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Synchronization in Mutually Coupled Chaotic Circuits

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1 Introduction

Chaotic phenomena have been drawing an extensive attention in various fields of natural science [1]. In the study on autonomous chaotic circuits, three-dimensional systems have reasonably well investigated and recently four or higher-dimensional systems attract our attentions [2]. A coupled system of some chaotic circuits is an example of higher-dimensional systems. Although there have been a few studies on coupled chaotic circuits [3], mathematical treatments seem to be almost impossible and very suggestive results have not been reported. Since mathematical theory for higher-dimensional dynamical systems has not been well established, we should pay our attentions to a subject of interest when we analyze nonlinear phenomena in such higher-dimensional systems.

On the other hand, coupled oscillator circuits are one of good models of some kinds of physical, chemical or biological systems. Therefore, it is important problems to clarify various nonlinear phenomena observed in coupled oscillator circuits. There have been many investigations of mutual synchronization of oscillator circuits ([4][5] and therein). We have also investigated synchronization phenomena observed from N oscillators with the same natural frequency mutually coupled by one resistor [6]. In the system various synchronization phenomena can be stably observed, because the system tends to minimize the current through the coupling resistor. Especially, we have confirmed that N-phase oscillation ($N = 2 \sim 13$) can be stably excited for the case that the nonlinearity of each oscillator is strong. Since there are many real physical oscillators exhibiting chaotic oscillations, it is interesting to investigate what kind of phenomena are observed from coupled chaotic circuits.

In this study, we fix our eyes upon synchronization phenomena of chaotic signals observed in a coupled chaotic circuits. The synchronization of chaotic systems is extremely interesting, because in chaotic systems a slight difference of initial values or parameter values leads two same orbits to entirely different ones. Several studies on synchronization of chaotic systems have been reported [7]-[9] and excellent results have been obtained. However, in such systems a chaotic signal of drive system is injected to response system. Therefore, the systems are essentially different from mutually coupled systems. We investigate synchronization phenomena observed from two or three chaotic circuits coupled by one resistor. Each chaotic circuit is proposed by Inaba and us [10] and this is one of the simplest autonomous chaotic circuits. We carry out circuit experiments and computer calculation and confirm the following.

1. When 2 chaotic circuits are coupled, two systems are synchronized at the opposite phase.
2. When 3 chaotic circuits are coupled, almost three-phase oscillation can be stably excited.

It should be noted that the signal obtained from each circuit is chaotic even if the above synchronizations occur. We also confirmed that the similar results can be observed from the same type of coupled systems consisting of another other chaotic circuit. Hence, this phenomenon is not considered to be generated for special type of chaotic circuits.

Although each circuit is generating chaos, these chaotic signals are almost synchronized each other. The explication of mechanism of this extremely interesting result seems to be very difficult. However, the result would give the effective suggestion to make clear the nonlinear phenomena in higher-dimensional systems.

2 Circuit Model

The circuit model is shown in Fig. 1. In our system N same chaotic circuits are coupled by one resistor. Each chaotic circuit is symmetric version of the circuit proposed by Inaba and us [10] and it consists of three memory elements, one linear negative resistor and one nonlinear resistor. Fig. 2 shows an example of chaotic attractors obtained from this chaotic circuit. In the following, the values of the circuit elements in each chaotic circuit are fixed to the values in Fig. 2.

At first, we approximate the $i - v$ characteristics of the nonlinear resistor consisting of diodes by the following function.

$$v_d(i_k) = \sqrt[9]{r_d i_k}. \tag{1}$$

By changing the variables,

$$\begin{gathered} t = \sqrt{L_1 C}\tau, \quad I_k = a\sqrt{\frac{C}{L_1}}x_k, \quad i_k = a\sqrt{\frac{C}{L_1}}y_k, \quad v_k = az_k, \quad \text{"}\cdot\text{"} = \frac{d}{d\tau}, \\ \alpha = \frac{L_1}{L_2}, \qquad \beta = r\sqrt{\frac{C}{L_1}}, \qquad \gamma = R\sqrt{\frac{C}{L_1}}, \qquad \left(\text{where} \quad a = \sqrt[8]{r_d\sqrt{\frac{C}{L_1}}}\right) \end{gathered} \tag{2}$$

