CHAOTIC PHENOMENA IN NONLINEAR CIRCUITS WITH TIME-VARYING RESISTORS

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Abstract: In this article, four simple nonlinear circuits with time-varying resistors are analyzed. These circuits are a kind of parametric excitation circuits and chaotic attractors are observed in three circuits. In order to analyze these circuits a degeneration technique is used, that is, diodes in the circuits are assumed to operate as ideal switches. Thereby the Poincaré maps are derived as one-dimensional maps and chaotic phenomena are well explained. Moreover, justifiability of the analyzing method is confirmed by circuit experiments.

INTRODUCTION

Recently, many chaos-generating systems have been proposed and analyzed in various fields. For parametric excitation circuits, the Duffing-Mathieu equation have been confirmed to generate chaotic attractors by computer calculations [1]. However, there are few discussions about the chaos in parametric excitation systems.

In this article, simple nonlinear circuits with time-varying resistors are analyzed. All these circuits consist of two memory elements, a diode and a time-varying resistor. These circuits are a kind of parametric excitation circuits and chaotic attractors are observed in three circuits. In order to analyze these circuits in detail, we use a degeneration technique, that is, we consider the case that the diodes in the circuits are assumed to operate as ideal switches. In this case, the Poincaré maps can be derived as one-dimensional maps and chaotic phenomena are well explained. This analyzing method have been proposed by Inaba et al.[2] and have been confirmed to be extremely effective to analyze some circuits including a diode. Moreover, the justifiability of the analyzing method is confirmed by circuit experiments.

CIRCUIT MODEL

Consider the circuits satisfying the following conditions.

1. The circuit consists of two memory elements, a diode, and a Time-Varying Resistor (TVR).
2. The circuit is governed by a two-dimensional nonlinear differential equation.
3. Both of the diode and the TVR are connected in parallel with a capacitor or in series with a inductor.

There are four circuits satisfying the above conditions and they are shown in Fig. 1. In the figures TVRs represent Time-Varying Resistors. The resistance $R_i(t)$ and the conductance $G_i(t)$ of TVRs vary with time as shown in Fig. 2. The function representing the variation of the TVRs is square wave with angular frequency ω_{ti} and duty ratio p_i.

At first, we approximate the $v - i$ characteristics of the diodes by the following two-segment piecewise linear functions (see Fig. 3).

$$\begin{aligned} i_{di}(v_i) &= \frac{G_{di}}{2}(v_i - V_i + |v_i - V_i|) \qquad (i=1,2) \\ v_{di}(i_i) &= \frac{R_{di}}{2}\left(i_i + \frac{V_i}{R_{di}} - \left|i_i - \frac{V_i}{R_{di}}\right|\right) \quad (i=3,4). \end{aligned} \tag{1}$$

By changing the variables,

$$v_i = V_i x_i, \quad i_i = \sqrt{\frac{C_i}{L_i}}V_i y_i, \quad t = \sqrt{L_i C_i}\tau, \quad \omega_{ti} = \frac{\omega_i}{\sqrt{L_i C_i}},$$

$$" \cdot " = \frac{d}{d\tau}, \quad \frac{1}{\varepsilon_i} = \begin{cases} G_{di}\sqrt{\dfrac{L_i}{C_i}} & (i=1,2) \\ R_{di}\sqrt{\dfrac{C_i}{L_i}} & (i=3,4), \end{cases}$$

$$a_i = \begin{cases} r_{i+}\sqrt{\dfrac{C_i}{L_i}} & (i=1,4) \\ g_{i+}\sqrt{\dfrac{L_i}{C_i}} & (i=2,3), \end{cases} \qquad b_i = \begin{cases} r_{i-}\sqrt{\dfrac{C_i}{L_i}} & (i=1,4) \\ g_{i-}\sqrt{\dfrac{L_i}{C_i}} & (i=2,3). \end{cases} \tag{2}$$

the normalized circuit equations are given as follows.

