

Multimode Chaos in Two Coupled Chaotic Oscillators with Hard Nonlinearities

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ABSTRACT

In this study, multimode chaos observed from two coupled chaotic oscillators with hard nonlinearities is investigated. At first, a simple chaotic oscillator with hard nonlinearities is realized. It is confirmed that in this chaotic oscillator the origin is always asymptotically stable and that the solution, which is excited by giving relatively large initial conditions, undergoes period-doubling bifurcations and bifurcates to chaos. Next, four different modes of oscillations are observed from two coupled chaotic oscillators with hard nonlinearities by both of computer calculations and circuit experiments. One of the modes of oscillation is a nonresonant double-mode oscillation and this oscillation is stably generated even in the case that oscillation is chaotic. Namely, for this oscillation mode, chaotic oscillation and periodic oscillation can be simultaneously excited. We call this phenomena as double-mode chaos. Finally, the beat frequency of the double-mode chaos is confirmed to be changed by tuning the value of the coupling capacitor.

INTRODUCTION

Coupled oscillators systems are good model to describe various nonlinear phenomena in the field of natural science and a number of excellent studies on mutual synchronization of oscillators have been carried out (e.g. [1]-[4]). Oscillators containing a nonlinear resistor whose $v - i$ characteristics are described by fifth-power nonlinear characteristics are known to exhibit hard excitation [5][6]. Namely, the origin is asymptotically stable and an proper initial condition, which is larger than a critical value, is necessary to generate the oscillation. Such an oscillator is often called as hard oscillator or said to have hard nonlinearity. Datardina and Linkens have investigated two identical oscillators with hard nonlinearities coupled by an inductor [2]. They have confirmed that the nonresonant double-mode oscillations, which could not occur for the case of third-power nonlinearity, were stably excited in the coupled system. They have also confirmed that four different modes coexist for some

range of parameter values; zero, two single-mode, and a double-mode. Recently, Yoshinaga and Kawakami have investigated the double-mode oscillation observed from an arbitrary number of identical oscillators with hard nonlinearities coupled by inductors as a ring [4]. They confirmed the envelopes of the double-mode oscillations were synchronized. Namely for the case that N oscillators coupled, N -phase synchronization with respect to the envelopes occurs and the phase shift between the envelopes of two adjacent oscillators is $2\pi/N$. As these studies, coupled systems of hard oscillators exhibits interesting synchronization phenomena of double-mode oscillations as well as single-modes.

On the other hand, many nonlinear dynamical systems in the various fields have been clarified to exhibits chaotic oscillations and recently applications of chaos to engineering systems attract many researchers' attentions [7]. Among the studies on such applications, synchronization of chaotic systems or signals is significant because such a technique is necessary to realize communication systems using chaos. Since the chaotic solution is unstable and small error of initial values are expanded as time goes, the synchronization of chaos seems to be extremely interesting phenomena. Further, because coupled chaotic oscillators are good example of higher-dimensional systems which have been drawing recent attentions, investigating what kind of phenomena are observed from such systems would contribute to develop the study of nonlinear dynamical systems. We have already studied synchronization phenomena observed from some types of coupled chaotic oscillators [8][9]. In the studies it has been confirmed that quasi-synchronization occurred even if each oscillator exhibits chaos. However, since we did not consider oscillators with hard nonlinearities in the previous studies, we have not obtained any results about double-mode oscillations.

In this study, we investigate multimode chaos observed from two coupled chaotic oscillators with hard nonlinearities. At first, a simple chaotic oscillator with hard nonlinearities is realized. Next, various modes of oscillations are investigated observed from two cou-

pled chaotic oscillators with hard nonlinearities by both of computer calculations and circuit experiments. Especially, a nonresonant double-mode oscillation is confirmed to be stably generated even in the case that oscillation is chaotic. Namely, for this oscillation mode, chaotic oscillation and periodic oscillation can be simultaneously excited. We call this phenomena as double-mode chaos. Finally, the beat frequency of the double-mode chaos is confirmed to be changed by tuning the value of the coupling capacitor.

CIRCUIT MODEL

Fig. 1(a) shows the realization of chaotic oscillator with hard nonlinearity. If we remove a resistor R_d and a pair of diodes connected in parallel with R_d , the circuit is the symmetric version of the chaotic circuit proposed by Inaba *et al.* [10]. We confirmed that this circuit have the following properties. The origin is always stable and it is surrounded by an unstable limit cycle. Out of the unstable limit cycle, there exists the other stable attractor. As a parameter $-r$ increases continuously, this attractor bifurcates from one-periodic limit cycle to chaos as shown in Fig. 1(b) via period-doubling bifurcations.

