MUTUALLY COUPLED OSCILLATORS WITH AN EXTREMELY LARGE NUMBER OF STEADY STATES

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ABSTRACT : There have been many investigations of the mutual synchronization of oscillators. In this article, N oscillators with the same natural frequency mutually coupled by one resistor are analyzed. In this system various synchronization phenomena can be observed. Especially, we pay our attention to N-phase osillation. It is confirmed that our system can stably take $(N-1)!$ phase states by both of computer calculations and circuit experiments.

INTRODUCTION

There have been many investigations of the mutual synchronization of oscillators ([1]-[4] and therein). Kimura et al. have confirmed that two oscillators coupled by resistors are synchronized at opposite phase[1]. Endo and we have analyzed a large number of coupled van der Pol oscillators[2]-[4]. In such systems various interesting synchronization phenomena are observed.

In this study, we investigate N oscillators with the same natural oscillating frequency mutually coupled by one resistor. When the nonlinearity of each oscillator is weak and N is large, each oscillator cannot synchronize. However, in the case of strong nonlinearity, various modes of synchronization can be stably observed. Especially, we pay our attention to N-phase oscillation. We take the waveform observed in one oscillator as reference signal. Then, other oscillators can take any phase differences among $\phi_k = 2k\pi/N$ $(k = 1, 2, \cdots N-1)$. Therefore, our system can stably take $(N-1)!$ phase states. This means that when $N = 13$, our system can take 479001600 steady states. Therefore, our system may be utilized as an extremely large memory.

CIRCUIT MODEL

The circuit model is shown in Fig. 1. The nonlinear resistor with strong nonlinearity is realized as shown in Fig. 2 and its $v-i$ characteristics is approximated by the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \qquad (g_1, g_3 > 0) \tag{1}$$

The circuit equation is described as follows,

$$\begin{aligned} C\frac{dv_k}{dt} &= -i_k - i_r(v_k) \\ L\frac{di_k}{dt} &= v_k - R\sum_{j=1}^{N} i_j \end{aligned} \tag{2}$$

$$(k=1,\ 2,\ \cdots,\ N)$$

By changing the variables,

$$\begin{aligned} &t = \sqrt{LC}\tau, \\ &v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \quad i_k = \sqrt{\frac{Cg_1}{3Lg_3}}y_k, \\ &\alpha = R\sqrt{\frac{C}{L}}, \qquad \beta = g_1\sqrt{\frac{C}{L}}, \end{aligned} \tag{3}$$

(2) is normalized as

$$\begin{aligned} \dot{x_k} &= -y_k - f(x_k) \\ \dot{y_k} &= x_k - \alpha\sum_{j=1}^{N} y_j \end{aligned} \tag{4}$$

$$(k=1,\ 2,\ \cdots,\ N)$$

where

$$f(x_k) = -\beta\left(x_k - \frac{x_k^3}{3}\right). \tag{5}$$

N-PHASE OSCILLATION

At first, we consider the case of $N = 5$. Figs. 3 and 4 show an example of numerical results obtained by using the Runge-Kutta method and the corresponding experimental results, respectively. For computer calculations, in

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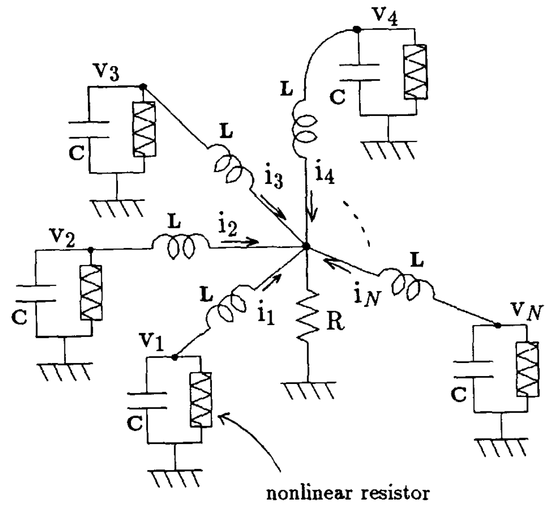

Fig. 1 Circuit model.