I. Circuit 1.

$$\begin{cases} \dot{x}_1 = y_1 - D_1(x_1) \\ \dot{y}_1 = -x_1 - y_1 f_1(\tau) \end{cases} \tag{3}$$

II. Circuit 2.

$$\begin{cases} \dot{x}_2 = -x_2 f_2(\tau) - y_2 - D_2(x_2) \\ \dot{y}_2 = x_2 \end{cases} \tag{4}$$

III. Circuit 3.

$$\begin{cases} \dot{x}_3 = -x_3 f_3(\tau) - y_3 \\ \dot{y}_3 = x_3 - D_3(y_3) \end{cases} \tag{5}$$

IV. Circuit 4.

$$\begin{cases} \dot{x}_4 = y_4 \\ \dot{y}_4 = -x_4 - y_4 f_4(\tau) - D_4(y_4) \end{cases} \tag{6}$$

where $f_i(\tau)$ corresponds to the function switching the TVRs and is shown in Fig. 4. The function $D_i(\cdot)$ corresponds to the

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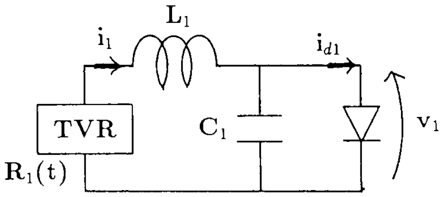
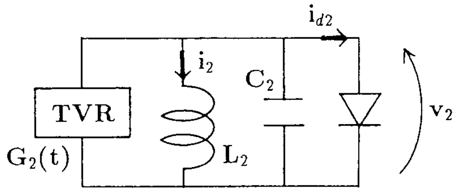
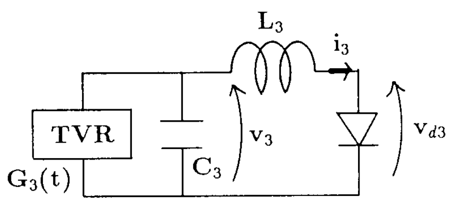


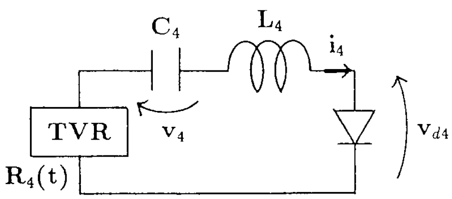
(a) Circuit 1.



(b) Circuit 2.



(c) Circuit 3.



(d) Circuit 4.

Fig. 1 Circuit model (TVR is Time-Varying Resistor).

characteristics of the diodes and is represented as follows.

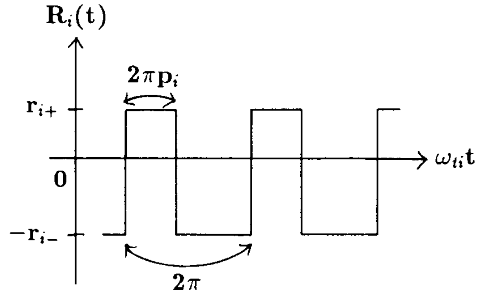
$$D_i(x_i) = \frac{1}{\varepsilon_i}\left(\frac{x_i - 1 + |x_i - 1|}{2}\right) \quad (i = 1, 2),$$
$$D_i(y_i) = \frac{1}{\varepsilon_i}\left(\frac{y_i + \varepsilon_i - |y_i - \varepsilon_i|}{2}\right) \quad (i = 3, 4). \tag{7}$$

Define the following subspaces.

