CHAOTIC PHENOMENA IN AN LCR OSCILLATOR WITH A HYSTERESIS INDUCTOR

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ABSTRACT: In this article, an extremely simple LCR oscillator with a hysteresis inductor is analyzed. It is of great interest that this two-dimensional autonomous circuit generates chaos. This circuit is described by second order differential equation piecewisely. Therefore, the Poincaré map can be derived as one-dimensional map strictly. For this Poincaré map the parameter region for which chaotic attractors are generated is obtained under simple assumption where the definition of chaos is that Poincaré map possesses the unique absolutely continuous invariant measure.

INTRODUCTION

Recently, various chaotic phenomena have been analyzed theoretically. Wider knowledge of this interesting nonlinear phenomena will be needed for many fields. For example, when engineering systems need very accurate control, we have to consider the influence of the slight nonlinearity of the characteristics of the elements, transfer function and so on. As one effective approach toword these problems it is important to analyze simple and native chaos generating systems and to make clear the condition of the generation of chaos.

In this aticle, an LCR oscillatot including a nonlinear inductor with hysteresis saturation characteristics is analyzed. This circuit model consists of only three elements and it is extremely simple two-dimensional autonomous circuit. Generally, for autonomous systems to have chaotic behaviour, the dimension of the phase space must be three or more due to the non-crossing trajectory property. However, this circuit model generates chaos due to the existence of the hysteresis loop. Newcomb et al. and Saito have proposed two-dimensional chaos-generating autonomous circuits with hysteresis resistor [1]~[3]. However, such circuits are not considered to be in substance two-dimensional because the hysteresis loop of resistors generates reactive power. Moreover, in their models current through the hysteresis resistor or voltage across it jumps when the solution on a branch moves to the other branch. On the other hand, in our circuit the dimension of the system does not increase because the hysteresis loop of inductors corresponds to resistance. Moreover, any variables in this circuit does not jump and hence this circuit is considered to be very native circuit. It is great interesting that this simple and native circuit generates chaos.

This circuit model is described by second order differential equation piecewisely. Therefore, the Poincaré map can be derived as one-dimensional map strictly. For this Poincaré map we obtain the parameter region for which chaotic attractors are generated under simple assumption where the definition of chaos is that Poincaré map possesses the unique absolutely continuous invariant measure [4].

CIRCUIT MODEL

The circuit model is shown in Fig. 1.

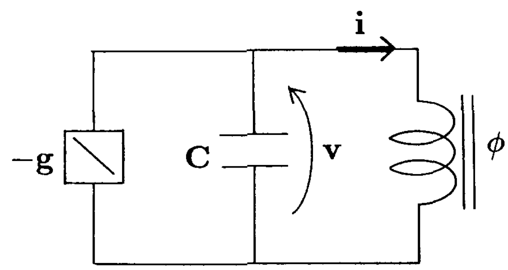

Fig. 1 Circuit model.

This circuit consists of only three elements, that is, a linear negative resistor, a capacitor, a nonlinear inductor with hysteresis saturation characteristics. The circuit equation is represented as follows.

$$\begin{cases} \dfrac{d\phi}{dt} = v \\ C\dfrac{dv}{dt} = gv - i(\phi) \end{cases} \quad (1)$$

where v is the voltage across the capacitor, ϕ is the flux of the hysteresis inductor and $i(\phi)$ is the current through the hysteresis inductor. The $\phi - i$ characteristics of the hysteresis inductor is shown in Fig. 2. This nonlinear inductor exhibits constant hysteresis saturation characteristics as Fig. 2; no minor loops exist. We call each linear branch as P^+, P^-, O^+ and O^-.

By changing the valuablues;

$$\phi = \Phi_1 x, \quad v = \frac{\Phi_1}{\sqrt{L_1 C}} y, \quad t = \sqrt{L_1 C}\tau, \quad " \cdot " = \frac{d}{d\tau},$$

$$\frac{L_1}{L_2} = \alpha\ (>1), \quad \frac{\Phi_2}{\Phi_1} = \beta\ (>1), \quad g\sqrt{\frac{L_1}{C}} = a\ (>0), \quad (2)$$

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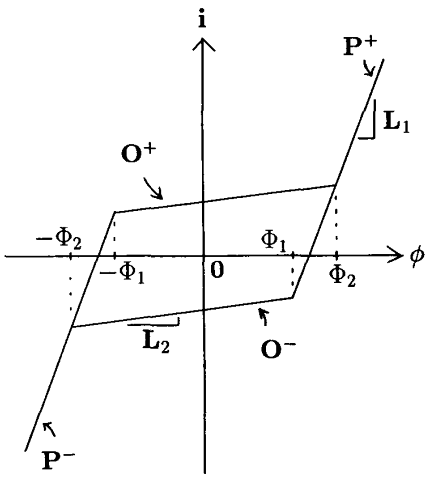

Fig. 2 $\phi - i$ characteristics of the histeresis inductor.