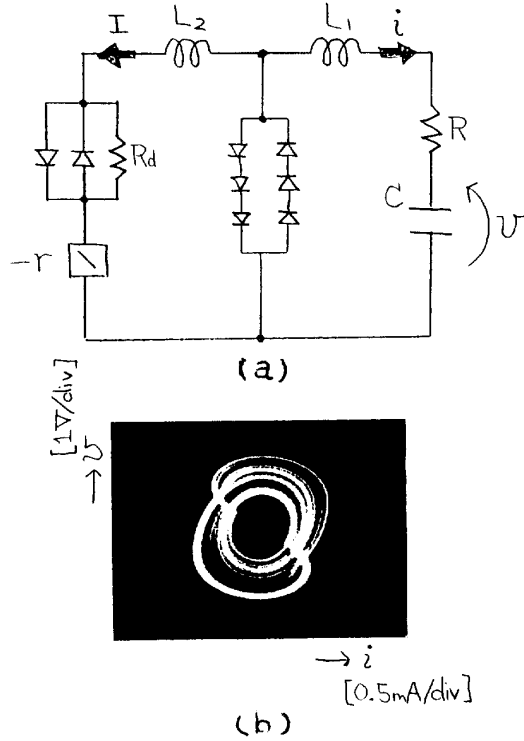


Fig. 1 (a) Realization of chaotic oscillator with hard nonlinearity. (b) Typical example of chaotic attractor observed from this circuit ($L_1 = 100mH$, $L_2 = 200mH$, $C = 0.068\mu F$, $R = 50\Omega$, $R_d = 2k\Omega$, $r = 1.44k\Omega$).

In this study we consider two coupled chaotic oscillators with hard nonlinearities as shown in Fig. 2(a). In this figure two same chaotic oscillators with hard nonlinearities are coupled by a capacitor C_0 . At first, we approximate the $i-v$ characteristics of the diode in the circuit as two-segment piecewise linear function as shown in Fig. 2(b). In this case, the $i-v$ characteristics of the nonlinear resistor including a linear negative resistor and of the nonlinear resistor consisting of six diodes are described by three-segment piecewise linear functions. The circuit equation is given as

$$\begin{aligned} L_1 \frac{di_k}{dt} &= -v_k - v_0 - Ri_k - v_D(i_k + I_k) \\ L_2 \frac{dI_k}{dt} &= -v_r(I_k) - v_D(i_k + I_k) \\ C \frac{dv_k}{dt} &= i_k \end{aligned} \quad (1)$$

$$(k = 1, 2)$$

$$(C_0 v_0 = C(v_1 + v_2))$$

where the functions v_D and v_r means the $i-v$ characteristics of the nonlinear resistors and are represented as follows.

$$v_r(I_k) = \begin{cases} V - rI_k & \left(I_k > \frac{2R_d + r_d V}{R_d r_d} \right) \\ \left(\frac{R_d r_d}{2R_d + r_d} - r \right) I_k & \left(|I_k| \leq \frac{2R_d + r_d V}{R_d r_d} \right) \\ -V - rI_k & \left(I_k < -\frac{2R_d + r_d V}{R_d r_d} \right) \end{cases} \quad (2)$$

$$v_D(i_k + I_k) = \begin{cases} 3V & \left(i_k + I_k > \frac{2}{r_d} V \right) \\ \frac{3}{2} r_d (i_k + I_k) & \left(|i_k + I_k| \leq \frac{2}{r_d} V \right) \\ -3V & \left(i_k + I_k < -\frac{2}{r_d} V \right) \end{cases} \quad (3)$$

The circuit equation (1) is calculated by using the Runge-Kutta method after normalizing the variables and parameters.