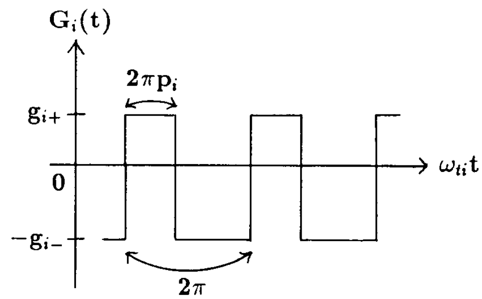
$$N_i \equiv \begin{cases} \{(x_i, y_i) | x_i < 1\} & (i = 1, 2) \\ \{(x_i, y_i) | y_i > \varepsilon_i\} & (i = 3, 4), \end{cases}$$

$$M_i \equiv \begin{cases} \{(x_i, y_i) | x_i \geq 1\} & (i = 1, 2) \\ \{(x_i, y_i) | y_i \leq \varepsilon_i\} & (i = 3, 4). \end{cases} \tag{8}$$

Because Eqs. (3)~(6) are piecewise linear and $f_i(\tau)$ takes only two constant values, the general solutions can be given.

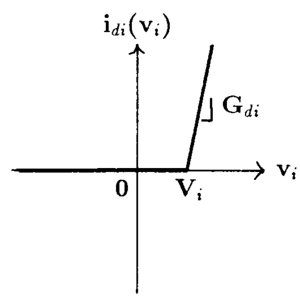


(a) i=1, 4.

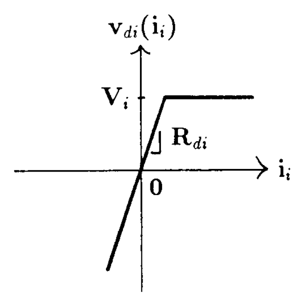


(b) i=2, 3.

Fig. 2 Characteristics of the TVRs.



(a) i=1, 2.



(b) i=3, 4.

Fig. 3 Characteristics of the diodes.

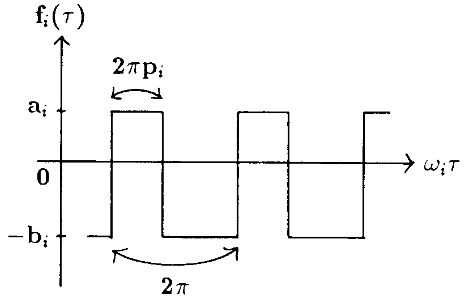
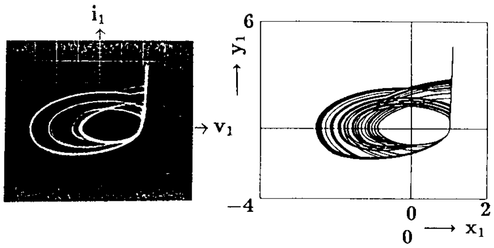


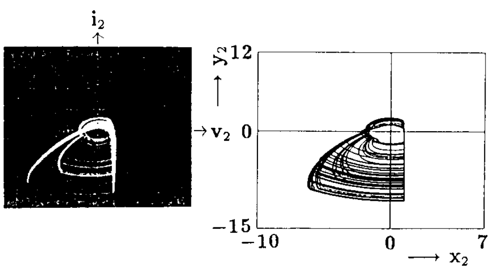
Fig. 4 $f_i(\tau)$ associated with $R_i(t)$ and $G_i(t)$.

(1)



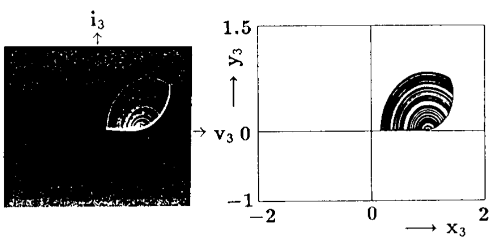
(a) ($a_1 = 0.04$, $b_1 = 0.58$, $\omega_1 = 1.4$, $p_1 = 0.5$)
(b) ($a_1 = 0.11$, $b_1 = 0.60$, $\omega_1 = 1.4$, $p_1 = 0.5$, $\varepsilon_1 = 0.02$)

(2)