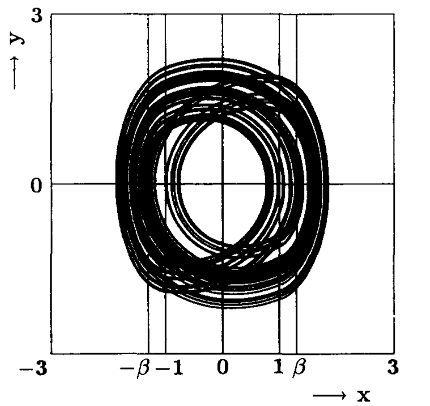

Fig. 3 An example of the chaotic attractors ($\alpha = 4$, $\beta = 1.3$, $a = 0.12$).

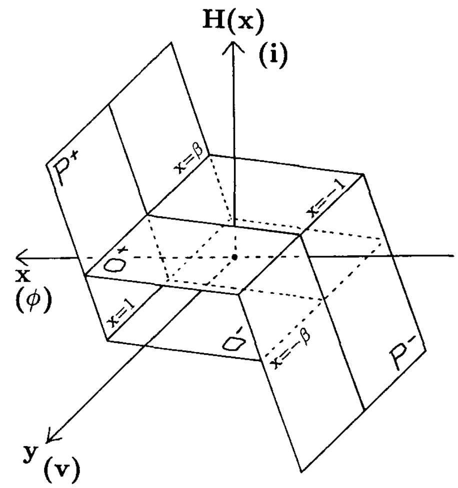

Fig. 4 State space with $H(x)$-axis.

the circuit equation is normalized as follows.

$$\begin{cases} \dot{x} = y \\ \dot{y} = ay - H(x) \end{cases} \quad (3)$$

where $H(x)$ represents hysteresis function and is given as follows.

$$\begin{cases} \text{on the branch } P^{\pm}: \quad H(x) = \alpha x \pm \dfrac{(1-\alpha)(1+\beta)}{2} \\ \text{on the branch } O^{\pm}: \quad H(x) = x \pm \dfrac{(1-\alpha)(1-\beta)}{2}. \end{cases} \quad (4)$$

The transitional conditions between each branch are given as follows.

$$\begin{array}{ll} P^+ \to O^-: & x = 1 \\ O^- \to P^+: & x = 1 \\ O^- \to P^-: & x = -\beta \\ P^- \to O^+: & x = -1 \\ O^+ \to P^-: & x = -1 \\ O^+ \to P^+: & x = \beta. \end{array} \quad (5)$$

Eq. (3) is linear on each branch and the general solution on each branch can be given. An example of the chaotic attractors obtained by calculating the general solution of Eq. (3) is shown in Fig. 3.

Though Eq. (3) is second order equation on the each branch, the phase space is not simple two-dimensional plane due to the existence of the hysteresis loop. Therefore, it is difficult to understand the motion of the solution. Then, consider the state space to which $H(x)$-axis is appended as shown in Fig. 4. Note that $H(x)$ corresponds to the current i through the hysterisis inductor and it is not a independent variable but a dependent variable.

Fig. 5 shows an example of the chaotic attractor in the state space in Fig. 4. The motion of the solution is explained as follows. Consider the solution starting from the initial point on the branch P^-. Because both equiribria on the branches P^- and O^+ exist near the boundary $x = -1$, the solution oscillates between two branches P^- and O^+. Here the amplitude of the solution becomes larger by energy supplied from the linear negative resistor. When the solution reaches the boundary $x = \beta$, it enters the branch P^+. For some proper parameter values, at this moment the amplitude of the solution decreases and it starts to oscillate between P^- and O^-. When the solution rotates the hysterisis loop, energy is consumed as hysteresis loss.

