

Synchronization Phenomena in RC Oscillators Coupled by One Resistor

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ABSTRACT

There have been many investigations of the mutual synchronization of oscillators ([1]–[6] and therein). Especially, we have analyzed N van der Pol type LC oscillators coupled by one resistor, and confirmed that this system can take a huge number of steady states. In this study, we analyze N Wien-bridge oscillators with the same natural frequency mutually coupled by one resistor by both of computer calculations and circuit experiments. Because there is no inductor in this system, it is suitable for VLSI implementation. Moreover, the system have much more phase states than that of the system with van der Pol type LC oscillators when N is not so large. So we can utilize this system as a structural element of cellular neural networks.

INTRODUCTION

There have been many investigations of the mutual synchronization of oscillators ([1]–[6] and therein). Endo and we have analyzed the systems of large number of coupled van der Pol oscillators[1]. Kimura et al. have confirmed that two oscillators coupled by resistors are synchronized at opposite phase[2] and three oscillators coupled by resistors are synchronized at 3-phase[3]. Moreover, we have investigated the synchronization phenomena in N oscillators coupled by one resistor[4][6]. When the nonlinearity is the third-power characteristic, N -phase oscillations are stably excited and the system has $(N - 1)!$ phase states[4]. When the nonlinearity is the fifth-power characteristic, not only N but also $N - 1, N - 2, \dots, 2$ -phase oscillations are stably excited and the system has much more phase states than that of system with the third-power nonlinear characteristics[6]. Because of the coupling structure and extremely large number of steady states of the systems [4] and [6], these systems could be used as a structural element of cellular neural networks[5].

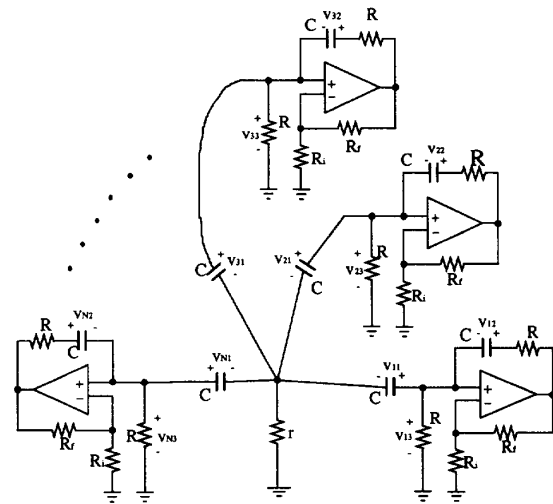


Fig. 1: Circuit model.

In this study, we investigate the synchronization phenomena in the RC Wien-bridge oscillators coupled by one resistor with both of computer calculations and circuit experiments. Because the Wien-bridge oscillator doesn't include inductors and consists of only capacitors, resistors and op-amp but doesn't include inductors, the system is suitable for VLSI implementation. And in this system, we can see different type of synchronization from N -phase oscillation and we can get much more phase states than that of the system of van der Pol oscillators when N is not so large. So this system is much more suitable for a structural element of cellular neural networks than the system of van der Pol type LC oscillators.

SYNCHRONIZATION PHENOMENA IN WIEN-BRIDGE OSCILLATORS' SYSTEM

Recently, we have reported synchronization phenomena in N van der Pol oscillators with the same natural frequency coupled by one resistor. In such systems, various synchronization phenomena can be stably observed, because they tend to minimize the current through the coupling resistor.

When the nonlinear characteristics are the third-power, we can see N -phase oscillations. When we take the waveform observed in one oscillator as reference signal, the other oscillators can take any phase differences among $\phi_k = 2k\pi/N$ ($k = 1, 2, \dots, N-1$). Therefore, this system can take $(N-1)!$ phase states. Because of this coupling structure and extremely large number of steady states, the systems described above would be suitable for an extremely large memory or a structural element of cellular neural networks.

But because the systems of van der Pol oscillators include inductors, these systems are not available for VLSI implementation. So we need to get many steady states on RC oscillators coupled by one resistor. We show an example of such systems as follows.

The circuit model is shown in Fig. 1. N Wien bridge oscillators with the same natural frequencies are coupled by one resistor. Op-amps have nonlinear gain characteristics approximately as follows.

$$v'_k = g_1 v_{k3} - g_3 v_{k3}^3 \quad (1)$$

So circuit equations are described as follows,

$$\begin{cases} C \frac{dv_{k1}}{dt} = C \frac{dv_{k2}}{dt} - \frac{1}{R} v_{k3} \\ C \frac{dv_{k2}}{dt} = \frac{1}{R} (g_1 v_{k3} - g_3 v_{k3}^3 - v_{k2} - v_{k3}) \\ C \sum_{j=1}^N \frac{dv_{k1}}{dt} = \frac{1}{r} (v_{k3} - v_{k1}). \end{cases} \quad (2)$$

