EXTREMELY SIMPLE HYPERCHAOS GENERATORS INCLUDING ONE DIODE

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ABSTRACT: In this article, two extremely simple hyperchaos generators including one diode are analyzed. In the case that we regard the diodes as ideal switches, the 2-dimensional return map can be derived rigorously and its Lyapunov exponents can be calculated by using piecewise exact solutions. Especially, it is confirmed that the resonant frequencies governing the diod are deeply related with the generation of hyperchaos.

INTRODUCTION

In the study on autonomous chaotic circuits, three-dimensional systems have reasonably well investigated, but four-dimensional systems have only recently begun to investigated [1]-[5]. Higher-dimensional chaotic circuits include various important problems, among them; physical meaning of chaos, classification of chaos and mathematical evidence for chaos. In order to approach such problems, we consider a hyperchaos generating circuit family which consist of four linear memory elements, some positive and negative linear resistors, and a diode. Hyperchaos is usually defined as a chaotic attractor with more than one positive Lyapunov exponent. It cannot be observed from three-dimensional autonomous circuits. Our circuit family are extremely simple hyperchaos generators, because nonlinear element in the circuit is only one diode. In [3][4], we have proposed another hyperchaos generators. However, the circuit analyzed in [3][4] are more complicated than our circuit family. Because the circuit in [3] includes a hysteresis resistor and the solutions jump one branch to the other one. Also, the circuit in [4] includes two diodes and hence it is three-region system.

In this study we consider two ladder circuits in the family. At first, we derive the 2-dimensional Poincaré map by using the simplification of the diode. This simplification have been proposed by Inaba and the authors [6] and its effectiveness has been confirmed for some circuits [4][5]. We can verify that these circuits generate hyperchaos by calculating the Lyapunov exponents of the Poincaré map. Especially, we can confirm that the resonant frequencies governing the diodes are deeply related with the generation of hyperchaos. There are very few discussions related with such physical explanation of the generations of hyperchaos.

CIRCUIT MODEL

Fig. 1 shows two circuit models. These circuits consist of only four memory elements, a linear negative resistor and one diode. In the circuits only the position of the diode differs. At first, we approximated the $v-i$ characteristics of the diode by

$$i_D = 0.5G(|v_k - E| + v_k - E) \qquad (1)$$

Using the following variables and parameters:

$$t = \sqrt{L_1C_1}\tau, \quad v_1 = Ex_1, \quad v_2 = Ex_2,$$

$$i_1 = \sqrt{\frac{C_1}{L_1}}Ex_3, \quad i_2 = \sqrt{\frac{C_1}{L_1}}Ex_4, \quad \varepsilon = \frac{1}{G}\sqrt{\frac{C_1}{L_1}}$$

$$\gamma_C = \frac{C_1}{C_2}, \quad \gamma_L = \frac{L_1}{L_2}, \quad \alpha = R\frac{\sqrt{L_1C_1}}{L_2}, \qquad (2)$$