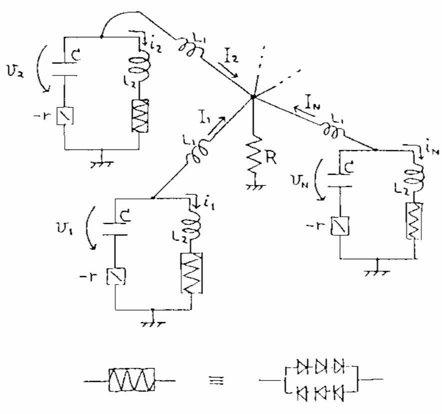

Fig. 1 Circuit model.

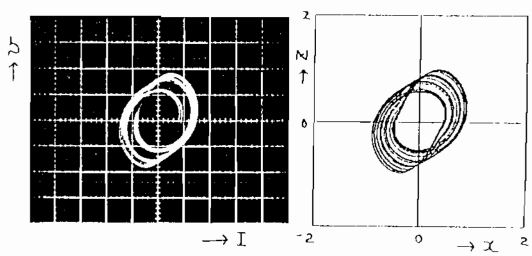

Fig. 2 Chaos from each circuit
($L_1 = 204mH$, $L_2 = 9.9mH$,
$C = 0.034\mu F$, $r = 790\Omega$).

the equation governing the circuit in Fig. 1 is represented as follows,

$$\begin{aligned} \dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma \sum_{j=1}^{N} x_k \\ \dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \qquad (k = 1, 2, \cdots, N) \\ \dot{z}_k &= x_k + y_k \end{aligned} \tag{3}$$

where

$$f(y_k) = \sqrt[9]{y_k}. \tag{4}$$

For computer calculations, in order to consider the difference of real circuit elements, (3) is rewritten as follows.

$$\begin{aligned} \dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma \sum_{j=1}^{N} x_k \\ \dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \qquad (k = 1, 2, \cdots, N) \\ \dot{z}_k &= (1 + \Delta\omega_k)(x_k + y_k) \end{aligned} \tag{5}$$

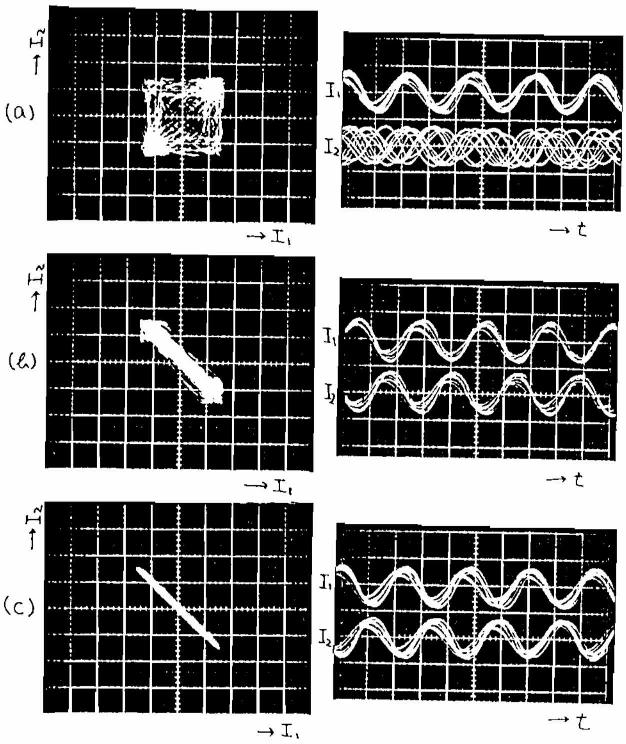

Fig. 3 Circuit experimental results for $N = 2$
($L_1 = 204mH \pm 0.5\%$, $L_2 = 9.9mH \pm 0.2\%$, $C = 0.034\mu F \pm 1.0\%$, $r = 790\Omega \pm 0.8\%$).
(a) $R = 0\Omega$, (b) $R = 188.5\Omega$, (c) $R = 465\Omega$.