By changing variables,

$$\begin{aligned} t &= RC\tau, \\ x_k &= \sqrt{\frac{g_1-3}{3g_3}} v_{k1}, \quad y_k = \sqrt{\frac{g_1-3}{3g_3}} v_{k2}, \\ z_k &= \sqrt{\frac{g_1-3}{3g_3}} v_{k3}, \quad \alpha = \frac{r}{R}, \quad \varepsilon = g_1 - 3, \end{aligned} \quad (3)$$

(2) is normalized as

$$\begin{cases} \dot{x}_k = \dot{y}_k - z_k \\ \dot{y}_k = \varepsilon \left(z_k - \frac{z_k^3}{3} \right) - y_k + 2z_k \\ z_k = x_k + \alpha \sum_{j=1}^N \dot{x}_j. \end{cases} \quad (4)$$

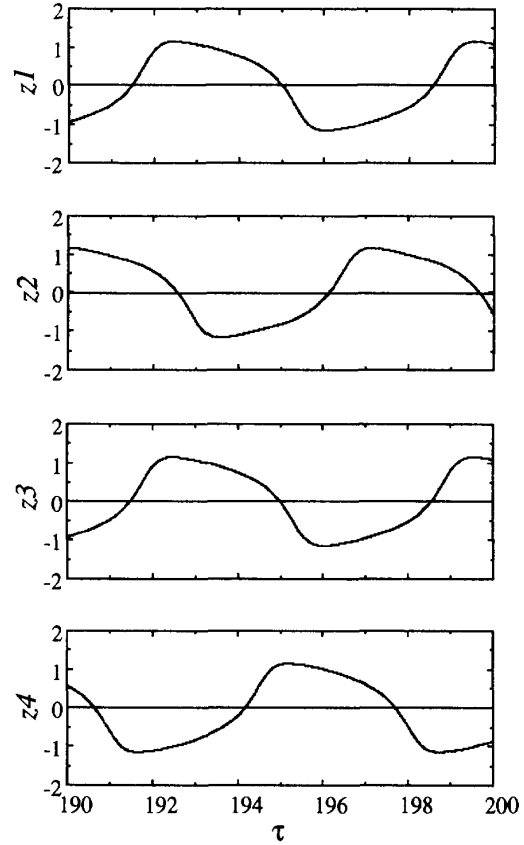


Fig. 2: Numerical results for the case of $N = 4$ ($\alpha = 0.01, \varepsilon = 1.5$).

In equations (4), α is coupling factor and ε is the strength of nonlinearity. Because the condition for oscillation of the Wien-bridge oscillators is $g_1 > 3$, the strength of nonlinearity ε must be larger than 0.

Next, we show an examples of numerical results with Runge-Kutta-Gill method and corresponding circuits experimental results for the case of $N = 4, 5$ (Figs. 2~5). On computer calculations, we have to consider the differences among the natural frequencies of real oscillators. On circuit experiments, we take $R_i = 4.7k\Omega$ and $R_f = 14.7k\Omega$.

From these results, we can see the different type of synchronization from N -phase oscillation. In these figures, we can see only 3-phase oscillation. In figs. 2 and 3, oscillators 1, 2 and 4 synchronize at almost 3-phase but oscillator 3 synchronize with oscillator 1 at in-phase. Similarly, in figs. 4 and 5, oscillators 1, 2 and 5 synchronize at almost 3-phase, but oscillators 1 and 3 synchro-

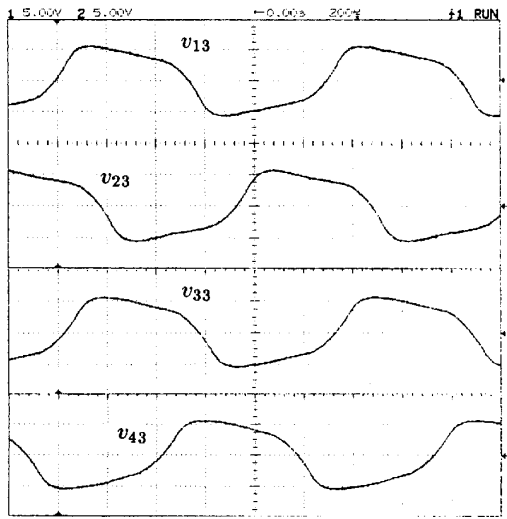


Fig. 3: Experimental results for the case of $N = 4$ ($R = 100\text{k}\Omega$, $C = 0.0015\mu\text{F}$, $r = 2.0\text{k}\Omega$, horizontal scale: 0.2msec/div. , vertical scale: 5V/div.)

nize at in-phase and oscillators 2 and 4 synchronize at in-phase.