the circuit dynamics are described by

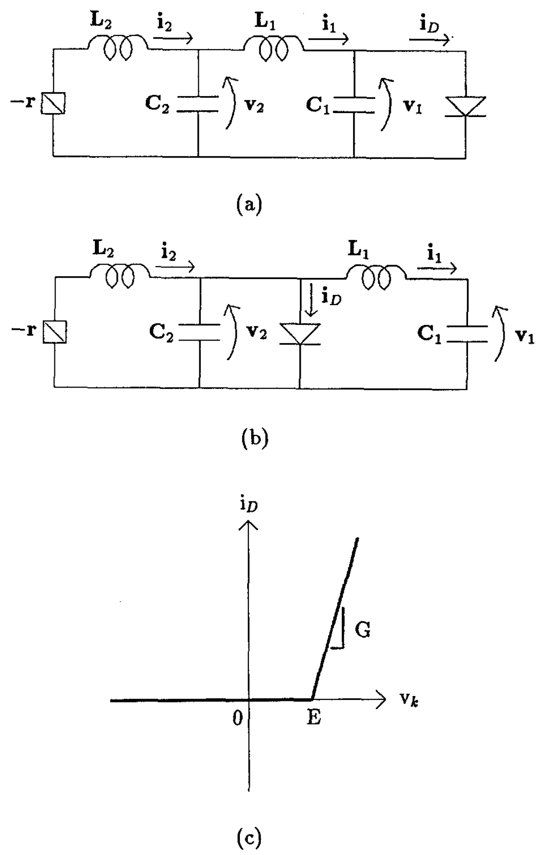

Fig. 1 Circuit model.
(a) Circuit 1.
(b) Circuit 2.
(c) $v-i$ characteristics of the diode.

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1. Circuit 1

$$\begin{cases} \dot{x_1} &= x_3 - f(x_1) \\ \dot{x_2} &= \gamma_C(x_4 - x_3) \\ \dot{x_3} &= x_2 - x_1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4 \end{cases}$$

2. Circuit 2

$$\begin{cases} \dot{x_1} &= x_3 \\ \dot{x_2} &= \gamma_C(x_4 - x_3 - f(x_2)) \\ \dot{x_3} &= x_2 - x_1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4 \end{cases} \quad (3)$$

where $$f(x) = 0.5\varepsilon^{-1}(|x-1| + x - 1) \quad (4)$$

Here, we regard the diodes as ideal switches; the diode becomes $E[V]$ voltage source if its branch voltage reaches $E[V]$ and it is opend if the branch current reaches zero. It corresponds to $\varepsilon \to 0$ and (3) and (4) are simplified into

1. Circuit 1

For $x_1 < 1$

$$(a)\begin{cases} \dot{x_1} &= x_3 \\ \dot{x_2} &= \gamma_C(x_4 - x_3) \\ \dot{x_3} &= x_2 - x_1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4 \end{cases}$$

For $x_1 = 1$

$$(b)\begin{cases} \dot{x_2} &= \gamma_C(x_4 - x_3) \\ \dot{x_3} &= x_2 - 1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4 \end{cases} \quad (5)$$

2. Circuit 2

For $x_2 < 1$

$$(a)\begin{cases} \dot{x_1} &= x_3 \\ \dot{x_2} &= \gamma_C(x_4 - x_3) \\ \dot{x_3} &= x_2 - x_1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4 \end{cases}$$

For $x_2 = 1$

$$(b)\begin{cases} \dot{x_1} &= x_3 \\ \dot{x_3} &= 1 - x_1 \\ \dot{x_4} &= -\gamma_L + \alpha x_4 \end{cases} \quad (6)$$

where the transitional conditions are follows.

$$\begin{array}{lcl} (6a) \to (6b) & : & x_1 = 1 \\ (6b) \to (6a) & : & x_3 = 0 \\ (7a) \to (7b) & : & x_2 = 1 \\ (7b) \to (7a) & : & x_4 - x_3 = 0 \end{array} \quad (7)$$

POINCARÉ MAP

When (6b) switches into (6a), x_1=1 and x_3=0 are satisfied. Also when (7b) switches into (7a), x_2=1 and x_3-x_4=0 are satisfied. Therefore, 2-dimensional Poincaré map T can be derived using the exact solutions of (6) and (7) rigorously. Also, the Jacobi matrix DT of T can be derived.

