Chaotic Phenomena in a Switched Capacitor Phase-Locked Loop

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ABSTRACT: Chaos in Phase-Locked Loops have been studied over the years[1], [2]. In those papaers, the existence of horseshoe chaos has been proved via the Melnikov method or the perturbation methods based on the idea of Melnikov. In this papaer, we discuss the chaotic phenomena in a Switiched-Capacitpor Phase-Locked Loop proposed by D. Asta et al. in [3]. We prove the existence of the logistic chaos by the one-dimensional map. And it has turned out to be cleared that in the map, there exist parameter volues where attractors coexist. Moreover 2-parameter space in which the chaotic phenomena occur is calculated. The analytical estimate is verified by circuit experiments.

INTRODUCTION

A Switched-Capacitor Phase-Locked Loop proposed by D. Asta et al. exhibits a fundamental behavior inherently diffrent from any previosly studied. The diffrence between this system and conventional PLL's is due to its structure as both continuous- and discrete-time conponents. And this loop is characterized by both nonuniform sampling and a sinusoidal nonlinearity. In this papaer, we will show that a Switched-Capacitor Phase-Locked Loop can exhibit chaotic behavior. Though this system allow for a filter of arbitrary order, we ristric our discussion only a first-order loop. The reason for above limitation is that a rigorous analysis of the chaotic phenomena can be achived by discribing the governing equation as a one-dimensinal map.

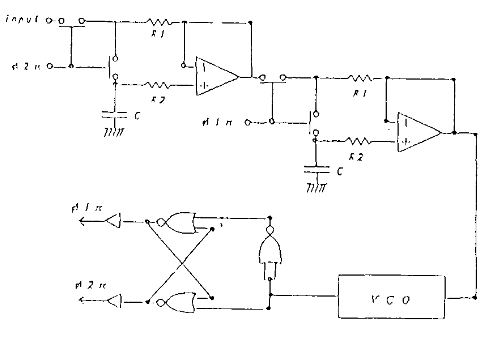

Fig. 1 Block diagram of the switched capacitor PLL

CIRCUIT MODEL

Figure 1 shows the block diagram of a Switched Capacitor Phase-Locked Loop. This circuit contains two sample-hold circuits as a sampler and one VCO. The loop input is modeled here as a continuos-time sinusoid with amplitude A and frequency ω. The phase is an arbitrary time function, θ.

$$V_{in}(T) = A\sin(\omega_0 t + \theta(t)) \tag{1}$$

The loop operates by tracking the zero crossings of the input waveform. The output of sampler is to be a sinusoidal function of the phase diffrence between the input waveform and waveform controlling the sampler. The period of VCO, $T_k = t_k - t_{k-1}$, is controlled by the output $u(t_k)$ of the sampler as follows:

$$T_k = \frac{2\pi}{\omega_0}\left(1 - \frac{\beta u(t_k)}{\omega + \beta u(t_k)}\right) \tag{2}$$

where ω_0 is VCO's free-running frequency

β is the VCO characteristic

The phase error is denoted by the diffrence between the positive-going zero crossing of the input signal and the rising edge of the VCO output(see Fig. 2), as follows.

$$\phi(t_k) = -2\pi\sum_{j=1}^{k}\frac{\beta u(t_{j-1})}{\omega_0 + \beta u(t_{j-1})} + \theta(t_k) \tag{3}$$

The scalor first-order equation describing the system can be written

$$\begin{aligned} \phi(k) &= \phi(k-1) + 2\pi\Delta \\ &- 2\pi(1+\Delta)\frac{\sin(\phi(k-1))}{\mu + \sin(\phi(k-1))} \\ & \quad where \quad \phi(k) = \phi(t_k) \\ & \quad \mu = \frac{\omega_0}{A\beta} \end{aligned} \tag{4}$$

The above equation exhibit a circular symmetry in $\phi(k)$. We can therefore ristrict $\phi(k)$ to $[-\pi, \pi]$ where we identify $-\pi$ and π.