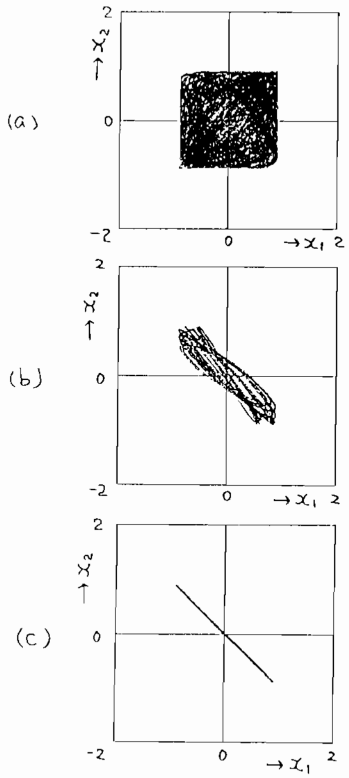

Fig. 4 Computer calculated results for $N = 2$
($\alpha = 20$, $\beta = 0.3$, $\Delta\omega_1 = 0$, $\Delta\omega_2 = 0.01$).
(a) $\gamma = 0$, (b) $\gamma = 0.01$, (c) $\gamma = 0.2$.

3 Synchronization of Chaotic Signals

At first, we consider the case of $N = 2$. Fig. 3 shows circuit experimental results. As R increases, two chaotic signals I_1 and I_2 become to be synchronized at the opposite phase. When $R = 465\Omega$; in Fig. 3(c), two signals seem to be completely synchronized. However, if we observe I_1 vs. v_1 or I_2 vs. v_2, we can see the chaotic attractor which is the same as that in Fig. 2. Fig. 4 shows the corresponding numerical results obtained by using the Runge-Kutta method. Similar results are obtained by computer calculations.

Next, we consider the case of $N = 3$. In this case, two types of almost three-phase oscillation can be stably excited. Figs. 5 and 6 show circuit experimental results. In Fig. 5 the phase of each waveform is ordered as (I_1, I_2, I_3). In Fig. 6 the phase of each waveform is ordered as (I_1, I_3, I_2). According to the initial conditions, one of two states can be observed. In the case of $N = 3$, Lissajous figure is not a definite ellipse. Namely, almost three-phase oscillation simultaneous oscillates irregularly. Figs. 7 and 8 show numerical results corresponding to Figs. 5 and 6, respectively. The almost three-phase

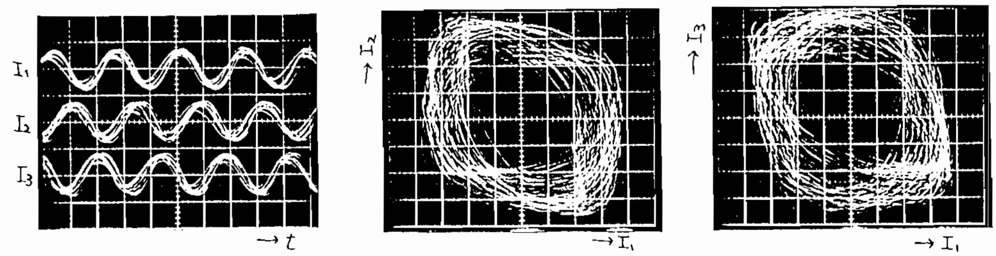

Fig. 5 Almost 3-phase oscillation : (I_1, I_2, I_3)
($L_1 = 204mH \pm 0.5\%$, $L_2 = 9.9mH \pm 0.2\%$,
$C = 0.034\mu F \pm 1.0\%$, $r = 790\Omega \pm 0.8\%$, $R = 61.1\Omega$).