(a) ($a_2 = 0.05$, $b_2 = 1.30$, $\omega_2 = 1.5$, $p_2 = 0.5$)
(b) ($a_2 = 0.10$, $b_2 = 1.30$, $\omega_2 = 1.5$, $p_2 = 0.5$, $\varepsilon_2 = 0.01$)

(3)



(a) ($a_3 = 0.01$, $b_3 = 0.48$, $\omega_3 = 1.25$, $p_3 = 0.5$)
(b) ($a_3 = 0.01$, $b_3 = 0.35$, $\omega_3 = 1.25$, $p_3 = 0.5$, $\varepsilon_3 = 0.02$)

(a) (b)

Fig. 5 Chaotic attractors obtained from (1) Circuit 1, (2) Circuit 2, (3) Circuit 3.
(a) Experimental results. (b) Computer calculated results.

The experimental and computer calculated results are shown in Figs. 5(a) and (b), respectively. In circuit experiments, TVR is realized by using an analog switch. We found chaotic attractors from the circuits 1∼3. In the circuit 4, we cannot found chaotic attractors and for any parameter values the solution converges to the origin or diverges. This reason is explained as follows. Because the diode in the circuit 4 is connected in series with the TVR, the stability of the origin is decided by the sum of resistance of the diode when it is off-state and resistance of the TVR. The resistance of the diode when it is off-state is relatively large. Hence, the negative resistance of the TVR is needed to be larger value in order to generate the oscillation, that is, $R_{d4} < r_{4-}$. However, if the oscillation amplitude becomes larger and the diode turns on, the solution diverges immediately because the resistance of the diode when it is on-state is relatively small.

IDEALIZATION OF DIODES

In the following, we analyze the circuits 1∼3 by using a degeneration technique, that is, we consider the case that the diodes in the circuits are assumed to operate as ideal switches. This degeneration technique have been proposed by Inaba et al. and have been verified to be effective to analyze chaos in some circuits including a diode.

Idealized diodes operate as follows.

1. The diode takes either ON-state or OFF-state.
2. The diode at ON-state operates as a constant voltage source with voltage V_i and the diode at OFF-state operates as open.
3. The diode turns off when the current through it becomes zero and turns on when the voltage across it reaches V_i.

This idealization corresponds to the limit $(G_{di}, R_{di}) \to (\infty, \infty)$ in Fig. 3.

In the case that the above idealization technique is used, the circuit equations in the regions M_i are degenerated as follows.

I. Circuit 1.

$$\begin{cases} \dot{x}_1 = 1 \\ \dot{y}_1 = -1 - y_1 f_1(\tau) \end{cases} \tag{9}$$

II. Circuit 2.

$$\begin{cases} \dot{x}_2 = 1 \\ \dot{y}_2 = 1 \end{cases} \tag{10}$$

III. Circuit 3.

$$\begin{cases} \dot{x}_3 = -x_3 f_3(\tau) \\ \dot{y}_3 = 0 \end{cases} \tag{11}$$

The transitional conditions to the regions M_i from N_i are given as follows.

I. Circuit 1.

$$y_1 = 0 \tag{12}$$

II. Circuit 2.

$$y_2 + f_2(\tau) = 0 \tag{13}$$

III. Circuit 3.

$$x_3 = 1 \qquad (14)$$

These conditions are derived from the relation $i_{di}(v_i) = 0$ (i=1, 2) or $v_{di}(i_i) = V_i$ (i=3).

In the case that this idealization method is used, the circuit equations are degenerated piecewisely and one-dimensional Poincaré maps can be derived strictly. We explain the derivation of the Poincaré map corresponding each circuit model.

1. In the circuit 1, when the solution enters the region N_1 from M_1, namely when the diode turns off, (x_1, y_1)=(1, 0) is satisfied. Hence, only the phase of the square wave switching the TVR decides the following motion of the solution. Then the Poincaré map can be derived as one-dimensional map which transforms the n-th phase ϕ_{1n} into the $(n+1)$-th phase $\phi_{1(n+1)}$ as shown in Fig. 6.