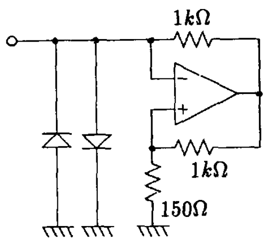

Fig. 2 Realization of the nonlinear resistor with strong nonlinearity.

order to consider the difference between the natural frequencies of real oscillators, (4) is rewritten as follows.

$$\begin{aligned} \dot{x_k} &= -y_k - f(x_k) \\ \dot{y_k} &= (1+\Delta\omega_k)x_k - \alpha \sum_{j=1}^{N} y_j \end{aligned} \qquad (7)$$

$$(k=1, 2, \cdots, N)$$

where $\Delta\omega_k$ corresponds to the difference between the natural oscillating frequency of the reference oscillator and those of another oscillators.

We can observe that 5-phase oscillation is stably excited. In Figs. 3 and 4 each oscillator's waveform are ordered as (x_1,x_4,x_2,x_5,x_3). Since x_1 is reference signal, the number of the states of $x_2 \sim x_5$ is equal to $(5-1)! = 24$. We confirmed that $(5-1)! = 24$ steady states are stably excited by both of computer calculations and circuit experiments.

Next, we consider the case of $N = 7$. Figs. 5 and 6 show a example of numerical results and the corresponding experimental results, respectively. For the case of strong nonlinearity, namely for large β, 7-phase oscillation is stably observed. In Figs.5 and 6, each oscillator's waveform are ordered as $(x_1,x_6,x_5,x_2,x_3,x_7,x_4)$. We do not check that all of $(7-1)! = 720$ steady states are stably excited, but a large number of steady states are stably observed from both of computer calculations and circuit experiments.

ANOTHER SYNCHRONIZATIONS

For the case that N is not a prime number, N-phase oscillation seems to be difficult to observe and another types of syncronization are observed. For example, Fig. 7 shows an example of experimental results for the case of $N = 4$. In this case, x_1 and x_4 almost synchronize at opposite phase and x_2 and x_3 almost synchronize at opposite phase. However, the phase between x_1 and x_3 is independent. This result is similar to that in [3]. The combination of the synchronized oscillators seem to be decided by the differences between the natural frequencies of real oscillators.

Similarly, in the case of $N = 6$, three pairs of two almost synchronized oscillators are observed. For example, x_1 and x_2, x_3 and x_4, and x_5 and x_6 almost synchronize, respectively. However, the phase between x_1 and x_3, the phase between x_1 and x_5 and the phase between x_3 and x_5 are independent. Moreover, in this case we can observe from the circuit experiments that two independent 3-phase oscillations are excited.

We consider that our system tends to minimize the current through the coupling resistor. Therefore, in the case that N is a prime number, N-phase oscillation stably excited. While in the case that N is not a prime number, various combinations to minimize the current can exist and

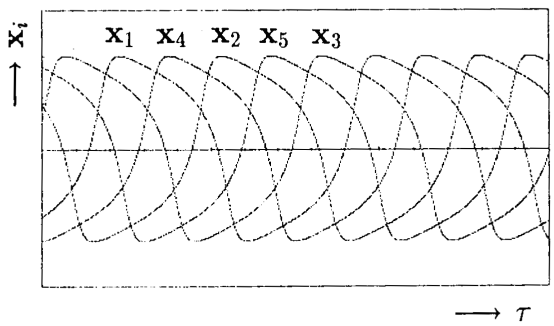

Fig. 3 Numerical result for the case of $N = 5$. ($\alpha = 2.0$, $\beta = 2.0$, $\Delta\omega_1 = 0$, $\Delta\omega_2 = 0.001$, $\Delta\omega_3 = 0.002$, $\Delta\omega_4 = 0.003$, $\Delta\omega_5 = 0.004$).

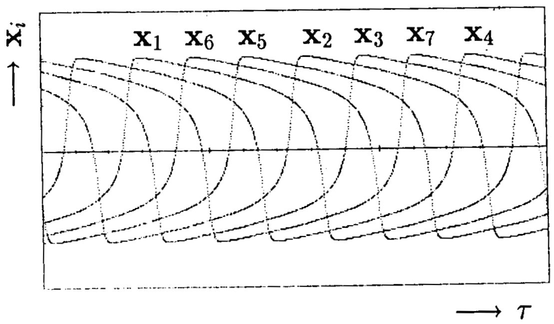

Fig. 5 Numerical result for the case of $N = 7$. ($\alpha = 2.0$, $\beta = 5.0$, $\Delta\omega_1 = 0$, $\Delta\omega_2 = 0.001$, $\Delta\omega_3 = 0.002$, $\Delta\omega_4 = 0.003$, $\Delta\omega_5 = 0.004$, $\Delta\omega_6 = 0.005$, $\Delta\omega_7 = 0.006$).