NUMERICAL AND EXPERIMENTAL RESULTS

We carried out computer calculations and circuit experiments. Parameters are fixed as $C_0 = 0.34\mu F$, $L_1 = 100mH$, $L_2 = 200mH$, $C = 0.068\mu F$, $R = 50\Omega$ and $R_d = 2k\Omega$ and only r is varied as a control parameter. For any r we obtained four different oscillation states; zero, in-phase single-mode, anti-phase single-mode, and double-mode. For the in-phase single-mode synchronization, the attractor is always one-periodic. For the anti-phase single-mode synchronization, the attractor bifurcates to chaos while holding quasi-synchronization via the following route. 1-period with symmetry \rightarrow 1-period with asymmetry \rightarrow 2-period with asymmetry

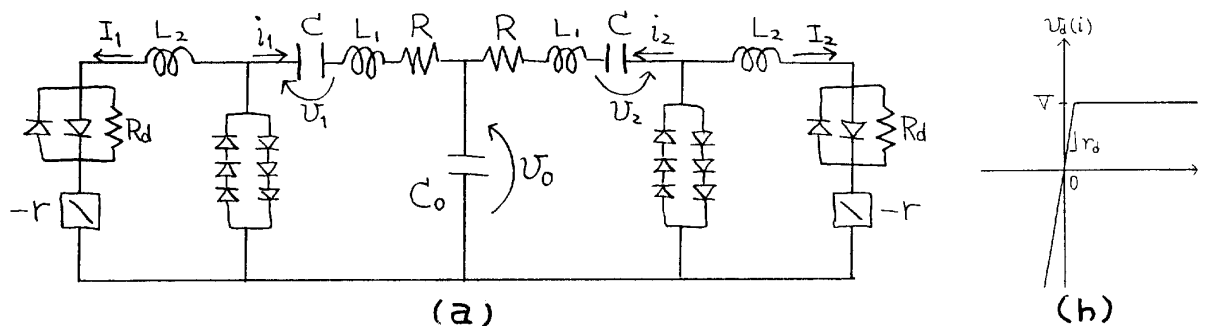


Fig. 2 (a) Two coupled chaotic oscillators with hard nonlinearities. (b) Piecewise linear approximation of the $i - v$ characteristics of the diode.

→ 2^n period with asymmetry → chaos with asymmetry → chaos with symmetry. For the double-mode oscillation, namely two single-mode oscillations (in-phase and anti-phase) simultaneously exist, attractor changes its character according to the character of contained single-mode oscillation. Namely, if the in-phase oscillation is periodic and the anti-mode oscillation is chaotic, the double-mode oscillation must contain both of periodic and chaotic oscillation.

Fig. 3 shows examples of numerical and experimental results. Figs. (a) and (b) show in-phase and anti-phase single-modes oscillation, respectively. For this parameter value, anti-phase oscillation is chaotic. Fig. (c) shows double-mode chaos; periodic in-phase oscillation and chaotic anti-phase oscillations simultaneously exist. By changing initial conditions we can obtain one of these figures or remaining trivial stable state; zero.

Finally, the beat frequency of double-mode chaos can be changed by varying the value of the coupling capacitor C_0 . Fig. 4 shows the results obtained by using another values of C_0 .

CONCLUSIONS

In this study, we have investigated multimode chaos observed from two coupled chaotic oscillators with hard nonlinearities. In this circuit model, in-phase single-mode was confirmed to be always one-periodic. If in-phase is chaotic as well as anti-phase, double-mode oscillation must contain two different kind of chaos. Searching such phenomena is one of our future researchs.