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Analysis of the one-dimensional map

Denote the mapping in (5) as $\phi_\pi(n+1) = F(\phi_\pi(n))$, which transform the n-th phase error $\phi_\pi(n)$ into the $(n+1)$-th phase error $\phi_\pi(n+1)$.

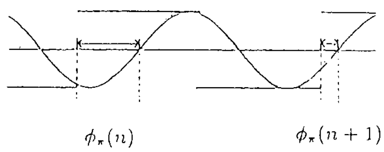


Fig. 2 Derivation of the one-dimensional map

An example of the one-dimensional map is shown in Fig. 3. For some parameter volues this map has two extram and chaotic solution is considered to appear.

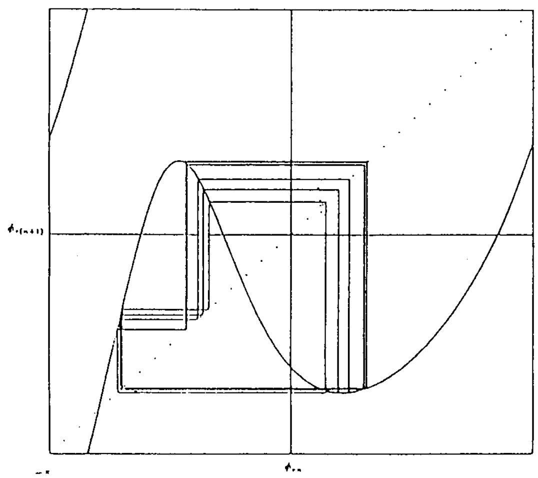


Fig. 3 One-dimensional map

Fig.4 and Fig.5 are examples of 1-parameter bifurcation diagram. The initial value are started from the right/left extram in Fig. 4/5, respectively. Period-doubling route chaos, periodic windows and crisis, are observed in these figures. The diffrence between Fig.4 and Fig.5 exists from the fact that there is within the bounds of possibility of coexisting diffrent type of attractors when there exist two extram in a one-dimensional map. Phisically, this means that for the parameter rigion where 1-periodic solution and n-periodic solution coexist, initial phase error decides the succesion of the locking state in the circuit. In logistic map there never exists a stable 1-periodic solution after chaotic phenomena have occured, while in the one-dimensional map which represents the circuit is not the same case. That is, even after crisis one-periodic solutions appear. We show this fact in Fig.6.

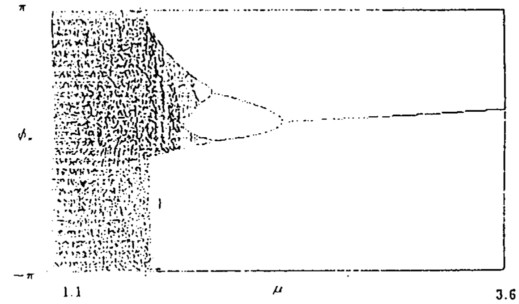


Fig. 4 1-parameter bifurcation diagram starting from the right extram ($\Delta = 0$)

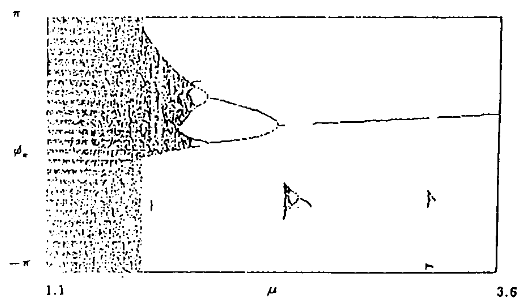


Fig. 5 1-parameter bifurcation diagram starting from the left extram ($\Delta = 0$)

The mechanism of the emergence of 1-periodic solution after crisis is as follows. With parameter μ decreases, the left extram increases and intersects 45°-line infinitely many times in the parameter space(see Fig. 7).

To determin the relation between the real system and the one-dimensional map, that is, n-priodic solution in the one-dimensional map corresponds to what periodic state in the real system, we should calculate rotation numbers. But the map usually don't satisfy homeomorphism, so it is no use evaluting rotation numbers. Instead we executed circuit simulations, and the simulations led us the fact that n-periodic solution in the one-dimensional map corresponds to n/m-periodic state in the system, where the ratio n/m denotes the output of VCO oscillation times divided by the input signal oscillation times. We prsent 2-parameter diagram and boundries of the basion of attraction of several periodic solution associated the switched-capacitor phase-looked loop circuit, moreover we will show chaotic regions in the parameter space. With this diagram, we confirm routes to chaos. Now, let denote the Liapunov exponents λ as equation (5).