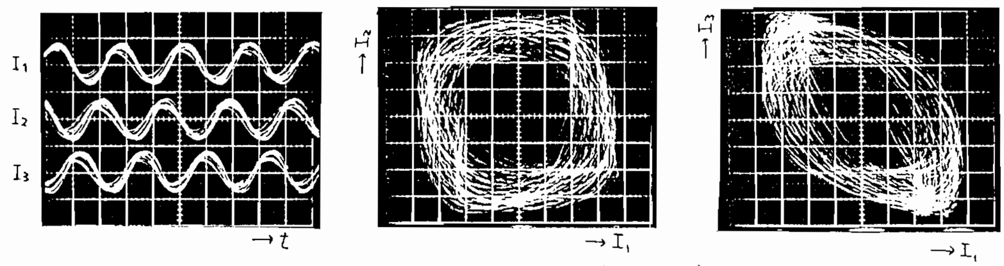

Fig. 6 Almost 3-phase oscillation : (I_1, I_3, I_2)
(parameter values are the same as those in Fig. 5).

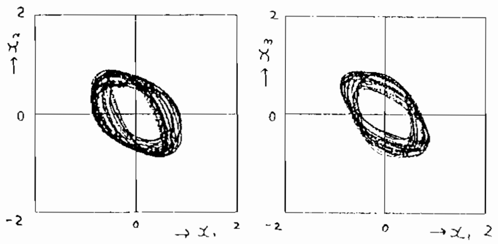

Fig. 7 Computer calculated results corresponding to Fig. 5.

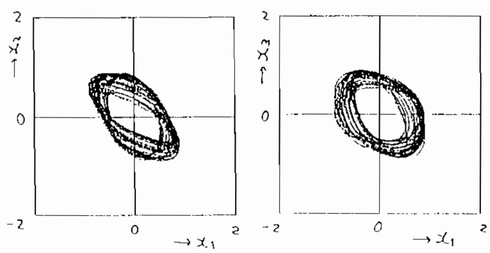

Fig. 8 Computer calculated results corresponding to Fig. 6.

($\alpha = 20$, $\beta = 0.3$, $\gamma = 0.02$, $\Delta\omega_1 = 0$, $\Delta\omega_2 = 0.005$, $\Delta\omega_3 = 0.01$)

oscillation can be also confirmed by computer calculations.

When $N \geq 4$, the system in Fig. 1 cannot be synchronized. Even in the case of the coupled oscillators [3], four or more oscillators with weak nonlinearity cannot be synchronized. Therefore, the chaotic circuit generating substantially sinusoidal waveform is difficult to be synchronized for the case of $N \geq 4$.

4 Another Example

We carry out circuit experiments for the case of another chaotic circuit in order to confirm that the synchronization phenomena observed in the previous section are not special example. The circuit model is shown in Fig. 9(a). In this system, each chaotic circuit is asymmetric version of the circuit proposed by Yamamoto and us [11][12] and it exhibits chaos as shown in Fig. 9(b).

Figs. 9(c) shows circuit experiments for the case of $N = 2$. This chaotic circuit