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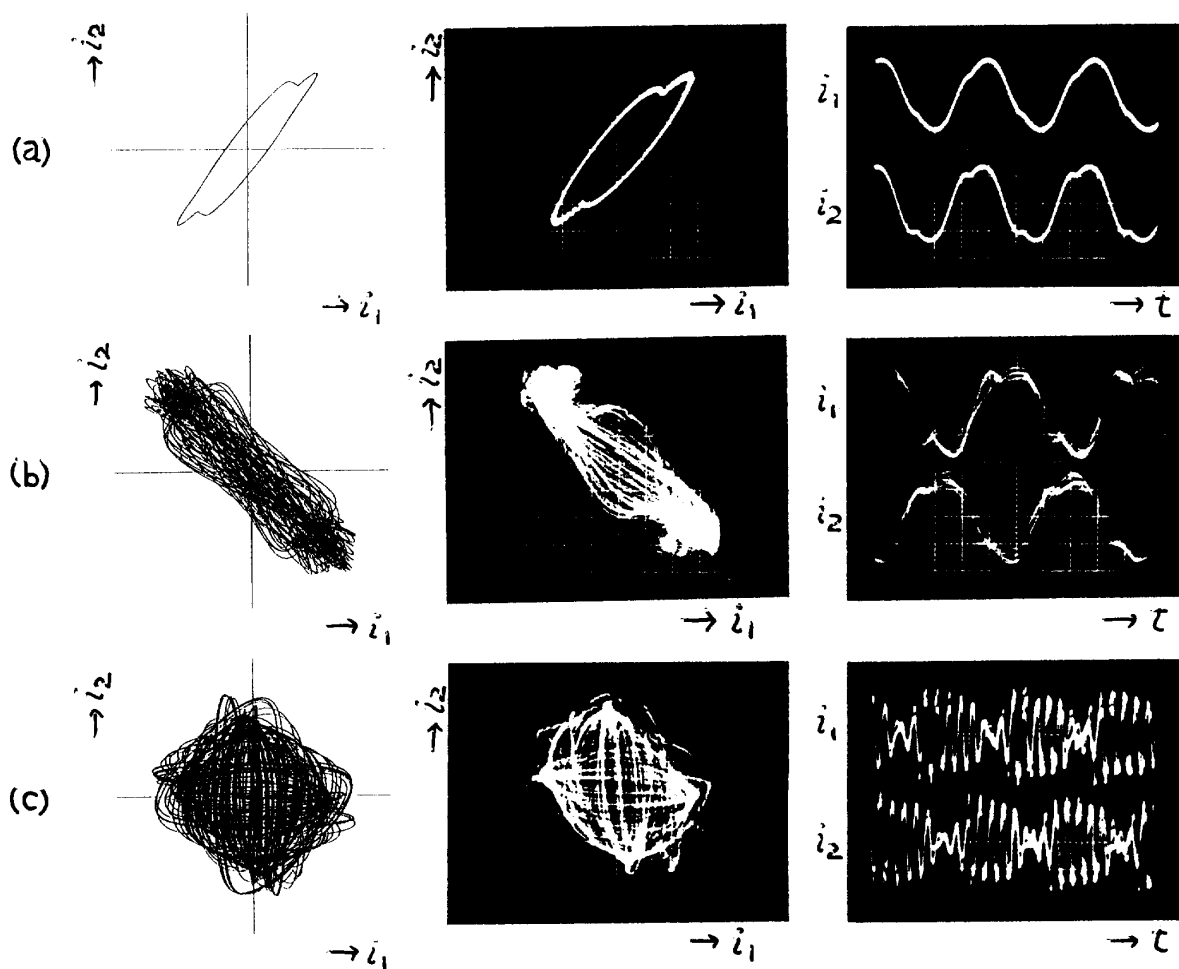


Fig. 3 Three different modes of oscillations obtained numerically and experimentally for $r = 1.37k\Omega$. (a) In-phase single-mode. (b) Anti-phase single-mode chaos. (c) Double-mode chaos. (For the lissajous $H=V=0.5\text{mA/div.}$ For time waveforms $V=1\text{mA/div.}$, $H=0.2\text{ms/div.}$ in (a)(b) and $H=2\text{ms/div.}$ in (c).)

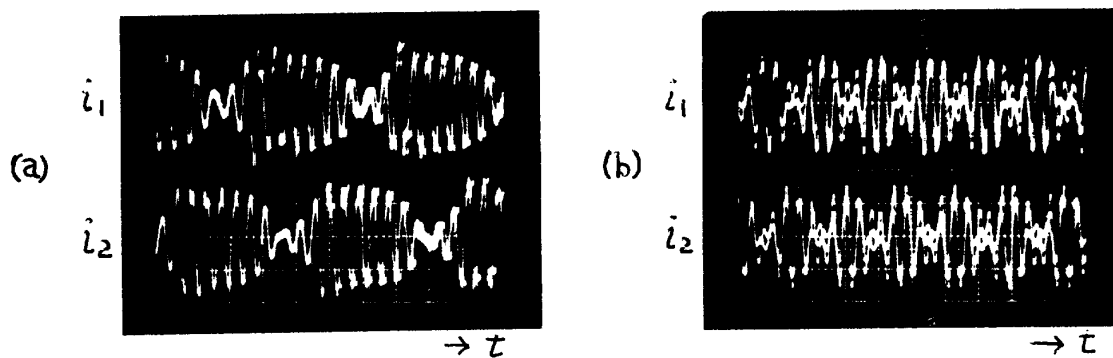


Fig. 4 Change of the beat frequency of double-mode chaos. (a) $C_0 = 0.408\mu F$. (b) $C_0 = 0.204\mu F$. ($V=1\text{mA/div.}$, $H=2\text{ms/div.}$)