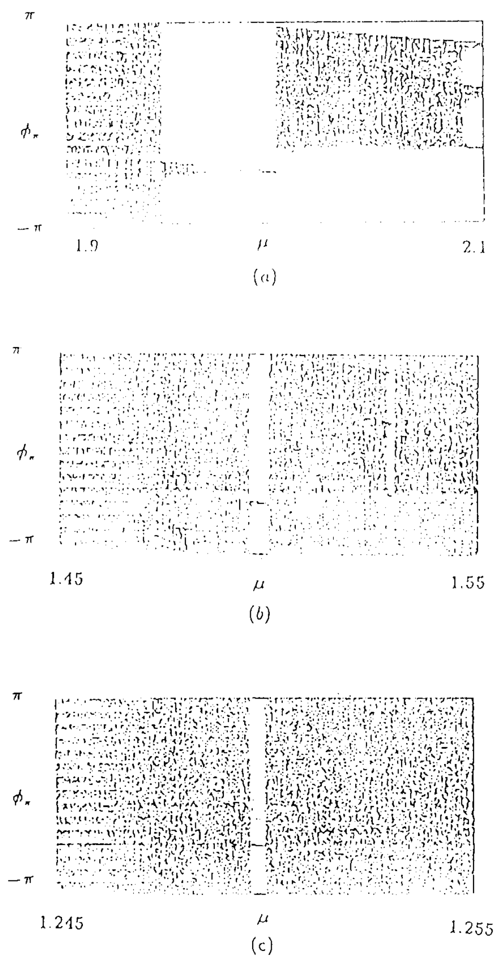


Fig. 6 1-parameter bifurcation diagrams ($\Delta = 0$)

(a) μ; 1.9 ~ 2.1
(b) μ; 1.45 ~ 1.55
(c) μ; 1.245 ~ 1.255

$$\lambda = \lim_{n \to \infty} \frac{1}{n \log 2} \log \mid DF^n(x_0) \mid \qquad (5)$$

The Liapunov exponents formalizes the concept of stretching rates and it represent the figures that distiguish between periodic-orbits ($\lambda < 0$), quasi-periodic orbit ($\lambda = 0$) and chaos ($\lambda > 0$). So, in this diagram we evoluated chaotic phenomena with the Liapunov numbers. Fig. 8 is an example of 2-parameter bifurcation diagram. In this figure, period-n-orbits are in the shape of "Arnold's tongue", and we confirmed that infinitly many "Arnold's tongue" exist in the parameter space.