Fig. 2 shows chaotic attractors of the 2-dimensional mapping obtained from the Circuit 1 and corresponding trajectories. Note that Circuit 1 tends to exhibit chaos only for $L_1C_1 < L_2C_2$. Fig. 3 shows the Lyapunov exponents. 1st and 2nd Lyapunov exponents, $\mu_1 \geq \mu_2$, are calculating by the following [7]

$$\begin{aligned} \mu_1 + \mu_2 &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \ln |DT_i|, \\ \mu_1 &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \ln |DT_i \cdot e_i|, \\ & \left(e_{i+1} = \frac{DT_i \cdot e_i}{|DT_i \cdot e_i|} \right). \end{aligned} \quad (8)$$

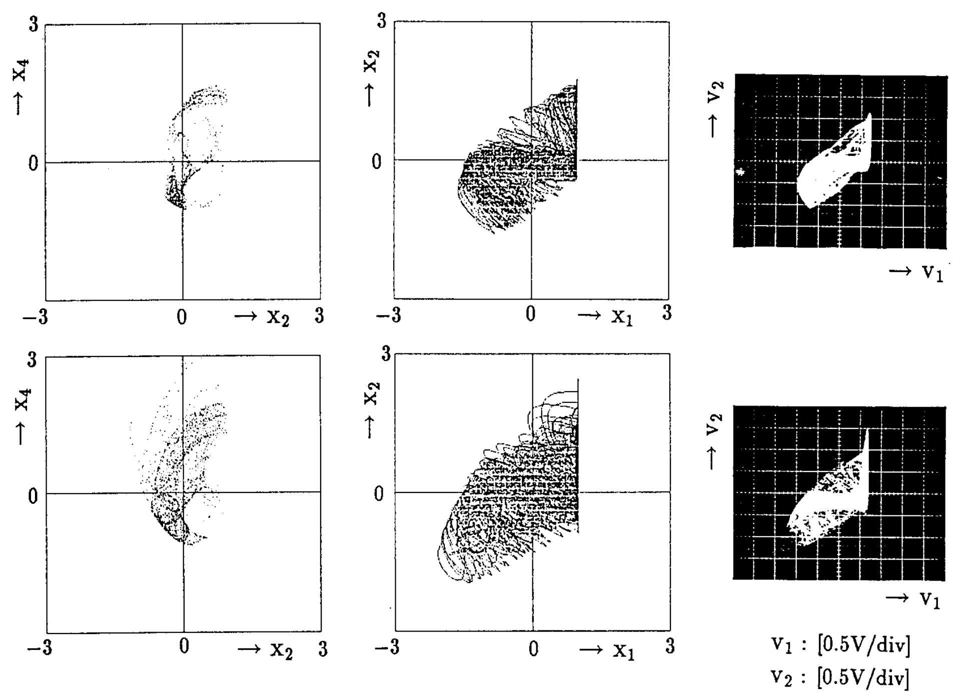
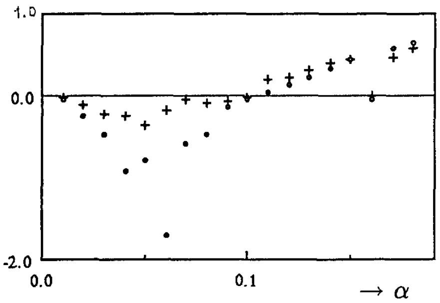


Fig. 2 Chaotic attractors obtained from the Circuit 1 for $L_1C_1 < L_2C_2$.
($\gamma_L = \gamma_C = 0.5$ ($L_1 = 10mH$, $L_2 = 20mH$, $C_1 = 0.022\mu F$, $C_2 = 0.044\mu F$))
(a) $\alpha = 0.13$, (b) $\alpha = 0.18$.

μ_1>0 implies chaos and μ_2>0 implies hyperchaos. For $\alpha \geq 0.19$ the solution diverges to infinity. We can confirm that the Circuit1 exhibits hyperchaos for $0.17 \leq \alpha \leq 0.18$.

Fig. 4 shows chaotic attractors of the 2-dimensional mapping obtained from the Circuit 2 and corresponding trajectories for $L_1C_1 < L_2C_2$. Fig. 5 shows the Lyapunov exponents. In this case chaotic attractors can be observed. However, Fig. 5 denotes that hyperchaos cannot be generated, because μ_2<0 for any α.