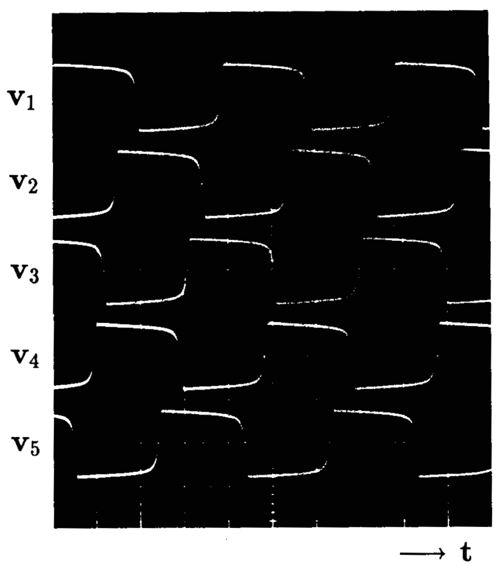

Fig. 4 Experimantal result for the case of $N = 5$. ($L = 10$mH, $C = 0.068\mu$F, $R = 241\Omega$, horizontal scale: 0.1msec/div., vertical scale 1V/div.)

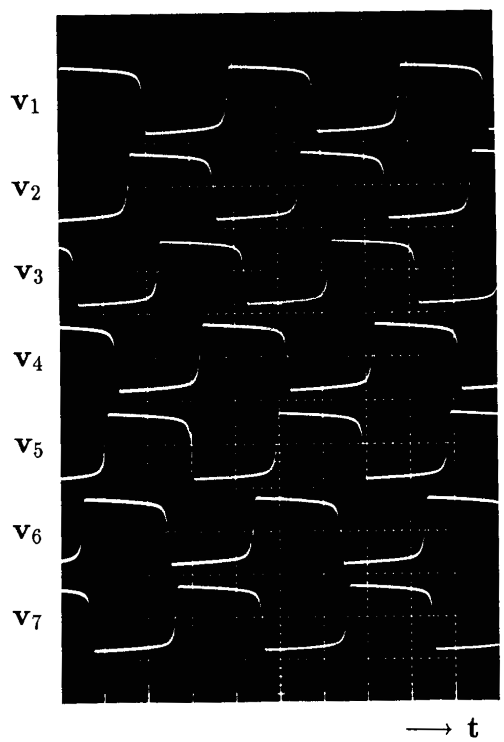

Fig. 6 Experimantal result for the case of $N = 7$. ($L = 10$mH, $C = 0.068\mu$F, $R = 241\Omega$, horizontal scale: 0.1msec/div., vertical scale 1V/div.)

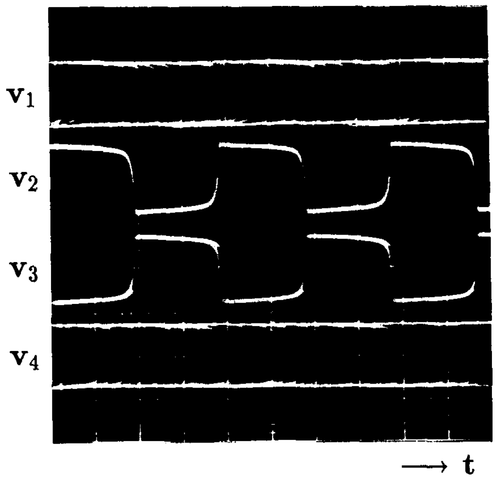

Fig. 7 Experimantal result for the case of $N = 4$. ($L = 10\text{mH}$, $C = 0.068\mu\text{F}$, $R = 241\Omega$, horizontal scale: 0.1msec/div., vertical scale 1V/div.)

various syncronization can be observed. Detailed analysis of such complicated synchronization phenomena is our future research.

If N is a prime number and the nonlinear resistor with strong nonlinearity is used, N-phase oscillations seems to be stably excited. Therefore, we believe that we can make a system with an extremely large number of steady states by using only 13 oscillators.

CONCLUSIONS

In this study, we have investigated N oscillators with the same natural oscillating frequency mutually coupled by one resistor. In the case of strong nonlinearity, we can observe that N-phase oscillations are stably excited from both of computer calculations and circuit experiments. Because this system can take $(N-1)!$ phase states, it may be utilized as an extremely large memory. Our future research are follows. Experiments for large N. Investigation of another types of synchronization. Theoretical analysis. Investigation for the case of another types of oscillators.

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