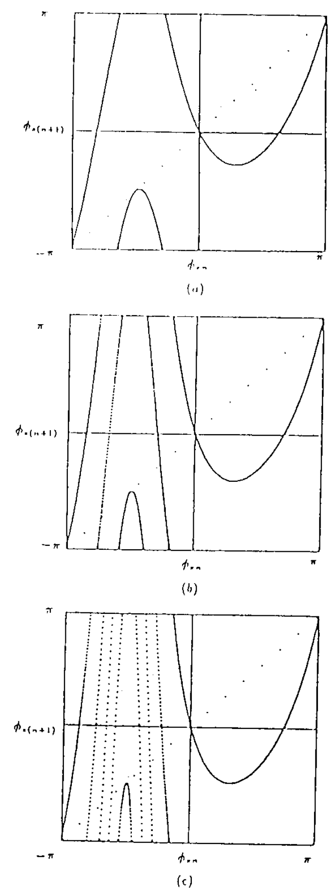


Fig. 7 One-dimensional maps
(a) $\mu = 2.0$ (b) $\mu = 1.5$ (c) $\mu = 1.25$

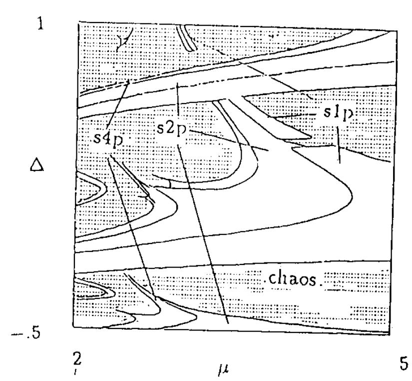
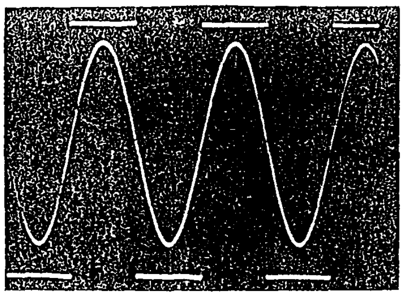


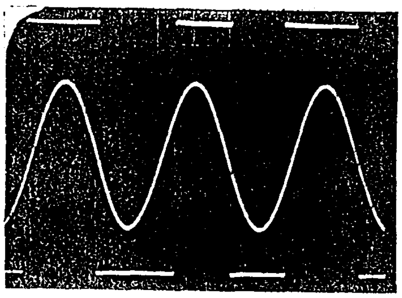
Fig. 8 2-parameter bifurcation diagram starting from the right extram snp;stable n-periodic solution

CIRCUIT EXPERIMENTS

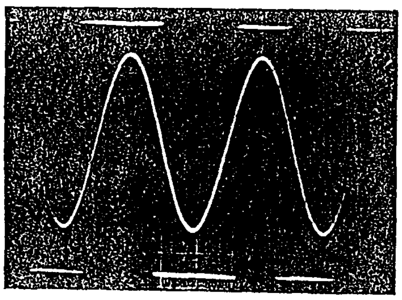
We confirmed the existence of logistic chaos and crisis by circuit experiments. Fig. 9 show experimental results. In these figures, parameter Δ was fixed to 0 ,that is, input signal frequency coinsides with PLL's free running frequency, and the parameter μ was changed with changing the input signal ampulitude A.



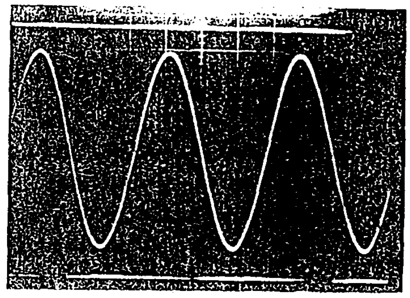
(a)



(b)



(c)



(d)

Fig. 9 Experimental results ($\Delta = 0$)

(a)$\mu = 4$;	stable one periodic solution
(b)$\mu = 2.8$;	stable two periodic solution
(c)$\mu = 2.5$;	chaos before crisis
(d)$\mu = 1.9$;	chaos after crisis

CONCLUTION

In this article, we have analyzed chaotic phenomena in a switched-capacitor phase-locked loop by the one-dimensional map. With analyzing the map, the existence of logistic chaos, coexistence of attractors, one-periodic solution after crisis and periodic solution in the shape of "Alnold's tongue" in the 2-parameter space were cleared. Moreover experimental results justified the analytical ones.

REFERENCES

[1] T. Endo and L. O. Chua, "Chaos from Phase-Locked Loop," IEEE Trans. Circuits Syst., Vol. CAS-35, No. 8, pp. 987-1003, August 1988.

[2] Y.-H. Chu, J.-H. Chou and S. Chang, "Chaos from Third-Order Phase-Locked Loops with a Slowly Varying Parameter," IEEE Trans. on Circuits Syst., Vol. CAS-37, No. 9, pp. 1104-1115, September 1990.

[3] D. Asta and D. Green, "Analysis of a Hybrid Analog/Switched-Capacitor Phase-Locked Loop," IEEE Trans. Circuits Syst., Vol. CAS-37, No. 2, pp. 183-197, February 1990.

[4] P. Collet and J.-P. Eckmann, *Iterated Maps on the Interval as Dynamical Systems*, Boston, MA: Birkhäuser, 1980.