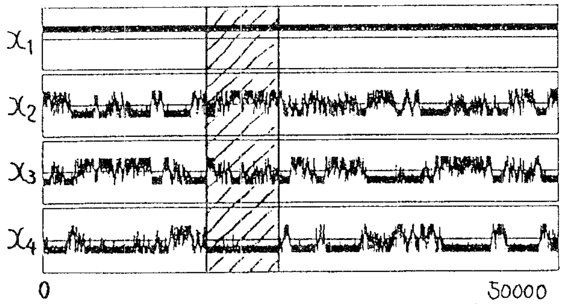

Fig. 8 Spatio-temporal chaos: Self-switching of two-pairs of opposite-phases quasi-synchronizations. $\beta = 0.295$, $\gamma = 0.01$ and $\Delta\omega = 0.0$.

Because of the limited space, we cannot explain detailed results on the effect of $\Delta\omega$. But we confirmed that both of the appearing frequency and switching speed are deeply related with the value of $\Delta\omega$. We show one example in Fig. 9.

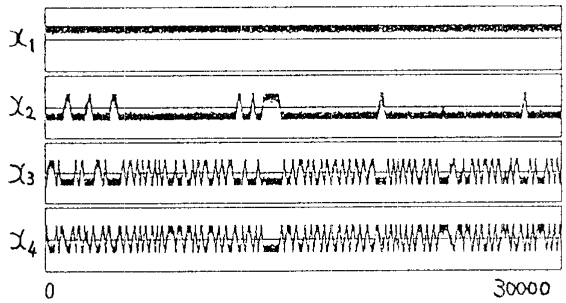

Fig. 9 Effect of the value of $\Delta\omega$. $\beta = 0.295$, $\gamma = 0.10$ and $\Delta\omega = 0.005$.

Further we confirmed that intermittency of one subcircuit sometimes causes breakdown of a spatial pattern.

VI. Concluding Remarks

In this study, we investigated four simple autonomous chaotic circuits coupled by one resistor. By carrying out computer calculations and circuit experiments, we found that our very simple coupled circuit can exhibit spatiotemporal chaos as well as two types of quasi-synchronizations of chaos in spite of that the number of chaotic subcircuits is only four. We wolud like to emphasize again that the circuit model in this article is the simplest real physical systems exhibiting spatiotemporal chaos.

Our future research is theoretical approach to spatiotemporal chaos including statistical study and extension to large number of circuits case or another coupling structures case.

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