+ : 1st-exponent (μ_1)
o : 2dim-exponent ($\mu_1 + \mu_2$)

Fig. 3 Lyapunov exponents obtained from the Circuit 1 for $L_1C_1 < L_2C_2$.
($\gamma_L = \gamma_C = 0.5$)

Fig. 6 shows chaotic attractors of the 2-dimensional mapping obtained from the Circuit 2 and corresponding trajectories for $L_1C_1 > L_2C_2$. Fig. 7 shows the Lyapunov exponents. In this case hyperchaos can be observed for $0.16 \leq \alpha \leq 0.26$.

We carried out many numerical experiments for some combinations of γ_L and γ_C and confirmed the following:

Circuit 1 generates hyperchaos only for $L_1C_1 < L_2C_2$.
Circuit 2 generates chaos for $L_1C_1 < L_2C_2$ and generates hyperchaos only for $L_1C_1 > L_2C_2$.

The above results denote that the resonant frequency governing the diode is deeply related with the generation of hyperchaos. Though this results may not be universal for any types of the circuits, we consider that this study would open the way to make clear the physical mechanism of the generation of hyperchaos.

CONCLUSIONS

In this article, two extremely simple hyperchaos generators including one diode have been analyzed. Especially, it is confirmed that the resonant frequencies governing the diod are deeply related with the generation of hyperchaos. Our future research are to make detailed bifurcation diagram and to investigate the relation between the resonant frequency governing the diode and the generation of hyperchaos for another circuit models.

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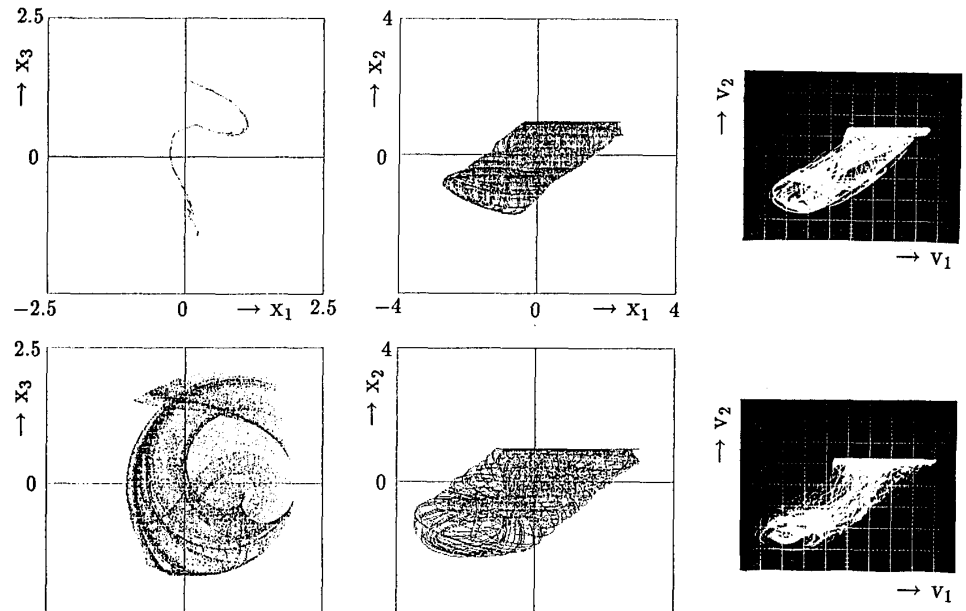

v_1 : [0.5V/div]
v_2 : [0.5V/div]

Fig. 4 Chaotic attractors obtained from the Circuit 2 for $L_1C_1 < L_2C_2$.
($\gamma_L = 1$, $\gamma_C = 0.3235$ ($L_1 = L_2 = 10mH$, $C_1 = 0.022\mu F$, $C_2 = 0.068\mu F$))
(a) $\alpha = 0.10$, (b) $\alpha = 0.17$.