Because the type of synchronization is different from the case with van der Pol oscillators, the number of phase states must be different from previous case. In this case, we can summarize the way of synchronization that 3 of N oscillators synchronize at 3-phase and the other oscillators synchronize at in-phase with the oscillator which synchronize at 3-phase. So when the number of oscillators are N , we can consider the number of the phase states P_N as follows.

$$P_N = 2 \cdot {}_N C_3 \cdot 3^{N-3} \quad (5)$$

From this equation, we can see this system have much more phase states than that of the system with van der Pol type LC oscillators. For comparison, we show the number of the phase states of this system and the systems with van der Pol type oscillators in Table 1.

When N is large (about over 10), the number of the phase states P_N becomes smaller than that of the system with van der Pol oscillators. But on cellular neural networks, the number of connections should be small. So when we use these oscillator system as a structural element of cellular neural networks, the number of coupled oscillators should be small. Therefore this system is very suitable for a structural element of cellular neural networks.

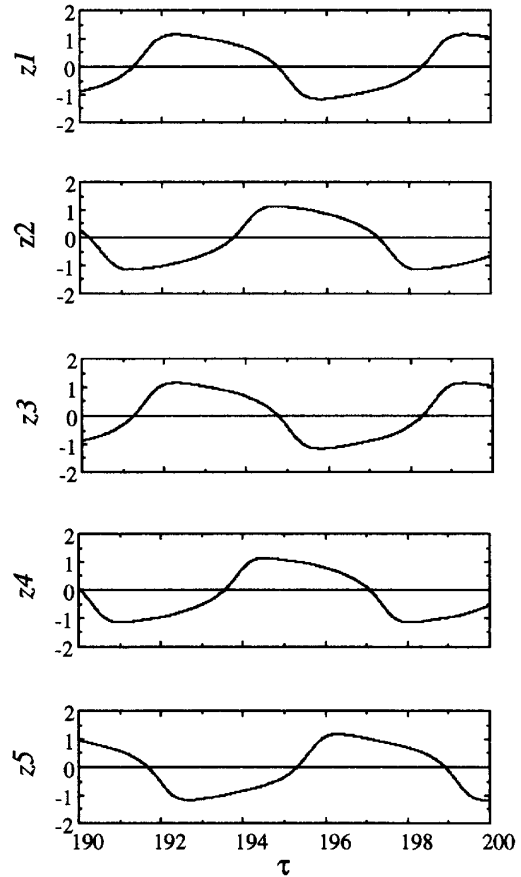


Fig. 4: Numerical results for the case of $N = 5$ ($\alpha = 0.01$, $\varepsilon = 1.5$).

CONCLUSIONS

In this study, we have investigated the synchronization phenomena of the Wien-bridge oscillators coupled by one resistor by both of computer calculations and circuit experiments. In this system, we can see only 3-phase oscillations while N -phase oscillations in the system of van der Pol oscillators. Because there is no inductor in Wien-bridge oscillators, this system is suitable for VLSI implementation. Moreover, we can get much more phase states than that of the system with van der Pol type oscillators.

In article [5] and [6], we show that the system of the oscillators coupled by one resistor would be utilized as a structural element of cellular neural networks because of their coupling structure and extremely large number of phase states. Moreover, the system has no inductor. So this system is much more suitable for a structural

Table 1: Comparison of the number of phase states.

N	with Wien-bridge oscillators	with LC oscillators	
		Third-power	Fifth-power
3	2	2	5
4	24	3	17
5	180	24	69
6	1080	45	169
⋮	⋮	⋮	⋮
13	33,776,028	479,001,600,	792,712,283

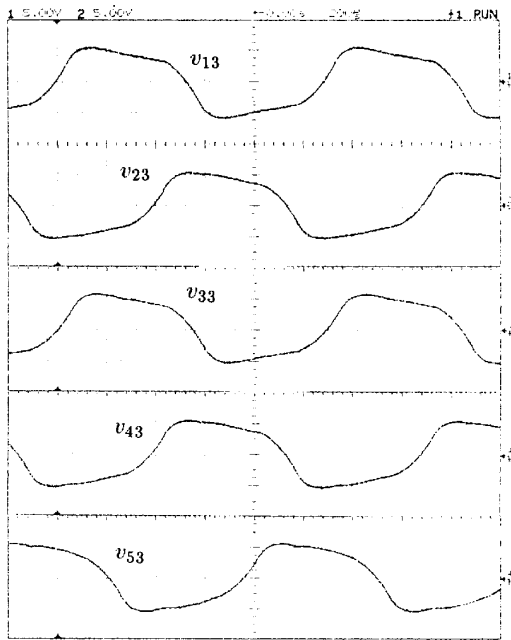


Fig. 5: Experimental results for the case of $N = 5$ ($R = 100\text{k}\Omega$, $C = 0.0015\mu\text{F}$, $r = 2.0\text{k}\Omega$, horizontal scale: 0.2msec/div. , vertical scale: 5V/div.)

element of new type of VLSI chip for cellular neural networks.

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