POINCARÉ MAP

Define the following two half line as Fig. 6.

$$\begin{cases} L^+ : x = -1, \quad y > 0. \\ L^- : x = 1, \quad y < 0. \end{cases} \quad (6)$$

where L^+ is the transitional condition from P^- to O^+ and L^- is that from P^+ to O^-. Consider the solution having

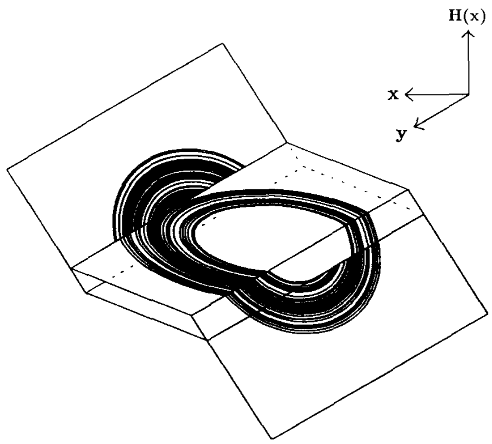

Fig. 5 An example of the chaotic attractors ($\alpha = 4$, $\beta = 1.3$, $a = 0.12$).

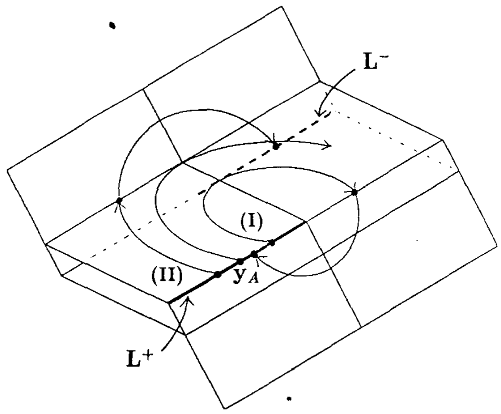

Fig. 6 Derivation of the Poincaré map.

the initial condition on L^+, that is, $(x, y) = (-1, y_0)$ ($y_0 > 0$). At first, define y_A as the y-coordinate of the initial point whose flow should be tangent to the transitional condition from O^+ to P^+; $x = \beta$. The value of y_A is given implicitly.

The motion of the solution is distinguished to the following cases (see Fig. 6).

(I) when $y_0 < y_A$: The solution reaches $x = -1$ without reaching the trsnsitional condition $x = \beta$ and enters P^-. The solution in P^- reaches L^+ again.
(II) when $y_0 \geq y_A$: The solution hits $x = \beta$ and enters P^+. The solution in P^+ reaches L^-.

Namely, the solution starting from L^+ never fail to reach L^+ or L^- at some time. Since L^- and L^+ locate symmetrically with respect to the origin, the solution starting from L^- also reaches L^+ or L^-. Moreover, a point on L^+ or L^- can be represented by its y-coordinate. Therefore, we can define the Poincaré map as the following one-dimensional map.

$$F : L^+ \cup L^- \rightarrow L^+ \cup L^-, \quad y_0 \rightarrow F(y_0). \tag{7}$$

An example of the Poincaré map is shown in Fig. 7.

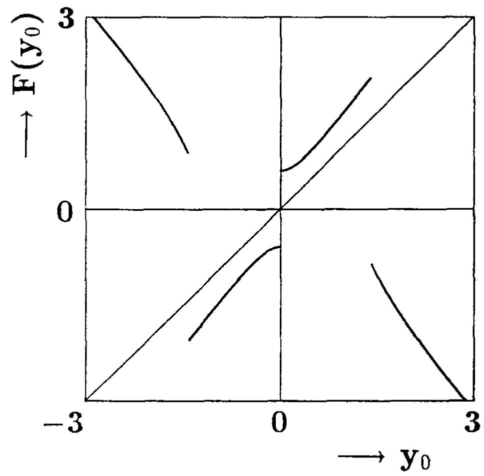

Fig. 7 Poincaré map F ($\alpha = 4$, $\beta = 1.3$, $a = 0.12$).

We define y_{max} and y_{min} as Fig. 8. The value of y_{max} (or $-y_{min}$) can be obtained as the y-coordinate of the point at which the solution starting from $(x, y) = (\beta, 0)$ reaches L^+ (or L^-) via O^+ and P^- (or P^+). Here we define the following intervals on L^+ and L^-.

$$\begin{aligned} J^+ &= [y_{min}, y_{max}] \subset L^+, \\ J^- &= [-y_{max}, -y_{min}] \subset L^- \end{aligned} \tag{8}$$

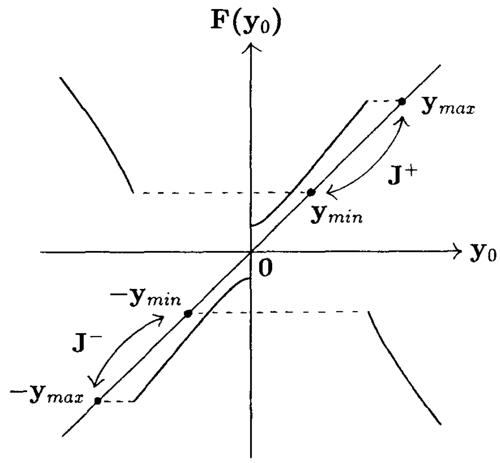

Fig. 8 Invariant intervals.