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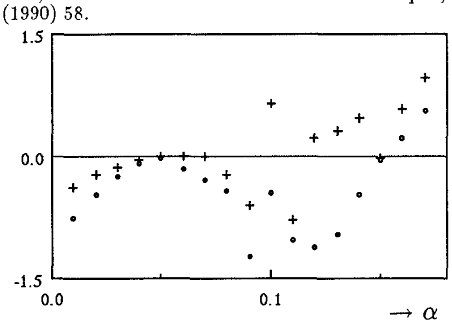
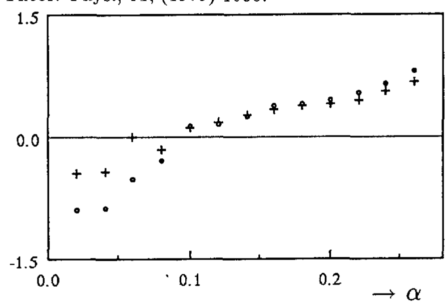


+ : 1st-exponent (μ_1)
o : 2dim-exponent ($\mu_1 + \mu_2$)

Fig. 5 Lyapunov exponents obtained from the Circuit 2 for $L_1C_1 < L_2C_2$.
($\gamma_L = 1$, $\gamma_C = 0.3235$)



+ : 1st-exponent (μ_1)
o : 2dim-exponent ($\mu_1 + \mu_2$)

Fig. 7 Lyapunov exponents obtained from the Circuit 2 for $L_1C_1 > L_2C_2$.
($\gamma_L = 1$, $\gamma_C = 1.545$)

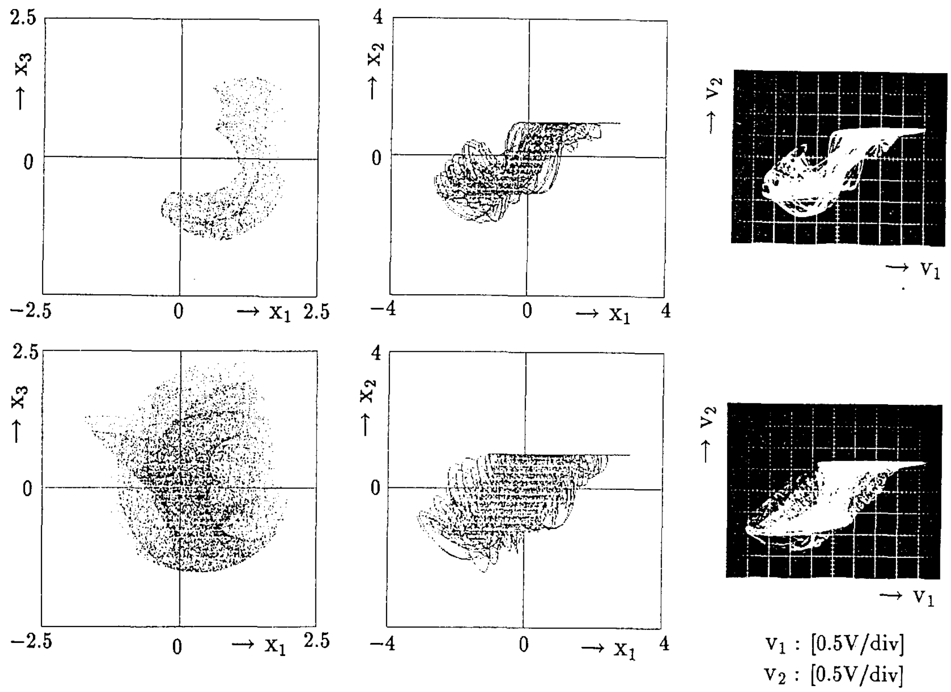


Fig. 6 Chaotic attractors obtained from the Circuit 2 for $L_1C_1 > L_2C_2$.
($\gamma_L = 1$, $\gamma_C = 1.545$ ($L_1 = L_2 = 10mH$, $C_1 = 0.068\mu F$, $C_2 = 0.044\mu F$))
(a) $\alpha = 0.14$, (b) $\alpha = 0.26$.