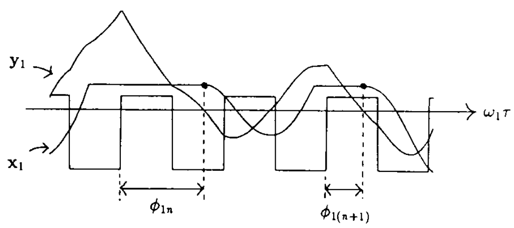


Fig. 6 Derivation of the Poincaré map.

2. In the circuit 2, the solution in the region M_2 must go through the point (x, y)=$(1, -a_2)$. Hence, when the solution is on the point $(1, -a_2)$, only the phase of the square wave decides the following motion of the solution. Therefore, the Poincaré map can be derived as one-dimensional map.

3. In the circuit 3, when the solution enters the region N_3 from M_3, namely when the diode turns on, (x_3, y_3)=(1, 0) is satisfied. Hence, only the phase of the square wave decides the following motion of the solution. Therefore, the Poincaré map can be derived as one-dimensional map.

An example of the Poincaré maps obtained from the circuit 1 is shown in Fig. 7. Though we omit the figures of those from the circuits 2 and 3, their Poincaré maps have many discontinuity and the orbits seem to be more complicated. One-parameter bifurcation diagram is shown in Fig. 8. We can see that the chaotic states and the periodic states appear in a complicated manner. Moreover, for $\omega \approx 3$ the Poincaré map is homeomorphic and quasi-periodic attractors are also generated.

CONCLUSIONS

In this article, we have analyzed four simple circuits with time-varying resistors. By using a degeneration technique we derived the one-dimensional Poincaré maps and confirmed that three circuits model generate chaos. Moreover we carried out circuit experiments and confirmed the justifiability of the analyzing results.

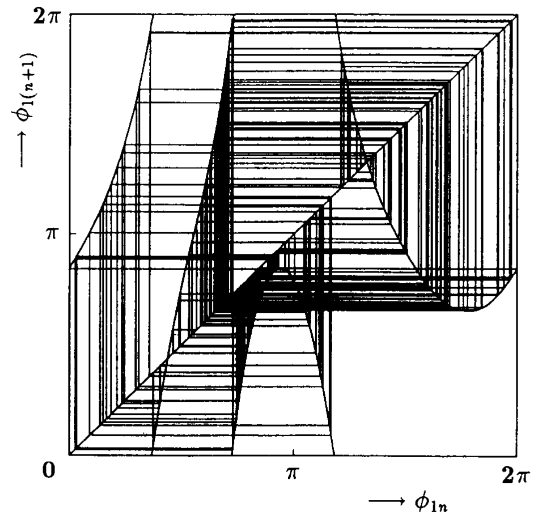


Fig. 7 Poincaré map (circuit 1).
($a_1 = 0.11$, $b_1 = 0.6$, $\omega_1 = 1.4$, $p_1 = 0.5$)

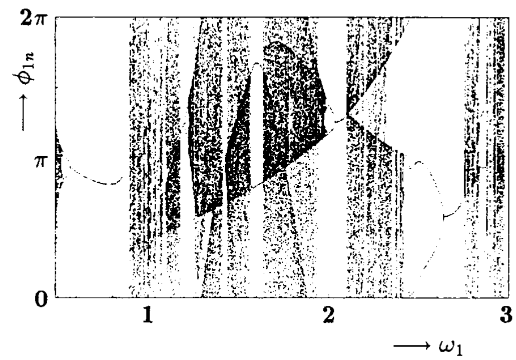


Fig. 8 1-parameter bifurcation diagram (circuit 1).
($a_1 = 0.11$, $b_1 = 0.6$, $p_1 = 0.5$)

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