In the case of $F(y_{max}) < -y_{max}$, the solution diverges to infinity. For these parameter values the energy supplied from linear negative resistor is considered to be larger than that as hysteresis loss.

In the case of $F(y_{max}) > -y_{max}$, interval $J^+ \cup J^-$ is invariant with respect to the mapping F and the solution starting from $J^+ \cup J^-$ make an attractor on $J^+ \cup J^-$. The gradient of F on $J^+ \cup J^-$ determins the stability of the attractor.

The differential coefficient of F is shown in Fig. 9.

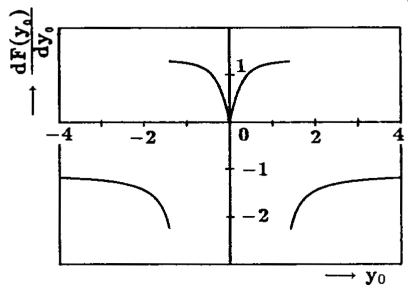

Fig. 9 Differential coefficient of F
($\alpha = 4$, $\beta = 1.3$, $a = 0.12$).

Then, we assume that the following condition is satisfied.

[Condition 1]

For all parameter values the following equation is satisfied for almost all y_0.

$$\frac{d^2F(y_0)}{dy_0^2} > 0. \qquad (9)$$

In this case the following theorem is satisfied.

[Theorem 1]

If the Poincaré map F satisfies the following conditions, F possesses the unique absolutely continuous invariant measure and chaos is generated.

$$\left.\frac{dF(y_0)}{dy_0}\right|_{y_0 = y_{min}} > 1, \quad \text{and} \qquad (10)$$

$$\left.\frac{dF(y_0)}{dy_0}\right|_{y_0 = y_{max}} < -1. \qquad (11)$$

The chaos generating region is shown in Fig. 10. In this figure the boundary between Divergence region and the region under Divergence region represents the parameter values satisfying $F(y_{max}) = -y_{max}$. The boundary between Chaos region and the region located on the left side of Chaos region represents the parameter values satisfying that the left side of Eq. (10) equals to one. As far as we carry out computer simulations, Eq. (11) is always satisfied in Chaos region. When α or β is large, that is, the hysteresis loss is large, chaos is generated for relatively large parameter region.

CONCLUSIONS

In this article, an extremely simple LCR oscillator with a hysteresis inductor have been analyzed. Though this circuit model is two-dimensional autonomous system, it generates chaos. Since this circuit is described by second order differential equation piecewisely, the Poincaré map can be derived as one-dimensional map strictly. For this Poincaré map the parameter region for which chaotic attractors are generated under simple assumption. Our future research is to carry out circuit experiments to confirm the theoretical results.

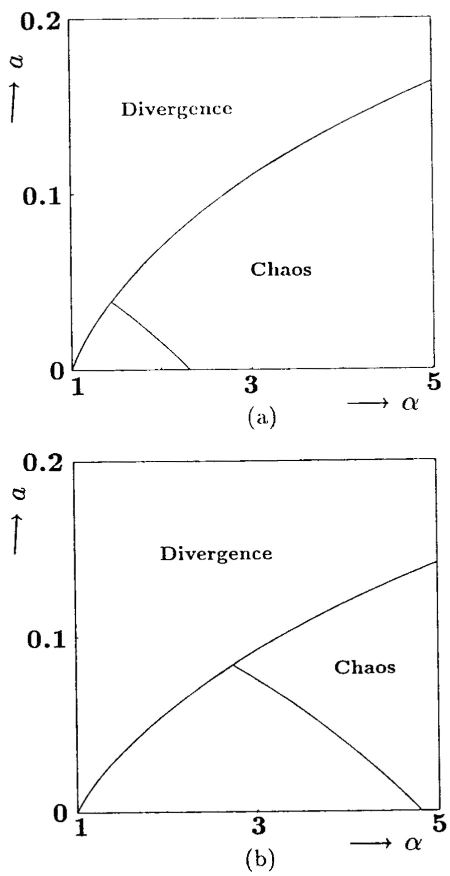

Fig. 10 Chaos generating region
(a) $\beta = 1.3$, (b) $\beta = 1.2$.

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