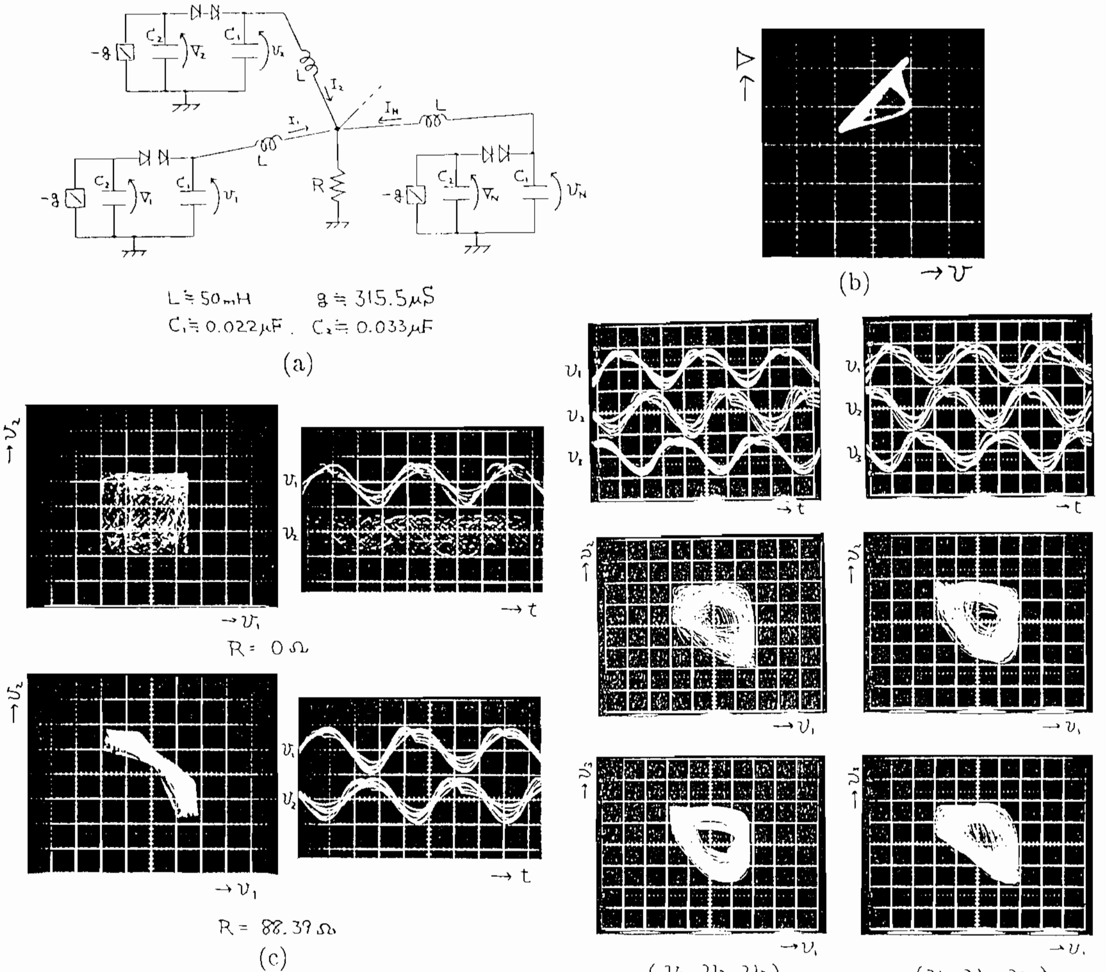

Fig. 9 Another example.
(a) Circuit model. (b) Chaos from each circuit.
(c) Experimental results for $N = 2$. (d) Almost 3-phase oscillations for $N = 3$.

cannot be synchronized at the opposite phase completely. Namely, almost opposite phase synchronization occurs. This is considered to be due to the asymmetry of the circuit.

Figs. 9(d) shows circuit experiments for the case of $N = 3$. In this case, almost three-phase synchronization which is similar to the results in the previous section can be observed.

5 Conclusions

In this study, we have investigated synchronization phenomena of chaotic signals observed in a coupled chaotic circuits; two or three chaotic circuits are coupled by one resistor. We carry out circuit experiments and computer calculation and confirm the following.

1. When 2 chaotic circuits are coupled, two systems are synchronized at the opposite phase.
2. When 3 chaotic circuits are coupled, almost three-phase oscillation can be stably excited.

Although each circuit is generating chaos, these chaotic signals are almost synchronized each other. This result is extremely interesting and would give the effective suggestion to make clear the nonlinear phenomena in higher-dimensional systems.

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